Sixth International Conference on Analysis and Applied Mathematics

ABSTRACT BOOK

of the conference ICAAM 2022

Edited by

Charyyar Ashyralyyev,

Abdullah S. Erdogan,

Mahmud A. Sadybekov

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We, the participants of Sixth International Conference on Analysis and Applied Mathematics (ICAAM 2022), all are very blessed to meet in-person after the pandemic and this abstract book is the valuable outcome of this gathering. As organizers, we are also fortunate because we received a very high number of abstracts submitted.

ICAAM 2022 is the continuation of our biannual conference that has been held in various locations in Turkey and Kazakhstan. The conference aims to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. As the knowledge of different branches of mathematics open new perspectives, it is important to learn more about the developments and advancements in the field of applied mathematics and analysis. As organizers we are proud to see that ICAAM provides a forum for researchers and scientists to share their recent developments and to present their original results in various fields of mathematics.

We welcome you to Antalya, Turkey and look forward to seeing you in upcoming conferences.

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Conference Sections and Minisymposiums :

- General Section
- Minisymposium MS1: Functional Analysis in Interdisciplinary Applications
- Minisymposium MS2: Fractional Chaotic Systems: Singular and Non-Singular Kernels

Invited Speakers

- Prof. Eberhard Malkowsky, State University of Novi Pazar, Serbia
  Title: Some Classes of Operators Between Certain BK Spaces
- Prof. Arsen Pskhu, Institute of Applied Mathematics and Automation KBSC RAS, Russian Federation
  Title: Boundary Value Problems for Fractional PDEs with the Liouville Fractional Derivatives
- Prof. Galina Kurina, Voronezh State University, Russian Federation
  Title: Singularly Perturbed Problems with Multi-Tempo Fast Variables
- Prof. Krassimira Vlachkova, Sofia University, Bulgaria
  Title: Interpolation of Scattered Data in R^3 Using Smooth Curve Networks
- Prof. Fadi Awawdeh, The Hashemite University, Jordan
  Title: Deferred Correction Methods for Solving IVPs

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FOREWORD

On behalf of the Organizing Committee of ICAAM, we are pleased to invite you to the Sixth International Conference on Analysis and Applied Mathematics, ICAAM 2022. The meeting will be held on October 31 - November 6, 2022 in Antalya, Turkey. The conference will consist of plenary lectures, minisymposiums and contributed oral presentations.

The conference is organized biannually. Previous conferences were held in Gumushane, Turkey (2012), Shymkent, Kazakhstan (2014), Almaty, Kazakhstan (2016), Lefkoşa (Nicosia), Turkeyc(2018), and Girne (Kyrenia), Turkey (2020).

The proceedings of all ICAAM conferences (ICAAM 2012-2020) were published in AIP (American Institute of Physics) Conference Proceedings. The proceedings of ICAAM 2022 will also be published in AIP Conference Proceedings. Also, selected full papers of this conference will be published in peer-reviewed journals.

We would like to thank our main sponsors Bahcesehir University, Turkey, Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Ghent Analysis & PDE Center, Belgium. We also would like to thank to all participants, invited speakers, Co-Chairs, Coordinating Committee, International Organizing Committee, International Organizing Committee, and Technical Program Committee Members.

With our best wishes and warm regards,

Prof. Allaberen Ashyralyev
Prof. Michael Ruzhansky
Prof. Makhmud Sadybekov

Chairs of ICAAM 2022

Содержание

GENERAL SECTION

Statistical models in the analysis of active fire-examples from south Asia

Jyoti U. Devkota[1]

[1] *Department of Mathematics, Kathmandu University, Nepal*
drjdevkota@ku.edu.np

Abstract: Active fires destroy the flora and fauna of a place. They also emit aerosols and green house gases. In this paper the behavior of active fires over a period of one year in Nepal, Bhutan and Sri Lanka is studied using spatial statistics. In these countries, these fires are mainly forest and vegetation fires. This study is based on data acquired through remote sensing data acquisition platform, NASA's MODIS. It is found that Bhutan has minimum incidences of such fires in contrast to Nepal, that has the highest incidences of such fire amongst the three countries. But these fires are of very high intensity in Bhutan. The distribution of intensity of such fires is symmetrical in Sri Lanka. The behavior of two variables namely Brightness and Fire radiative power is minutely analysed here. Spatial statistics is used here to study the incidence of such fires with respect to geographical location. The behavior of parameters of various autoregressive spatial models like Spatial Durban Model, Spatial Lag Model, Spatial Error Model, Manski Model and Kelegian Prucha Model are minutely analyzed. The best model with highest pseudo R^2 is selected. Such studies hold great significance, as important information on impact of forest fires can be indirectly assessed. This can be of a special significance for countries with limited and scarce data [1].

Keywords: Variogram, auto correlation, spatial correlation

2010 Mathematics Subject Classification: 62Jxx, 62Pxx, 62Hxx]62-08 medskip

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Some properties of Newton transformations and applications

Mohammed Abdelmalek[1]

[1] *Higher School of Management of Tlemcen, Algeria*

abdelmalekmhd@yahoo.fr

Abstract: Given M^{n+1} an $(n+1)-$dimensional Riemannian manifold and let M^n be an oriented hypersurface of M^{n+1} with regular boundary $\partial M \subset P^n$, where P^n is a totally geodesic hypersurface of M^{n+1}.

In this work we give some properties of the Newton transformations T_r. Wegive also some applications of these opeators. In particular we prove that the hypersurfaces M^n and P^n are transverse along the boundary ∂M if for some $1 \leq r \leq n$, the Newton transformation T_r is positif defined.

2010 Mathematics Subject Classification: 53A10, 53C42, 53C24.

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A description of ℓ^p–spaces symmetrically finitely represented in rearrangement invariant function and sequence spaces

Sergey Astashkin

Samara National Research University, Russia; Bahcesehir University, Turkey
astash56@mail.ru

Abstract: It is well known that there is an essential difference between *global* and *local* properties of Banach spaces, that is, between properties of their infinite-dimensional subspaces and subspaces of finite (though large) dimension. The report will be devoted to the problem of a description of local properties of rearrangement invariant function and sequence spaces.

Let X be a rearrangement invariant space on $(0, \infty)$, $1 \le p \le \infty$. We say that ℓ^p (c_0 if $p = \infty$) is *symmetrically finitely represented* in X if for every $n \in \mathbb{N}$ and each $\varepsilon > 0$ there exist equimeasurable functions $x_k \in X$, $k = 1, 2, \dots, n$, such that $\operatorname{supp} x_i \cap \operatorname{supp} x_j = \varnothing$, $i \ne j$, and for any $a = (a_k)_{k=1}^n$

$$(1) \qquad (1+\varepsilon)^{-1}\|a\|_p \le \Big\| \sum_{k=1}^n a_k x_k \Big\|_X \le (1+\varepsilon)\|a\|_p.$$

In [1], for a separable rearrangement invariant space X on $(0, \infty)$ of fundamental type we identify the set of all $p \in [1, \infty]$ such that ℓ^p is symmetrically finitely represented in X. This characterization hinges upon a description of the set of approximate eigenvalues of the doubling operator $x(t) \mapsto x(t/2)$ in X. We prove that this set is surprisingly simple: depending on the values of some dilation indices of such a space, it is either an interval or a union of two intervals.

Similar problems were considered in [2] for rearrangement invariant function spaces on $[0, 1]$ and in [3] for rearrangement invariant sequence spaces. We apply these results to the Lorentz and Orlicz spaces.

Keywords: ℓ^p, finite representability, Banach lattice, rearrangement invariant space, dilation operator, shift operator, approximate eigenvalue, Boyd indices, Orlicz space, Lorentz space

2010 Mathematics Subject Classification: 46B70, 46B42

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The order of convergence of difference schemes approximating fractional differential equations

Sergey Piskarev

Lomonosov Moscow State University, Russia

piskarev@gmail.com

Abstract: This report is dedicated to approximations of the abstract Cauchy problem:

(1) $$D^{\alpha}u(t) = Au(t),\ 0 < \alpha < 1,$$

(2) $$u(0) = u^0.$$

where D^{α} is a Caputo derivative and A generates an exponentially bounded resolvent family.

The classical Banach–Steinhaus theorem states that stability and consistency on smooth elements implies strong convergence of approximating operators. However, firstly, this does not imply a rate of convergence, and secondly, in the case of fractional problems, often even establishing consistency is not a trivial task. When approximating fractional problems, many authors [1–4] did not indicate the rate of convergence of the proposed difference schemes, not to mention the speed of convergence to the problem (1)-(2), which, as is known, on a uniform grid depends on α.

We will discuss the real rates of convergence of difference schemes proposed by various authors for solving problem (1)–(2).

Keywords: difference schemes, order of convergence, fractional equations, stability, consistency

2010 Mathematics Subject Classification: 35K99, 65M12, 65N06

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Existence theorem for a weak solution of an initial-boundary value problem for a model of an inhomogeneous Kelvin-Voigt fluid of an arbitrary finite order

Victor Zvyagin[1,2], Mikhail Turbin[1,3]

[1] *Voronezh State University, Russian Federation*

[2] *zvg_vsu@mail.ru*
[3] *mrmike@mail.ru*

Abstract: We consider the following initial-boundary value problem:

$$\rho\frac{\partial v}{\partial t}+\rho\sum_{i=1}^{n}v_i\frac{\partial v}{\partial x_i}-\mu_1\Delta v-\mu_2\frac{\partial \Delta v}{\partial t}-\int_0^t h(t,s)\Delta v(s)ds+\nabla p=\rho f; \tag{1}$$

$$\frac{\partial \rho}{\partial t}+\operatorname{div}(\rho v)=0; \quad \operatorname{div} v=0. \tag{2}$$

$$v|_{t=0}(x)=a(x), \quad \rho|_{t=0}(x)=\rho_0(x), \quad x\in\Omega; \quad v|_{\partial\Omega}=0. \tag{3}$$

Here $(t,x)\in[0,T]\times\Omega$ where $\Omega\subset\mathbb{R}^N$ is a bounded domain with smooth boundary. $v(t,x), p(t,x), \rho(t,x), f(t,x)$ are the velocity, the pressure, the density of the fluid and the vector of external forces, respectively. $\mu_2>0$ is some constant, h is a function responsible for the memory in the fluid and representing the sum of the exponents with different exponents.. We suppose $0<m\leqslant\rho_0(x)\leqslant M$ where m, M are some constants.

The main result is the following theorem

Theorem 1.1. *There is at least one weak solution to initial-boundary value problem* (1)–(3)*.*

The proof is carried out on the basis of the approximation-topological approach to the study of fluid dynamic problems. Namely, we consider a problem with a small parameter that approximates the original one. For this new problem, using the Leray-Schauder theorem, we prove solvability and establish a priori estimates for solutions. Then the passage to the limit is carried out as the approximation parameter tends to zero, and thus the existence of weak solutions of the original initial-boundary value problem is proved.

Funding: The study was supported by the Russian Science Foundation (project 22-11-00103).

Keywords: Existence theorem, weak solution, initial-boundary value problem, inhomogeneous fluid

2010 Mathematics Subject Classification: 76A05, 35Q35

Pullback attractors for the Bingham model with periodical boundary conditions on spatial variables

Victor Zvyagin[1,2], Anastasiia Ustiuzhaninova[1,3]

[1] *Voronezh State University, Russia*

[2] *zvg_vsu@mail.ru*
[3] *nastyzhka@gmail.com*

Abstract: We consider the following problem for the Bingham model with periodical boundary conditions on spatial variables:

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial v}{\partial x_i} - \mathrm{Div}\,\sigma + \nabla p = f, \quad \mathrm{div}\, v = 0, \quad (x,t) \in \Omega \times (\tau; +\infty); \tag{1}$$

$$\sigma = \begin{cases} 2\mu\mathcal{E}(v) + \tau^* \dfrac{\mathcal{E}(v)}{|\mathcal{E}(v)|} & \text{for } |\mathcal{E}(v)| \neq 0, \quad (x,t) \in \Omega \times (\tau; +\infty); \\ |\sigma| \leqslant \tau^* & \text{for } |\mathcal{E}(v)| = 0, \quad (x,t) \in \Omega \times (\tau; +\infty). \end{cases} \tag{2}$$

$$v|_{t=\tau}(x) = a(x), \quad x \in \Omega. \tag{3}$$

Here $\Omega = \prod_{i=1}^{3}(0, l_i) \subset \mathbb{R}^3$; Div σ is the vector of column divergences of σ; $\mu > 0$ is the viscosity of a fluid, $\tau^* > 0$ is the constant describing the threshold of yield for a fluid; $\mathcal{E}(v) = \frac{1}{2}(\nabla v + (\nabla v)^T)$ is the strain rate tensor.

On the basis of the trajectory pullback attractors theory [1,2] the qualitative behavior of weak solutions for considered problem is investigated. The solvability of weak solutions on $\mathbb{R}_+$ is proved, a family of trajectory spaces is determined and the existence of minimal trajectory and minimal pullback attractors is proved.

Funding: The study was supported by the Russian Science Foundation (project 22-11-00103).

Keywords: Pullback attractor, trajectory space, Bingham model, weak solution, existence theorem

2010 Mathematics Subject Classification: 35Q35, 35B41, 76A05

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Backward and non-local problems for the Rayleigh-Stokes equation

Ravshan Ashurov[1]

[1] *V. I. Romanovskii Institute of Mathematics, Uzbekistan*

ashurovr@gmail.com

Abstract: The Fourier method is used to find conditions on the right-hand side and on the initial data in the Rayleigh-Stokes problem, which ensure the existence and uniqueness of the solution. Then, in the Rayleigh-Stokes problem, instead of the initial condition, consider the non-local condition: $u(x,T) = \beta u(x,0) + \varphi(x)$, where β is either zero or one. It is well known that if $\beta = 0$, then the corresponding problem, called the backward problem, is ill-posed in the sense of Hadamard, i.e. a small change in $u(x,T)$ leads to large changes in the initial data. Nevertheless, we will show that if we consider sufficiently smooth current information, then the solution exists and it is unique and stable. It will also be shown that if $\beta = 1$, then the corresponding non-local problem is well-posed and coercive type inequalities are valid.

Keywords: The Rayleigh-Stokes problem, the backward problem, non-local problem, the Fourier method.

2010 Mathematics Subject Classification: Primary 35R11; Secondary 34A12.

On the weak solution of third order of accuracy difference scheme for coupled sine-Gordon equations

Ozgur Yildirim[1], Burcu Batibay Yumurtaci[2]

[1] *Department of Mathematics, Yildiz Technical University, Istanbul, Turkey*
ozgury@yildiz.edu.tr

[2] *Department of Mathematics, Yildiz Technical University, Istanbul, Turkey*
burcu.batibay34@gmail.com

Abstract: In the present study, the nonlinear coupled system for sine-Gordon equations which describe the open states in DNA double helices is considered. The unconditionally stable third order of accuracy difference scheme is studied. The existence and uniqueness of weak solutions for the system of finite difference schemes for the coupled sine-Gordon equations are presented. Some energy estimates for the weak solvability of these equations are obtained.

The authors were supported in part by the Research Fund of the Yildiz Technical University. Project Number: FYL-2022-4894

Keywords: weak solutions, existence, uniqueness, energy estimates, difference equations

2010 Mathematics Subject Classification: 35D30, 35A01, 34A12, 35B45, 39A60

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Nonlinear Schrödinger Equation with Delay

V. Zh. Sakbaev

Keldysh Institute of Applied Mathematics, Moscow Institute of Physics and Technology, Moscow, Russian Federation

sakbaev.vzh@gmail.com

Joint work with A.D. Shyraeva

The properties of initial-boundary value problem for a nonlinear Schödinger equation including terms with delay of time argument in the nonlinear potential are studied. The conditions of global existence or arising of gradient blow up phenomenon are obtained. The relation of the gradient blow up phenomenon with the self-focusing and the destruction of pure quantum state are described. We consider the evolution of a quantum state generated by the solution of nonlinear Schrödinger equation as the curve in the set of pure quantum states. We introduce the regularization procedure which defines the continuation of the solution through the moment of blow up by a curve in the set of quantum states with transition into the set of mixed states.

A study on approximate solution of the two dimensional source identification telegraph problem with Neumann condition

Allaberen Ashyralyev[1,2,3], *Haitham Al-hazaimeh* [4] , *Charyyar Ashyralyyev*[1,5]

[1]Department of Mathematics, Bahcesehir University, 34353, Istanbul, Turkey
[2]Peoples Friendship University Russia, Ul Miklukho Maklaya 6, Moscow 117198, Russia
[3]Institute of Mathematics and Mathematical Modeling, 050010, Almaty, Kazakhstan
[4]Department of Mathematics, Near East University, Nicosia, TRNC, Mersin 10, Turkey
[5] National University of Uzbekistan, Tashkent, Uzbekistan
aallaberen@gmail.com, mathmatichhh@gmail.com, charyar@gmail.com

Abstract In this work, the source identification problem for the telegraph equation is investigated. We propose the first order of accuracy absolute stable difference scheme to find the approximation solution of the two dimensional identification problem for the telegraph equation with the Neumann boundary condition. Numerical results have been provided for the solution of the time-dependent source identification problems for the two dimensional telegraph differential equations with the Neumann boundary condition and compared with the exact solution for showing the efficiency of the proposed numerical method.

Keywords: Stability, time dependent, telegraph equation

[1] A. Ashyralyev, H. Al-Hazaimeh, H. A numerical algorithm for the source identification telegraph problem, *AIP Conference Proceedings,* 2334, No. 1, (060007) (2021) .

[2] A. Ashyralyev, A. Al-Hammouri, C. Ashyralyyev, On the absolute stable difference scheme for the space-wise dependent source identification problem for elliptic-telegraph equation, *Numerical Methods for Partial Differential Equations,* 37 962–986 (2021).

Mathematical modelling of traffic with regression analysis and machine learning

Ozgur Yildirim[1], Serdar Demir[2]

[1] *Department of Mathematics, Yildiz Technical University, Istanbul, Turkey ozgury@yildiz.edu.tr*
[2] *Department of Mathematics, Yildiz Technical University, Istanbul, Turkey and Department of Management Information Systems, Istinye University, Istanbul, Turkey*
demir.serdar@outlook.com.tr

Abstract: In this study, it is aimed to create a traffic model for a specific part of Istanbul. The main concern is to obtain a mathematical model that predicts the level of traffic jam and average speed of the cars where some parameters are given. In order to obtain that model, required traffic data in some specific locations are obtained from Istanbul Metropolitan Municipality traffic data sets and some other data sets are included to the data set for creating other parameters that may affect traffic jam. After collecting this data, statistical methods are used both manually and with the help of the machine learning algorithms created by ourselves. As a result, affecting factors on traffic jam is shared by correlation maps and a formula is created to predict the average speed and the level of traffic jam. Study results are compared with real world results.

This research was supported by Research Fund of Yıldız Technical University by grant No. FYL-2021-4399. This work is produced from MSc thesis of the second author.

Keywords: mathematical modelling, traffic modelling, machine learning, regression analysis, ridge regression.

2010 Mathematics Subject Classification: 62J07, 74S60, 97M10, 97R20, 97R40.

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Cauchy problem for nonlocal abstract Stokes equations and applications ***Veli Shakhmurov***

Antalya Bilim University Department of Industrial Engineering, Dosemealti, 07190 Antalya, Turkey, E-mail: veli.sahmurov@antalya.edu.tr

Azerbaijan State Economic University, Center of analytical-information resource 194 M. Mukhtarov AZ1001 Baku, veli.sahmurov@gmail.com

Abstract

In this talk, the Cauchy problem for the stationary and instationary nonlocal imcompressible abstract Stokes equations are considered. The equation involve the the convolution term and abstract operator in a Banach space E on lideang part. The exisistense and uniqueness in L^p spaces is derived. We can obtain a different classes Novier-Stokes equations by choosing the space E and the linear operator A which occur in a wide variety of physical systems. In application the existence, uniqueness and L^p estimates for solution of mixed problems for nonlocal degenerate Navier-Stokes equations and nonlocal Navier-Stokes equations with discontinuous coefficients are established. We consider the Cauchy problem for the nonlocal Stokes equation

(1) $$\frac{\partial u}{\partial t} - b * \Delta u + Au + \nabla\varphi = f\left(x,t\right), \; x \in \mathbb{R}^n, \; t \in \left(0,T\right),$$

(2) $$divu = 0, \; u\left(x,0\right) = a\left(x\right),$$

where A is a linear operator in a Banach space E, $u = \left(u_1\left(x,t\right), u_2\left(x,t\right), ..., u_n\left(x,t\right)\right)$ is an E-valued unknown solution $f = \left(f_1\left(x,t\right), f_2\left(x,t\right), ..., f_n\left(x,t\right)\right)$ is given and $a = \left(a_1\left(x\right), a_2\left(x\right), ..., a_n\left(x\right)\right)$ is a initial date. Moreover,

$$b = b\left(x\right) = \left(b_1\left(x\right), b_2\left(x\right), ...b_n\left(x\right)\right), \; b * u = \left(b_1 * u_1, b_2 * u_2, ..., b_n * u_n\right),$$

$b_i * u_i$ denotes the convolution of the functions b_i, u_i defined by

$$b_i * u_i =_{\mathbb{R}^n} b_i\left(x\right) u\left(x-\xi\right) d\xi$$

for smooth enough complex-valued functon b_i and E-valued functoun $u\left(x,t\right)$. Here, $\varphi = \varphi\left(x,t\right)$ is represent an E-valued unknown pressure. This problem is characterized with the presence of abstract operator A and the convolution term $b * \Delta u$. We obtain the well-posedeness of the problem $(1.1) - (1.2)$ in E-valued Bohner space.

Finite-time stability analysis of fractional time delay systems

Nazim Mahmudov, Gülbahar Akgün

Department of Mathematics
Eastern Mediterranean University
Famagusta, 99628, T. R. North Cyprus
via Mersin 10, Turkey
nazim.mahmudov@emu.edu.tr
gulbahar.akgun@emu.edu.tr

Abstract: The following nonhomogeneous systems of fractional differential equations with pure delay are considered:

$$\begin{cases} \left({}^{C}D_{0+}^{\alpha}y\right)(x) = Ay\,(x-\tau) + By\,(x) + f\,(x)\,, \quad \tau > 0, \quad x \in [0,T]\,, \\ y\,(x) = \psi\,(x)\,, \quad y'\,(x) = \psi'\,(x)\,, \quad -\tau \le t \le 0, \end{cases}$$

where ${}^{C}D_{0+}^{\alpha}$ is said to be the Caputo fractional derivative of order $1 \le \alpha \le 2$ with the lower index zero, $\tau > 0$ is a delay $\psi \in C^2\left([-\tau,0]\,,\mathbb{R}^n\right)$, $A, B \in \mathbb{R}^n,\ f \in C\left([0,\infty)\,,\mathbb{R}^n\right)$ is a given function. The representation of explicit solutions of these systems and the delayed Mittag-Leffler matrix functions are used to present the finite time stability results. Our results improve and extend the previous related results. Finally, to illustrate our theoretical results, we present some examples.

Throughout this note we mainly use techniques from the work [1].

Keywords: Mittag-Leffler function, finite time stability, fractional delay systems, fractional derivative.

2010 Mathematics Subject Classification: 34K20; 34K37; 34A08

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Stabilized finite element computations augmented with shock-capturing: 3D convection-diffusion equations

Suleyman Cengizci[1]

[1] *Computer Programming, Antalya Bilim University, Antalya, Turkey*

suleymancengizci@gmail.com

Abstract: We are interested in obtaining oscillation-free finite element solutions for three-dimensional convection-diffusion equations in the following form:

(1) $$-\nabla \cdot (\varepsilon \nabla u(\mathbf{x})) + \boldsymbol{b} \cdot \nabla u(\mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \in (0,1)^3 = \Omega,$$

(2) $$u(\mathbf{x}) = g^D(\mathbf{x}), \quad \mathbf{x} \in \quad \Gamma^D = \partial\Omega.$$

In convection dominance, i.e., for $0 < \varepsilon \ll \|\mathbf{b}\|$, classical discretization methods typically suffer from node-to-node nonphysical oscillations. Therefore, stabilized formulations are needed. Unfortunately, in many cases, stabilized formulations are also insufficient to capture localized oscillations around strong gradients where the solution exhibits sharp changes, necessitating additional numerical techniques.

In this report, we employ the streamline-upwind/Petrov–Galerkin (SUPG) [1] formulation for stabilizing the classical Galerkin finite element method (GFEM) for solving problems in the form of Eqs. (1)–(2). The SUPG-stabilized formulation is also complemented with the YZβ shock-capturing [2–4] operator. Numerical approximations obtained reveal that the proposed formulation achieves quite good solution profiles without any spurious oscillations.

Keywords: Convection-diffusion, finite elements, stabilization, shock-capturing

2010 Mathematics Subject Classification: 35Q35, 65N30, 76M10

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On numerical solution of second kind boundary value problem for parabolic equation with nonlocal integral condition

Charyyar Ashyralyyev [1,2], Rükiye Açıkel[3]

[1] *Department of Mathematics, Bahcesehir University, 34353, Istanbul, Turkey*

[2] *Department of Mathematics, National University of Uzbekistan, Tashkent, Uzbekistan*

[3] *Department of Mathematics, Graduate Education Institute, Gumushane University, 29100, Gumushane, Turkey*

E-mail: [1] *charyyar.ashyralyyev@eng.bau.edu.tr,* [3] *slmackl61@gmail.com*

Abstract: In this study, approximate solution of second kind boundary value problem (BVP) for parabolic partial differential equation with integral type nonlocal condition is discussed. The first order of accuracy absolute stable difference scheme for approximate solution of parabolic BVP with the Neumann boundary condition is investigated. Numerical analysiss for the solution of integral type nonlocal BVP for parabolic partial differential equation with the Neumann boundary condition are illustrated. The efficiency of the proposed numerical method is provided in test examples.

Keywords: parabolic equation, nonlocal, sdifference scheme, stability

2010 Mathematics Subject Classification: 35K60, 39A14, 65M06

medskip **References:**

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On solvability of a multipoint boundary value problem for loaded functional-differential equations with a conformable derivative

Usmanov Kairat[1], Nazarova Kulzina[1]

[1] *Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkestan, Kazakhstan*

kairat.usmanov@ayu.edu.kz
kulzina.nazarova@ayu.edu.kz

Abstract: In this paper, a multipoint boundary value problem is considered on a segment $[0,T]$ for a system of loaded functional-differential equations with a conformable derivative

(1) $$T_\alpha x(t) + AT_\alpha x(T-t) = \sum_{k=1}^{N} \int_0^T \varphi_k(t)\psi_k(s)x(s)\,ds + \sum_{j=0}^{m} K_j(t)x(\theta_j) + f(t), t \in [0,T], \; x \in R^n,$$

(2) $$\sum_{i=1}^{m} B_i x(\theta_i) = d, d \in R^n,$$

$$0 = \theta_0 < \theta_1 < \ldots < \theta_{m-1} < \theta_m = T,$$

where $0 < \alpha < 1$, matrices $\varphi_k(t)$, $\psi_k(s)$, $K_j(t)$ and n-dimensional vector function $f(t)$ are continuous on $[0,T]$, A is a symmetric matrix, B_i, $i = \overline{1,m}$ are constants of $n \times n$ matrices.

Using the property of an involutive transformation, the problem is reduced to a multipoint boundary value problem for loaded integro-differential equations. Further, the parametrization method proposed by Professor D. Dzhumabaev [1] is applied to the obtained problem.

Keywords: System of functional differential equations, parametrization method, multipoint boundary condition, unique solvability.

2010 Mathematics Subject Classification: 34K06, 34K37, 34K10

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Second order perturbations and stability of the solar system

Angel Zhivkov

Sofia University, Bulgaria

zhivkov@fmi.uni-sofia.bg

Abstract: We consider the Sun and the eight principal planets as mass points moving according to the Newton's law of gravitation. In a heliocentric coordinate system the planetary dynamics reads as a system of 48 first order ODE's, i.e. six equations for each planet [1]. If $\mathbf{r}$ and $\mathbf{r}'$ are the positions of two planets in $\mathbb{R}^3$, we expand the Newton's potential $|\mathbf{r}-\mathbf{r}'|^{-1}$ as a double Fourier's series in the mean longitudes. Next we estimate the corresponding amplitudes and apply a simple co-homology technics to prove the following theorem:

Suppose up to 10^{-4} *relative error in the initial data of the system of 48 first order ODE's, i.e. for the observed orbital elements and masses [2,3]. Then the dynamics of the eight principal planets remains stable in sense that the semi-major axes vary within 1% and eccentricities and inclinations remain bounded.*

Keywords: Solar system dynamics, perturbations, stability

2010 Mathematics Subject Classification: 70F15, 34D10, 34D20

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A Note for Compact Operators on Infinite Tensor Products

Meltem Sertbaş[1], Fatih Yılmaz[2]

[1] *Karadeniz Technical University, Faculty of Sciences, Department of Mathematics, 61080, Trabzon, Turkey*

m.erolsertbas@gmail.com

[2] *Karadeniz Technical University, Graduate Institute of Natural and Applied Sciences, 61080, Trabzon, Turkey*

fatih.yilmaz@ktu.edu.tr

Abstract: In this study, it is mentioned firstly that the infinite tensor product space of Hilbert spaces H_n denoted by $\underset{n\in\mathbb{N}}{\otimes^c} H_n$ which was established by giving the definition of $\otimes_x$, including H_n separable Hilbert space and is given the definition of the infinite tensor product of A_n, denoted by $\underset{n\in\mathbb{N}}{\otimes} A_n$, on $\underset{n\in\mathbb{N}}{\otimes^c} H_n$ from [2] and [1]. Secondly, the necessary conditions are investigated for this operator to be compact. Also, we prove that some compact operators have only point spectrum.

Keywords: Compact Operator; Infinite Tensor Product

2010 Mathematics Subject Classification: 47B07, 47A80, 46M05

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Fixed-point iteration method for solution first order differential equations with integral boundary conditions

Misir J. Mardanov[1,2], Yagub A. Sharifov[1,2]

[1] *Institute of Mathematics and Mechanics, ANAS, Baku, Azerbaijan*

[2] *Baku State University, Baku, Azerbaijan*

misirmardanov@yahoo.com
sharifov22@rambler.ru

Abstract: One of the most famous iterative methods to solve nonlinear problems is the quasi-linearization technique [1]. We consider the system of n-nonlinear coupled differential equations

(1) $$\dot{x}\left(t\right) = f\left(t, x\left(t\right)\right), t \in [0, T]$$

along with boundary conditions

x(0) $+\int\limits_0^T m(t)\,x(t)\,dt = C.$(2) Where $m(t) \in R^{n\times n}$ given matrices with $\det N \neq 0$, $N = E + \int\limits_0^T m(t)\,dt$; $f : [0,T] \times R^n \to R^n$ is some given continuous function. The aim of this thesis is to show that the sequence of functions x_n, which are solutions of

$$\dot{x}_{n+1}(t) = f(t, x_n(t)), \tag{3}$$

subject to the boundary conditions

$$x_{n+1}(0) + \int_0^T m(t)\,x_{n+1}(t)\,dt = C \tag{4}$$

converges to the solution of problem (1)-(2).

In [2] it is shown that any solution of the boundary value problem (1)-(2) can be represented as:

$$x(t) = N^{-1}C + \int\limits_0^T G(t,s)\,f(s, x(s))\,ds,$$

where

$$G(t,s) = sign(t-s)\begin{cases} N^{-1}\left(E + \int\limits_0^t m(\tau)\,d\tau\right), \; 0 \le s < t, \\ -N^{-1}\int\limits_t^T m(\tau)\,d\tau, \; t \le s \le T. \end{cases}$$

Theorem 1. Let x and x_n, respectively, be the solutions of (1)-(2) and (3)-(4). Assume that f is a nonlinear analytic function. Then, if $MKT < 1$, the sequence of functions x_n converges to the exact solution x in the L_2 norm, where $M = \max|f_x(t,x)|$, $K = \max\|G(t,s)\|$.

Keywords: Keywords: iteration method, integral boundary conditions, convergence.

2010 Mathematics Subject Classification: 34B37, 37C25, 37C75

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A collocation-shooting method for solving the boundary value problems for generalized Bagley-Torvik equation

Suzan Cival Buranay [1,a], Mtema J. Chin [1,b], Mahmudov I. Nazım [1,c]

[1] *Department of Mathematics, Faculty of Arts and Sciences, Eastern Mediterranean University Famagusta North Cyprus Via Mersin 10, Turkey.*

[a] *suzan.buranay@emu.edu.tr*
[b] *18600111@emu.edu.tr*
[c] *nazim.mahmudov@emu.edu.tr*

Abstract: We present a method for solving the boundary value problems for generalized Bagley-Torvik equation. The problem involves fractional derivatives of order β, $0 < \beta < 2$. The existence and the uniqueness of the solution is analyzed using Laplace transform approach. For the numerical solution of the boundary value problem for generalized Bagley-Torvik equation, we apply the collocation-shooting method (see also [1]). Numerical investigation is given on several examples with non-polynomial exact solutions. Results are presented in tables and figures illustrating the efficiency of the method.

Keywords: Generalized Bagley-Torvik equation, existence and uniqueness, collocation, shooting method, numerical investigation.

2020 Mathematics Subject Classification: 34A08, 65D07, 65L10.

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On the dynamics of Pluto

Ivaylo Tounchev

Sofia University, Bulgaria

Abstract: We study analytically the movement of the solar system planets and Pluto governed by Newton's law of gravitation.
After evaluating the Fourier coefficients of the perturbation function and supposing the stability of the eight principal planets, we prove that Pluto's orbit remains stable at least for the next 100 000 years, in sense that the semi-major axis varies in the range from 38.414 au to 40.316 au, eccentricity varies in the range from 0.227 to 0.276 and inclination varies in the range from $16.1°$ to $17.7°$. These results correspond to the numerical integration conducted in [1] and [2]. We also prove that the minimum distance between Neptune and Pluto will be within 2 au to 3 au, which is significantly less than 16.73 au estimated in [1], see also [3].

Keywords: Solar system dynamics, perturbations, stability

2010 Mathematics Subject Classification: 70F15, 34D10, 34D20

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Linear Fractional Programming Method for Optimizing of Inventory Model with Price-Dependent Demand Rate

Sema Akin Bas[1], Beyza Ahlatcioglu Ozkok[2]

[1] *Department of Mathematics, Yildiz Technical University, Istanbul, Turkey*

akins@yildiz.edu.tr

[2] *Department of Business Administration, Yildiz Technical University, Istanbul, Turkey*

bahlat@yildiz.edu.tr

Abstract: This report is devoted to an iterative approach to solve inventory model with price-dependent demand rate. A multi-product inventory problem under resources constraints is modelled with linear fractional programming (LFP). The inventory models with more than one objective functions in conflict with each other are reconstituted as a linear fractional inventory problem (LFIP)s. The proposed solution method to LFIP is based the (ϵ, δ) definition of continuity. The converge condition and fractional objective function are associated and introduced to a new iterative constraint for solving of LFIP. In the study, the optimal order quantity is designated while maximizing the total profit and minimizing the holding cost with the budget constraint, the space constraint, and the budgetary constraint on the ordering cost of each item. Moreover, the capacity of warehouse and the investment in inventories are limitted in the model had only one period in the cycle time. The proposed algorithm is tested with two examples from the literature.

Keywords: Inventory problem, linear fractional programming, continuity, iterative approach.

2010 Mathematics Subject Classification: 90C05, 90C32, 90B05

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On Stability of Hyperbolic Difference Equations on Circle

Allaberen Ashyralyev[1], Fatih Hezenci[2], and Yasar Sozen[3]

[1] *Department of Mathematics, Bahcesehir University, 34353 Istanbul, Türkiye,*

Peoples' Friendship University of Russia (RUDN University), Ul Miklukho Maklaya 6, Moscow 117198, Russia, and

Institute of Mathematics and Mathematical Modeling, 050010, Almaty, Kazakhstan

allaberen.ashyralyev@eng.bau.edu.tr

[2] *Department of Mathematics, Faculty of Science and Arts, Düzce University, 81620 Düzce, Türkiye*

fatihezenci@gmail.com

[3] *Department of Mathematics, Hacettepe University, 06800 Ankara, Türkiye*

ysozen@hacettepe.edu.tr

Abstract: Local and nonlocal boundary value problems for hyperbolic equations in the Euclidean space have been well investigated by various researchers (see, e.g. [1]). In [2], we considered the boundary value problems nonlocal type for parabolic and hyperbolic equations on smooth closed manifolds. We established the well-posedness of boundary value problems nonlocal type for parabolic equations in Hölder spaces. We also established the stability estimates for the nonlocal hyperbolic boundary value problems.

The present abstract considers nonlocal boundary value problems for the hyperbolic equations on the circle $\mathbb{T}^1$. The first order of accuracy difference scheme and modified difference scheme for the numerical solution of nonlocal boundary value problems for the hyperbolic equations on circle are presented. It establishes the stability estimates for the solutions of the difference schemes. Moreover, numerical examples are provided.

Keywords: Difference equations on manifolds, difference schemes, stability estimates

2010 Mathematics Subject Classification: 58Jxx, 58J32, 58J99.

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Euler - Bernoulli beam equation: invariant equations and solutions

Gulistan Iskenderoglu[1], Dogan Kaya[2]

Istanbul Commerce University, Faculty of Art and Science, Department of Mathematics, Turkey
[1]*giskenderoglu@ticaret.edu.tr*
[2] *dogank@ticaret.edu.tr*

Abstract: In this work, we present the Lie group classification of the dynamic fourth-order Euler-Bernoulli partial differential equation [1,2]. We construct all invariant equations for the beam equation and some classes of exact solutions, including similarity and hypergeometric solutions.

Keywords: Lie group classification, symmetry algebras, fundamental invariants, generalized nonlinear beam equation

2010 Mathematics Subject Classification: 70G65, 17B15, 34C14, 74K10

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Inverse time-dependent source problems in evolution equations

Dilyara Kuangaliyeva[1]

[1] *Department of Mathematics, Nazarbayev University, Kazakhstan*

dilyara.kuangaliyeva@nu.edu.kz

Abstract: This paper was written based on the joint work with D. Suragan and M. Karazym from the Department of Mathematics at Nazarbayev University, Kazakhstan [1]. We solve inverse problems of determining continuous time-dependent source terms in evolution equations. As a particular case, one inverse initial-boundary value problem with observation data at a spatial point is sufficient to recover the coefficient explicitly. The main feature of the method of recovering source terms is to solve inverse problems by considering one initial-boundary value problem with observation data at a spatial point. The concept is illustrated with analytical and numerical examples.

Keywords: inverse problem, initial-boundary value problem, classical solution

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On the second-order q-difference operator

Meltem Sertbaş

Karadeniz Technical University, Mathematics Department, Trabzon, Turkey

m.erolsertbas@gmail.com

Abstract: In this paper, the minimal and maximal operators defined by the second-order q-difference operator are discussed. Spectrum sets of these defined operators have been determined. In addition, the problem of extensions of the minimal operator is also mentioned.

Keywords: Second order q-difference operator, q-hyponormal operators, q-cohyponormal operators, minimal and maximal operators, spectrum.

2010 Mathematics Subject Classification: 39A13, 47A05, 47A10

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A criterion for minimality of the Laplace operator

Tynysbek Kal'menov[1], Nurbek Kakharman[2]

[1,2] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

[1]*kalmenov.t@mail.ru*, [2]*n.kakharman@math.kz*

Abstract: Let $\Omega \subset R^n$ be a finite domain with a smooth boundary $\partial\Omega$. Let us consider the Laplace equation

$$\Delta u = \sum_{k=1}^{n} \frac{\partial^2 u}{\partial x^2} = f(x), \qquad x \in \Omega. \tag{1}$$

A minimal operator Δ_0 is the closure of the differential operator Δ in $L_2(\Omega)$ on subset of the functions $u \in C^{2+\alpha}(\overline{\Omega})$

$$u\Big|_{x\in\partial\Omega} = \frac{\partial u}{\partial n_x}\Big|_{x\in\partial\Omega} = 0, \tag{2}$$

where $\frac{\partial}{\partial n_x}$ is a normal derivative. Δ_0^* is the adjoint operator of Δ_0 in $L_2(\Omega)$, then Δ_0^* is maximal operator, and by $\ker \Delta_0^*$ we denote the kernel of the maximal operator. Using the properties of the operator Δ_0 and $\ker \Delta_0^*$ M.I. Vishik [1] describes all regular boundary value problems for the Laplace equation (1) in the Hilbert space $L_2(\Omega)$. In [2], M. Otelbaev extended the results of M.I. Vishik to the case of Banach space and gave the description of the correct restriction of the maximal operator Δ_0^*, which includes not only boundary value problems, also problems with internal boundary conditions.

In the present work, we obtain a criterion solvability of the problem (1)-(2) in $L_2(\Omega)$, i.e. a necessary and sufficient condition on the right-hand side of the equation for the problem (1)-(2) is uniquely solvable in $L_2(\Omega)$. In this case, it mainly uses the boundary condition of the Newton (volume) potential constructed in [3] by T.Sh. Kalmenov, D. Suragan .

Keywords: Minimal operator, maximal operator, restriction, extension.

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Numerical solution of fractional diffusion equation with neural networks

Burcu Ece Alp[1], Mehmet Fatih Uçar[2]

[1] *Department of Mathematics,Istanbul Kultur University, Turkey*
1800005232@stu.iku.edu.tr

[2] *Department of Mathematics,Istanbul Kultur University, Turkey*
f.ucar@iku.edu.tr

Abstract: Differential equations arise in the modelling of various natural phenomena in computational science. For most of the scenarios, numerical methods are only options to solve such equations with some discretization. Semi-analytical and convergent solution functions are more desirable than the discrete numerical solutions. Therefore, artificial neural networks are increasingly used to construct continuous solution functions for solving various kinds of differential equations. Here we propose a physics informed neural network (PINN) method to solve fractional diffusion equation with variable coefficients on a finite domain. The PINN generate approximate solutions to the fractional PDE by training to minimize the physical loss function consisting of residual, boundary condition and initial condition parts. Riemann fractional derivative of the equation is discretized with the first order Grünwald formula and the resulted semi-discrete equation is used to construct the residual function of the PINN. Numerical experiments show that the present PINN method provides accurate solutions on the considered computational space-time domain.

Keywords: Fractional partial differential equation, Artificial neural network, Grünwald formula, Physics-informed neural networks, Diffusion equation

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On Hardy-Littlewood's theorem

Askhat Mukanov[1]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Kazakhstan Branch of Lomonosov Moscow State University, Kazakhstan*

mukanov.askhat@gmail.com

Abstract: This report is devoted to relation between integrability properties of functions and summability properties of their Fourier coefficients. In particular, we prove multidimensional Hardy-Littlewood type theorem.

For functions $f(x) \in L_1([0,1])$ with Fourier series $\sum\limits_{k=1}^{\infty} a_k \cos 2\pi kx$, where $\{a_k\}_{k=1}^{\infty}$ is nonincreasing sequence the Hardy-Littlewood theorem holds i.e., there exist $C_1, C_2 > 0$ such that

$$C_1 \|f\|_{L_p} \leq \left(\sum_{k=1}^{\infty} k^{p-2} a_k^p \right)^{\frac{1}{p}} \leq C_2 \|f\|_{L_p}.$$

Multidimensional analogues of Hardy-Littlewood's theorem were obtained in [1-4] (see also references therein).

We prove new multidimensional analogue of Hardy-Littlewood's theorem for functions with general monotone Fourier coefficients and give comparison with some known results.

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Keywords: Trigonometric Fourier series, Lebesgue spaces, general monotonicity, Hardy-Littlewood theorem

2010 Mathematics Subject Classification: 42A16, 42A32, 42B05

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Interpolation methods with parametric functions

Aigerim Kopezhanova

L.N.Gumilyov Eurasian National University, Kazakhstan

kopezhanova@mail.ru

Abstract: In this work we construct interpolation methods with parametric functions that can be used to study the interpolation properties of spaces with mixed metrics.

Let $1 \leq \bar{q} = (q_1, q_2) \leq \infty$, $\bar{\varphi}(t) = (\varphi_1(t), \varphi_2(t)) \geq 0$. We define anisotropic Lorentz spaces as follows:

$$\Lambda_{\bar{q}}(\bar{\varphi}) := \left\{ f : \left(\int_0^{+\infty} \left(\int_0^{+\infty} \left(f^{*_1*_2}(t_1, t_2) \varphi_1(t_1) \varphi_2(t_2) \right)^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}} < \infty \right\},$$

where $f^{*_1*_2} = f^{*_1*_2}(t_1, t_2)$ is the nonincreasing permutation of a function f [1]. In the paper [2] were studied one-dimensional generalized Lorentz spaces.

In this work the interpolation theorem for Lebesgue and Lorentz spaces with mixed metrics are obtained.

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Keywords: Lebesgue and Lorentz spaces, interpolation methods, interpolation theorem

2010 Mathematics Subject Classification: 46B70, 46E30

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New cubature formulas for Sobolev spaces $W_q^\alpha[0,1]^n$ with dominating mixed derivative

Nazerke Tleukhanova[1], Serzhan Basarov[2]

[1] *L.N.Gumilyov Eurasian National University, Kazakhstan*

tleukhanova@rambler.ru

[2] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

bassarov.serzhan98@gmail.com

Abstract: Let $W_q^\alpha[0,1]^n$ be the Sobolev spaces with dominating mixed derivative. In this paper we consider the problem of finding a grid M_k, $k = 1, \dots, n$ and coefficients c_k such that the error

$$\inf_{M,c} \sup_{||f||=1} \left| I(f) - \sum_{k=1}^{N} c_k f(M_k) \right|$$

is close to the optimal error.

A cubature formula is constructed for periodic functions with respect to each variable from space with a dominant mixed derivative (the Sobolev space $W_q^\alpha[0,1]^n$):

$$F_m(f;p) = \frac{1}{p^m} \sum_{\substack{k_1+\dots+k_n=m \\ k_j \geq 0}} \sum_{r_1=0}^{p^{k_1}-1} \cdots \sum_{r_n=0}^{p^{k_n}-1} (1-p)^{\sum_{j=1}^{n-1} sign k_j} \times$$

$$\times f\left(\frac{r_1}{p^{k_1}}, \dots, \frac{r_n}{p^{k_n}}\right) \frac{1}{(p-1)^{n-1}} \prod_{j=1}^{n-1} \left[\sum_{l=1}^{p-1} e^{2\pi i \frac{l r_j}{p}} \right],$$

where p is a prime number and $m \in \mathbb{N}$.

This cubature formula is exact for trigonometric polynomials with spectrum from the corresponding hyperbolic cross. In the case of $p = 2$ the cubature formula was given in [1].

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Keywords: Sobolev spaces, a quadrature formula, trigonometric polynomials

2010 Mathematics Subject Classification: 42B05, 46E30, 46E35

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[1] E.D. Nursultanov, N.T. Tleukhanova, Quadrature formulae for classes of functions of low smoothness, Sb. Math., vol 194, no 10, 133–160, 2003.

On solvability of some boundary value problems for a nonlocal polyharmonic equation with fractional order boundary operators

Batirkhan Turmetov,

Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkistan, Kazakhstan

batirkhan.turmetov@ayu.edu.kz

Abstract: In this work, solvability of some boundary value problems for a nonlocal polyharmonic equation is studied. The nonlocal polyharmonic operator is introduced with the help of mappings of involution type. Boundary value problems with Dirichlet conditions and boundary operators of fractional order are considered. Fractional order operators are defined by modified Hadamard derivatives. Theorems on the existence and uniqueness of solutions of the studied problems under stud are proved.

Previously in our papers [1,2], similar problems were studied for the nonlocal Laplace equation and the biharmonic equation

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Keywords: involution; nonlocal operator; polyharmonic equation; Hadamard derivatives; Dirichlet problem; Neumann problem

2010 Mathematics Subject Classification: 31B30,35J30,35J40

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On the convolution operator in Morrey spaces

Erlan Nursultanov[1], Durvudkhan Suragan[2]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

er-nurs@yandex.ru

[2] *Nazarbayev University, Kazakhstan*

durvudkhan.suragan@nu.edu.kz

Abstract: Let $0 \leq \lambda \leq \frac{n}{p}$ and $0 < p < \infty$. A set of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ is called the Morrey space if

$$\|f\|_{M_p^\lambda} \equiv \|f\|_{M_p^\lambda(\mathbb{R}^n)} = \sup_{x\in\mathbb{R}^n} \sup_{r>0} r^{-\lambda}\|f\|_{L_p(B_r(x))} < \infty.$$

Here $B_r(x)$ is the ball centered at the point x and with radius $r > 0$.

In 1938 Morrey introduced the function spaces now bearing his name. These spaces were studied as a consequence of questions of regular solutions of nonlinear elliptic equations and systems. In the last two decades, great interest has been shown in the study of the classical operators of function theory acting in these spaces. Riesz's potential in Morrey spaces are studied in papers [1], [2], [3].

We study estimates for the norm of the convolution operator

$$(Tf)(x) = (K * f)(x) = \int\limits_{\mathbb{R}^n} K(x-y)f(y)dy,$$

which is acted from one Morrey space to another Morrey space.

This paper is devoted to the study of upper bounds for the norm of the convolution operator in Morrey spaces. The spaces $M_{p,q}^\alpha$, which cover the classical Morrey spaces, are introduced. Moreover, their embedding properties are investigated, and their interpolation properties are described. Young-O'Neil type inequalities in Morrey spaces are proved. New results on the boundedness of Riesz's potential in Morrey spaces are established.

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Keywords: Morrey spaces, convolution operator, Riesz's potential

2010 Mathematics Subject Classification: 46B70, 46E30

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On the circulant matrices with Ducci sequences and conditional sequences

Cahit Kome

Nevǎ415ǎ45fehir
Hacǎ414± $Bekta$ǎ415ǎ45$fVeliUniversity, DepartmentofMathematics$, 50300, $Turkey.$
cahit@nevsehir.edu.tr

Abstract: The concept of Ducci sequence was introduced by Ciamberlini and Marengoni and their discovery was attributed to Prof. E Ducci in 1937 [1]. Ducci sequence, produced by $X = (x_1, x_2, \ldots, x_n)$, is the sequence $\{X, D(X), D^2(X), \ldots\}$ where $D : \mathbb{Z}^n \to \mathbb{Z}^n$ is defined by

$$D(x_1, x_2, \ldots, x_n) = (|x_2 - x_1|, |x_3 - x_2|, \ldots, |x_n - x_{n-1}|, |x_n - x_1|).$$

In this work, we investigate a new circulant matrix, $Circ(DQ)$, by applying the Ducci map to each row of the circulant matrix

$$Circ(Q) = Circ\left(\left(\frac{b}{a}\right)^{\frac{\xi(1)}{2}} Q_0, \left(\frac{b}{a}\right)^{\frac{\xi(2)}{2}} Q_1, \ldots, \left(\frac{b}{a}\right)^{\frac{\xi(n)}{2}} Q_{n-1}\right)$$

whose elements are the generalized conditional numbers. Moreover, we obtain several new identities by using the Binet formula of $\{Q_n\}_{n=0}^{\infty}$. By virtue of these identities, we get the Hadamard inequality, spectral norm, Euclidean norm, ℓ_p norm, determinants and eigenvalues of the circulant matrices with Ducci sequences and conditional sequences.

Keywords: Circulant matrix, Ducci sequence, Conditional sequences, Matrix Norm, Inequality.

2010 Mathematics Subject Classification: 11B83, 15A15, 15A60, 11B39.

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Pseudoinverse of some special singular matrices and their applications

Cahit KÖME

Nevşehir Hacı Bektaş Veli University, Department of Mathematics, 50300, Turkey.
cahit@nevsehir.edu.tr

Abstract: The topic of the generalized inverses emerge in many area such as physics, neural networks, statistics, numerical analysis, control system analysis, curve fitting, tensor computations and the solution of system of equations. In recent years, there have been several studies in different areas related to the Pseudoinverse and its applications [1, 2].

Let $\mathbb{C}^{m\times n}$ be the set of $m \times n$ complex matrices, for every $A \in \mathbb{C}^{m\times n}$, the Pseudoinverse of a matrix A is the unique $n \times m$ matrix $A^{\dagger}$ with the following properties:

$$(1) \qquad AA^{\dagger}A = A, \quad A^{\dagger}AA^{\dagger} = A^{\dagger}, \quad \left(AA^{\dagger}\right)^{*} = AA^{\dagger}, \quad \left(A^{\dagger}A\right)^{*} = A^{\dagger}A$$

where A^{*} denotes the conjugate transpose of A.

The purpose of this work is to provide novel results on the investigation of Pseudoinverse of some special singular matrices which are generated by the conditional sequences. We obtain explicit Pseudoinverses by using some analytical techniques. Furthermore, we investigate the correlations between such singular matrices and the $q-$Pascal matrices of the first and of the second kind. Also, we derive several combinatorial identities and provide more generalized results compared to the previous works.

Keywords: Pseudoinverse, Pascal matrix, Combinatorial identities, Singular matrix.

2010 Mathematics Subject Classification: 15A09, 11B39, 05A19.

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Extreme learning machine approach for solving ordinary differential equations arising in biology

Huseyin Tunc

Department of Biostatistics and Medical Informatics, School of Medicine, Bahcesehir University, Istanbul, Turkey huseyin.tunc@med.bau.edu.tr

Abstract: Mathematical models are widely used to understand the dynamics of various biological processes. Modelling of such processes generally yields nonlinear system of differential equations (ODE) that can only be solved with some numerical techniques. Here we propose an extreme learning machine (ELM) approach for solving such ODEs to find out continuous approximate solutions. The ELM approach is shown to have considerable advantageous over the existing neural network based approximate methods and numerical initial value problem solvers. Two test problems representing between host and within host viral dynamics are considered to measure the efficiency of the current approach.

Keywords: Machine learning, Artificial neural network, Mathematical biology, Viral dynamics, Ordinary differential equations

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The effect of groove on the outer wall of waveguide to the sound propagation

Hülya Öztürk

Gebze Technical University, Department of Mathematics, Kocaeli, Turkey
h.ozturk@gtu.edu.tr

Abstract: In this work, we will focus on the theoretical study of waveguide with a groove on the outer wall. We start by modelling this waveguide as a boundary value problem, then based on the boundary and continuity conditions, we will build Wiener-Hopf equation which is solved by classical factorization and decomposition procedures. Moreover, with the help of MATLAB programme numerical results for some parameters are presented.

Keywords: Fourier transform, boundary value problem, Wiener-Hopf technique, acoustics

2010 Mathematics Subject Classification: 34K10,42B10, 78AK0

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[1] N. Peake, I.D. Abrahams, Sound radiation from a semi-infinite lined duct, Wave Motion, vol. 92, 102407, 2020..

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Minimizing sequences for a linear-quadratic control problem with three-tempo variables under weak nonlinear perturbations

Galina Kurina[1], Margarita Kalashnikova[2]

[1] *Voronezh State University and Federal Research Center "Computer Science and Control"of RAS, Russia*

kurina@math.vsu.ru

[2] *Atos IT Solutions and Services, Russia*

margarita.kalashnikova@mail.ru

Abstract: An optimal control problem of the form

(1)
$$\int_0^T (1/2(w(t,\varepsilon)'W(t)w(t,\varepsilon)+u(t,\varepsilon)'R(t)u(t,\varepsilon))+ \\ +\varepsilon F(w(t,\varepsilon),u(t,\varepsilon),t,\varepsilon))\,dt \to \min_u,$$

(2)
$$\mathcal{E}(\varepsilon)\frac{dw(t,\varepsilon)}{dt} = A(t)w(t,\varepsilon)+B(t)u(t,\varepsilon)+ \\ +\varepsilon f(w(t,\varepsilon),u(t,\varepsilon),t,\varepsilon),\ t\in[0,T],\ w(0,\varepsilon)=w^0$$

is considered. Here ε is a non-negative small parameter, $T>0$ is fixed, the prime means transposition; $\mathcal{E}(\varepsilon)=diag(I_{n_1},\varepsilon I_{n_2},\varepsilon^2 I_{n_3})$.

Under some conditions, the algorithm of constructing asymptotic solution of problem (1), (2) by means of the direct scheme, consisting of immediate substituting a postulated asymptotic solution into a problem condition and determining a series of problems for finding asymptotics terms, is given in [1]. Estimates of the proximity between the asymptotic and exact solutions are proved for the control, state trajectory and minimized functional. It is established that the constructed asymptotic sequences, consisting of solutions of auxiliary problems, are minimizing.

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Keywords: optimal control problems, three-tempo variables, weak nonlinear perturbations, asymptotic solution, the direct scheme method, estimates

2010 Mathematics Subject Classification: 9370, 34H05, 34E13

References:

[1] M. Kalashnikova, G. Kurina, Direct scheme of constructing asymptotic solution of three-tempo linear-quadratic control problems with weak nonlinear perturbations, 16 International Conference on Stability and Oscillations of Nonlinear Control Systems (Pyatnitsky's Conference), Moscow, Russia, 01-03 June 2022.

A Method for Toric Resolution

Büşra Karadeniz Şen

Gebze Technical University, Turkey

busrakaradeniz@gtu.edu.tr

Abstract: The vanishing set of a function $f \in \mathbb{C}[x, y, z]$ is the set of common zeroes of f.

$$X := \{p = (p_1, p_2, p_3) \in \mathbb{C}^3 \mid f(p) = 0\}$$

This set define an hypersurface $X \subset \mathbb{C}^3$. We focus on a toric resolution of X with its jet space when X has non-isolated singularity. Actually a toric resolution of $X \subset \mathbb{C}^3$ can be obtained by constructing a regular subdivision of the dual Newton polyhedron of f [1]. Hence we first construct the jet space of X then using the vectors of this jet space we establish a regular subdivision of the dual Newton polyhedron of f. This gives us a toric resolution of $X \subset \mathbb{C}^3$.

More specifically, in this talk we will give an hypersurface X belonging to a special class of singularity and we will construct a toric resolution of X.

Keywords: non-isolated singularity, toric resolution, jet space.

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To numerical solution of the non-divergent diffusion equation in non-homogeneous medium with source or absorption

Mersaid Aripov[1], Makhmud Bobokandov[2]

[1,2] *National University of Uzbekistan, Tashkent, Uzbekistan*

mirsaidaripov@mail.ru[1]
m.boboqandov@nuu.uz[2]

Abstract: In the domain $Q = \left\{(t,x) : t \geq t_0 > 0, x \in \mathbb{R}^N\right\}$, we study the following Cauchy problem to the doubly nonlinear parabolic equation not in divergence form with source or absorption:

$$\partial_t u = u^q \nabla \left(|x|^n u^{m-1} |\nabla u^k|^{p-2} \nabla u\right) + \varepsilon t^\lambda u^\beta, \tag{1}$$

$$u_{t=t_0} = u_0(x) \geq 0. \tag{2}$$

Here $k, m \geq 1, p \geq 2, \beta \neq 0, \ \varepsilon = \pm 1, \ \lambda, \ n, \ q$ are the given numerical parameters, $\nabla(.) = \underset{x}{grad}(.)$.

The (1)-(2) arises in different applications [1]-[2]. Equation (1) is degenerate type. Therefore, in the domain Q, where $u = 0, \nabla u = 0$ it is degenerate type. Therefore, in this case, we need to consider a weak solution from having a physical sense class. The (1) for the particular value of numerical parameters intensively studied by many authors, (see [1]-[2] and literature therein). Numerical solutions to this problem are based on investigating qualitative properties of the problem such as Fujita type global solvability, asymptotic solution, localization of solution, finite speed propagation of distribution, blow-up solution, and so on by many authors (for example, see [1] and literature therein).

In this work we consider the problem numerical investigation. For this goal, it an important to find an initial approximation depending on the value of numerical for iterative processes. In the work consider slowly and fast diffusion, critical and singular cases. Using comparison, the $\beta > m + k(p-2) + \frac{p(1-q)}{n+N}$ established such Fujita type. Properties of self-similar solution analyzed. It is established asymptotic self-similar equation, including critical case.

Keywords: parabolic equation, self-similar solution, self-similar equation, critical Fujita, global solution, asymptotic behavior.

2010 Mathematics Subject Classification: 35K55,35D30,35B44,35B40

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Another approach to weighted iterated Hardy-type inequalities

Rza Mustafayev[1], Merve Gorgulu[2]

[1] *Mathematics Department, French-Azerbaijani University (UFAZ), Azerbaijan State Oil and Industry University, Baku, Azerbaijan*

rzamustafayev@gmail.com

[2] *Karamanoğlu Mehmetbey University, Turkey*

mervegorgulu@kmu.edu.tr

Abstract: In this talk, by using a combination of reduction and discretization techniques we present a solution to the inequality

$$\left(\int_0^\infty \left(\int_x^\infty \left(\int_0^t f\right)^q w(t)\,dt\right)^{r/q} u(x)\,dx\right)^{1/r} \le C\left(\int_0^\infty f^p v\right)^{1/p}.$$

for any non-negative measurable function f on $(0,\infty)$ where $1 \le p < \infty$, $0 < q$, $r < \infty$ and u, v, w are weights on $(0,\infty)$.

Keywords: quasilinear operators, iterated Hardy inequalities, weights

2010 Mathematics Subject Classification: 26D10, 26D15

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Analytical Analysis of the Assistive Devices for the Asthma Patients

Sabareela Viktory Moro[1], Berna Uzun[2]34, Ali Denker[4], Dilber Uzun Ozsahin[5]3

[1] *Department of Biomedical Engineering, Faculty of Engineering, Near East University, Nicosia 99138, TRNC Mersin 10, Turkey*

sabareela.moro@gmail.com

[2] *Carlos III University of Madrid, Statistics Department, Getafe 28903, Madrid, Spain* [3] Operational Research Center in Healthcare, Near East University, Nicosia 99138, TRNC Mersin 10, Turkey

buzun@est-econ.uc3m.es

[4] *Department of Mathematics, Faculty of Applied Sciences, Near East University, Nicosia 99138, TRNC Mersin 10, Turkey*

ali.denker@neu.edu.tr

[5] *Medical Diagnostic Imaging Department, College of Health Sciences, University of Sharjah, Sharjah, United Arab Emirates*

dozsahin@sharjah.ac.ae

Abstract: Multi-criteria decision analysis (MCDA) is a field that provides solution to the complex issue where multiple conflicting criteria occurs for determination of the alternatives [1]. Fuzzy logic is a process which enables the expert defining and analyzing the environment under vague conditions [1]. In this study the fuzzy based MCDA model is proposed for the analytical evaluation of the asthma medical devices. Asthma is a respiratory disease that affects the lungs. When the lungs are unable to inhale air properly, it becomes inflamed and swollen [2]. The inflammation makes it difficult to breathe in air into the lungs and when air is not properly inhaled into the lungs, it could lead to death. Dust, tobacco smoke, old buildings/paintings, stress, anxiety, obesity and low weight, pollen grains, genetics, occupational chemicals, are the main causes of the asthma [2]. These causes needed to properly managed else it may cause the asthma attack. The attack of the asthma arises when any of the causes of asthma triggers the lungs which can make the airways become swollen and inflamed. Coughing, wheezing, tightness of chest and inability to breathe properly are the main symptoms of asthma. It can be managed and treated with the appropriate medical devices. These devices are inhalers and they designed to give medication into the lungs through inhalation to allow the release of the lungs or expansion of the lungs to allow the flow of air properly. Inhalers are portable devices that are used to administer drugs to asthma patients. The drugs are bronchodilators and anti-inflammatory which release the lungs when they are inhaled for allow the air flow. Four 7 different inhalers are analyzed from the 4 types of inhalers which include: Soft mist inhaler, Respimat inhaler, Dry powder inhaler and Nebulizers. These inhalers were evaluated and compared using fuzzy PROMETHEE, a multi-criteria decision making technique.

The parameters are selected as side effects, specificity, efficiency, cost, practicability, treatment duration, limitation, advantages and disadvantages. The data are determined using the fuzzy sets and defuzzified with Yager inbdex then the PROMETHEE approach is applied using the Gaussian preference function for each criterion. Based on the selected parameters Accuhaler ranked as the best alternative with 0,2078 net flow value, soft mist inhaler ranked as the second with net flow value of 0,1943 and followed with Jet Nebulizer with 0,1912 net flow value.

Keywords: Asthma, inhalers, inflammation, asthma medical devices, decision making, fuzzy logic, PROMETHEE.

2010 Mathematics Subject Classification: 20F10, 03B52, 90B50

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The effect of groove on the outer wall of waveguide to the sound propagation

Hülya Öztürk

Gebze Technical University, Department of Mathematics, Kocaeli, Turkey
h.ozturk@gtu.edu.tr

Abstract: In this work, we will focus on the theoretical study of waveguide with a groove on the outer wall. We start by modelling this waveguide as a boundary value problem, then based on the boundary and continuity conditions, we will build Wiener-Hopf equation which is solved by classical factorization and decomposition procedures. Moreover, with the help of MATLAB programme numerical results for some parameters are presented.

Keywords: Fourier transform, boundary value problem, Wiener-Hopf technique, acoustics

2010 Mathematics Subject Classification: 34K10,42B10, 78AK0

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[2] H. Öztürk, Wiener-Hopf approach for the coaxial waveguide with an impedance-coated groove on the inner wall, Journal of Engineering Mathematics, vol. 124, no. 1, 75-88, 2020.

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On estimates for linear widths of classes of functions of several variables in the Lorentz space

Gabdolla Akishev

Lomonosov Moscow University, Kazakhstan Branch and
Institute of Mathematics and Mathematical Modeling, Kazakhstan
akishev_g@mail.ru

Abstract: $L_{p,\tau}(\mathbb{T}^m)$ denotes the Lorentz space of all real-valued Lebesgue measurable functions $f(\overline{x})$ that have a 2π-period in each variable and for which the quantity

$$\|f\|_{p,\tau} = \left[\frac{\tau}{p}\int\limits_0^1 \left(f^*(t)\right)^{\tau} t^{\frac{\tau}{p}-1} dt\right]^{1/\tau}, \ 1 \leqslant p < \infty, 1 \leqslant \tau < \infty,$$

is finite, where $f^*(y)$ is a non-increasing rearrangement of the function $|f(2\pi\overline{x})|$, $\overline{x} \in \mathbb{I}^m$ (see [1]). Let us consider an analog of the Nikol'skii–Besov class (see [2]) $\mathbb{S}^{\overline{r}}_{p,\tau,\theta}B = \left\{f \in \mathring{L}_{p,\tau}(\mathbb{T}^m): \ \left\|\left\{2^{\langle\overline{s},\overline{r}\rangle}\left\|\delta_{\overline{s}}(f)\right\|_{p,\tau}\right\}_{\overline{s}\in\mathbb{Z}^m_+}\right\|_{l_\theta} \leqslant 1\right\}$. In case when $\tau = p$ the class $\mathbb{S}^{\overline{r}}_{p,\tau,\theta}B$ is the well-known Nikol'skii–Besov class $S^{\overline{r}}_{p,\theta}B$ in the space $L_p(\mathbb{T}^m)$ (see, for example, [3], p. 32).

The report will present estimates of the linear width $\lambda_n\left(\mathbb{S}^{\overline{r}}_{p,\tau_1,\theta}B, \ \ L_{q,\tau_2}\right)$ of the class $\mathbb{S}^{\overline{r}}_{p,\tau_1,\theta}B$ in the space $L_{q,\tau_2}(\mathbb{T}^m)$ for different relations between the parameters $p, q, \tau_1, \tau_2, \theta$ (the definition of the linear width could be found, for example, in [3], p. 50). In particular,

Theorem 1. Let $1 < p < 2 \leqslant q < p' = \frac{p}{p-1}$, $1 < \tau_1, \tau_2 < \infty$, $1 \leqslant \theta \leqslant \infty$, $r_1 = ... = r_\nu < r_{\nu+1} \leqslant \ldots \leqslant r_m$ and $r_1 > \frac{1}{p}$. Then

$$\lambda_n\left(\mathbb{S}^{\overline{r}}_{p,\tau_1,\theta}B, \ \ L_{q,\tau_2}\right) \asymp n^{-(r_1+\frac{1}{2}-\frac{1}{p})}(\log^{\nu-1} n)^{r_1+\frac{1}{2}-\frac{1}{p}}\left(\log n\right)^{(\nu-1)\left(\frac{1}{2}-\frac{1}{\theta}\right)_+},$$

where $a_+ = \max\{a, 0\}$.

Theorem 2. Let $1 < p \leqslant 2$, $p' = \frac{p}{p-1} < q < \infty$, $1 < \tau_1, \tau_2 < \infty$, $1 \leqslant \theta \leqslant \infty$, $\ r_1 = ...r_\nu < r_{\nu+1} \leqslant \ldots \leqslant r_m$ and $r_1 > 1 - \frac{1}{q}$. Then

$$\lambda_n\left(\mathbb{S}^{\overline{r}}_{p,\tau_1,\theta}B, \ \ L_{q,\tau_2}\right) \leqslant Cn^{-(r_1+\frac{1}{q}-\frac{1}{2})}\left(\log^{\nu-1} n\right)^{r_1+\frac{1}{q}-\frac{1}{2}}\left(\log n\right)^{(\nu-1)(\frac{1}{\tau_2}-\frac{1}{\theta})_+}.$$

Keywords: Lorentz space, linear width, Nikol'skii–Besov class

2010 Mathematics Subject Classification: 41A10, 41A25, 42A05

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Mathematical model of the dynamics of a liquid metal bridge at electrical contacts opening

Stanislav Kharin

Institute of Mathematics and Mathematical Modeling, Kazakhstan

staskharin@yahoo.com

Abstract: The mathematical model of the dynamics of the temperature field in a liquid metal bridge at electrical contacts opening is based on the heat transfer problem for the visible part of a bridge and its hidden part. Heating of the visible part of a bridge is described by the heat equation in the cylindrical domain with moving boundaries both in the radial and axial directions. The radial motion of the boundary is defined from the balance of the volumes of the visible and melted zones, while the axial motion is defined by the velocity of contact opening. Heating of the hidden part of the bridge is described by the Stefan problem for the spherical heat equation for the melted and solid zones. Conjugation of temperature field for visible and hidden zones is realized using the integral heat balance method which enables to reduce the considered problem to a system of nonlinear ordinary differential equations solvable by standard methods.

The obtained solution enables us to define the time of reaching of temperature boiling point and calculate the value of the contact bridge erosion. The analysis of obtained results may be very useful for the optimal choice of parameters of contact opening for various contact materials, values of electrical current, circuit parameters and other contact characteristics to minimize the bridge erosion.

The calculating values of bridge erosion for various contact materials is compared with experimental data.

Compactness of a class of integral operators with logarithmic singularity

Akbota Abylayeva[1], Askar Baiarystanov[1]

[1] *Department of Fundamental Mathematics,*

L.N.Gumilyov Eurasian National University, Astana, Kazakhstan

abylayeva_ b@mail.ru, oskar_ 62@mail.ru

Abstract: Let $I = (0,\infty)$ and let v, u be almost everywhere positive and locally integrable functions on the interval I. Let $1 < p, q < \infty$ and $p' = \frac{p}{p-1}$. Let us denote by $L_{p,v} \equiv L_p(v, I)$ the set of measurable functions f on I for which $\|f\|_{p,w} = \left(\int\limits_0^\infty |f(x)|^p w(x)dx\right)^{\frac{1}{p}} < \infty$. Let W be a strictly increasing and locally absolutely continuous function on the interval I. Let $\frac{dW(x)}{dx} = w(x)$, for almost all $x \in I$. Consider the operator

$$T_{\alpha,\beta}f(x) = \int\limits_0^x \frac{\left(ln\frac{W(x)}{W(x)-W(s)}\right)^\beta u(s)f(s)w(s)ds}{(W(x)-W(s))^{1-\alpha}}, \quad x \in I, \tag{1}$$

where $\alpha > 0$, $\beta \geq 0$.

The boundedness of the operator (1) from $L_{p,w}$ to $L_{q,v}$ when $\beta = 0$ is obtained in the paper [1] for $\alpha > \frac{1}{p}$, $1 < p \leq q < \infty$ and $0 < q < p < \infty$. Further, we assume that W is non-negative on I and $\lim\limits_{x\to 0^+} W(x) = 0$. The following theorem holds.

Theorem. Let $0 < \alpha < 1$, $\frac{1}{\alpha} < p \leq q < \infty$ and $\beta \geq 0$. Let the function u be non-increasing on I. Then the operator $T_{\alpha,\beta}$, defined by formula (1), is bounded from $L_{p,w}$ to $L_{q,v}$ if and only if $A_{\alpha,\beta} = \sup\limits_{z>0} A_{\alpha,\beta}(z) < \infty$,

$$A_{\alpha,\beta}(z) = \left(\int\limits_z^\infty v(x)W^{q(\alpha-\beta-1)}(x)dx\right)^{\frac{1}{q}} \left(\int\limits_0^z W^{\beta p'}(s)u^{p'}(s)w(s)ds\right)^{\frac{1}{p'}},$$

and operator $T_{\alpha,\beta}$ is compact from $L_{p,w}$ to $L_{q,v}$ if and only if $A_{\alpha,\beta} < \infty$ and $\lim\limits_{z\to 0^+} A_{\alpha,\beta}(z) = \lim\limits_{z\to\infty} A_{\alpha,\beta}(z) = 0$. Moreover $\|T_{\alpha,\beta}\| \approx A_{\alpha,\beta}$, where $\|T_{\alpha,\beta}\|$ is the norm of the operator (1) from $L_{p,w}$ to $L_{q,v}$.

Keywords: Compactness, boundadness, operators with logarithmic singularity

2010 Mathematics Subject Classification: 26A33, 26D10, 47G10

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Asymptotic behavior of discrete dynamical systems of Voltaire type

Dilfuza Eshmamatova[1], Mohiniso Toshmatova[2]

[1] *Tashkent State Transport University, Tashkent, Uzbekistan*
24dil@mail.ru

[2] *Tashkent State Transport University, Tashkent, Uzbekistan*
toshmatova-mm@mail.ru

Abstract: Let $V : S^{m-1} \to S^{m-1}$ be a Voltaire type operator [1] and $x^0 \in S^{m-1}$. The sequence $\{x^{(n)}\}$, where $x^{(n)} = V^n x^0$ is called the trajectory at $n \in Z$, the positive (negative) trajectory at $n \in N(-n \in N)$. Through $\omega^+(x^0)$ and $\omega^-(x^0)$ denote the set of limit points, respectively, of positive and negative trajectories.

A continuous functional $\varphi : S^{m-1} \to R$ is called a Lyapunov function for a discrete dynamical system

$$x_k^{(n+1)} = x_k^{(n)} \left(1 + \sum_{i=1}^{m} a_{ki} x_i^{(n)}\right), k = \overline{1, m}, n \in Z, \tag{1}$$

if there are limits for any starting point $x^0 \in S^{m-1}$

$$\lim_{n\to+\infty} \varphi(x^{(n)}), \qquad \lim_{n\to-\infty} \varphi(x^{(n)})$$

Throughout this note we mainly use techniques from our works [1].

Keywords: The Volterra mapping, simplex, fixed point, positive trajectory, negative trajectory.

2010 Mathematics Subject Classification: 37B25, 37C25, 37C27

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On the inverse problems for a 2-D system of Navier-Stokes

Muvasharkhan Jenaliyev[1], Madi Yergaliyev[2], Bekzat Orynbasar[3]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

muvasharkhan@gmail.com

[2] *Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Al-Farabi Kazakh National University, Kazakhstan*

ergaliev.madi.g@gmail.com

[3] *Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Al-Farabi Kazakh National University, Kazakhstan*

qairatulybekzat@gmail.com

Abstract: Let $\Omega = \{|y| < 1\} \subset R^2$ be an open bounded domain with boundary $\partial\Omega$, $Q_{yt} = \Omega \times (0, T)$, $\Sigma_{yt} = \partial\Omega \times (0, T)$. The following inverse problem of determining functions $\{w(y, t), P(y, t), f(y)\}$ is considered:

$$\partial_t w - \nu\Delta w = g(t)f(y) - \nabla P, \quad (y, t) \in Q_{yt}, \tag{1}$$

$$\operatorname{div} w = 0, \quad (y, t) \in Q_{yt}, \tag{2}$$

$$w(y, t) = 0, \quad (y, t) \in \Sigma_{yt}, \;\; w(y, 0) = 0, \quad y \in \Omega, \tag{3}$$

with overdetermination condition:

$$w(y, T) = w_T(y), \tag{4}$$

where $g(t) = \{g_1(t), g_2(t)\}$ and $w_T(y) = \{w_{T1}(y), w_{T2}(y)\}$ are given functions.

For a biharmonic operator in a circle, a generalized spectral problem has been posed. For the latter, a system of eigenfunctions and eigenvalues is constructed, which is used in the report for the numerical solution of the inverse problem in a circular cylinder with specific numerical data. Graphs illustrating the results of calculations are presented.

Some of our results are published in [1]. The report discusses the development of the obtained results for the nonlinear 2-D system of Navier-Stokes.

Keywords: Navier-Stokes equations, inverse problem, numerical solution

2010 Mathematics Subject Classification: 35Q30, 35R30, 65N21

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Dynamics of Piecewise Differential Systems with a Pseudo Type Point

Rebiha Benterki

[1] *Department of Mathematics, University Mohamed El Bachir El Ibrahimi of Bordj Bou Arréridj 34000, El Anasser, Algeria*

r_ benterki@yahoo.fr

Abstract: This paper deals with the global dynamics of planar piecewise linear-quadratic and quadratic-quadratic differential systems with pseudo-focus points at the origin separated by the straight line $x = 0$. We make the classification of the global phase portraits in the Poincaré disk and we provide all the different topological phase portraits can exhibit for such piecewise differential systems.

Keywords: Phase portraits, Poincaré compactification, Piecewise differential systems, Pseudo focus point.

2010 Mathematics Subject Classification: 34C29, 34C25, 47H11

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Analytical solutions for Sturm-Liouville operator with transmission conditions

Daulet Nurakhmetov[1,2], Almir Aniyarov[1,2]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

[2] *Astana International University, Kazakhstan*

nurakhmetov@math.kz, aniyarov@math.kz

Abstract: We construct the analytical solution for Sturm-Liouville operator with transmission and boundary conditions:

(1) $$l(y) \equiv -k\, y''(x) + c(x)y'(x) + b(x)y(x) = f(x),\ x \in (0,\, 1) \setminus \{x_0\},$$

(2) $$y(0) = 0, \qquad y(1) = 0,$$

(3) $$[y(x)]_{x_0} = h_1, \qquad [y'(x)]_{x_0} = h_2.$$

Here $[\varphi(x)]_{x=x_0} = \lim_{\varepsilon \to +0} (\varphi(x_0 + \varepsilon) - \varphi(x_0 - \varepsilon))$ it means the jump of the function at the point $x = x_0$.

The Green function of discontinuous boundary value problem with transmission conditions was studied in [1]. The review of other investigation with respect to various questions of spectral and quality solutions for transmission or interface problems one can find in [2]. In this talk we will show the connection between solutions of the boundary value problem and transmissions problem. Analytical solutions of multi-point well-posed boundary value problems will be derived on the based of contraction theory, see [3].

Keywords: transmission conditions, boundary value problem, Green function, system of fundamental solutions

Funding: The authors were supported by grant AP08052239 of the Ministry of Education and Science of the Republic of Kazakhstan.

2010 Mathematics Subject Classification: 34B10, 34B24, 34B27

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Derivation of some Hom-algebras

Hadjer Adimi[1]

[1] *Faculty of Mathematics and Computer Science.*
Department of mathematics.
University of Bordj Bou Arreridj, Algeria.

h.adimi@univ-bba.dz

Abstract: Hom-algebra structures are given on linear spaces by multiplications twisted by linear maps. Hom-Lie algebras and general quasi-Hom-Lie and quasi-Lie algebras were introduced by Hartwig, Larsson and Silvestrov as algebras embracing Lie algebras, super and color Lie algebras and their quasi-deformations by twisted derivations. In this work we introduce the notion of derivation of some algebras, illustrated by some examples, and an appendix which describes the method of computing a derivation under the mathematica software.

Keywords: Algebras, Hom-algebras, derivation.

2010 Mathematics Subject Classification: 17A30, 16Y99, 17A01, 17A20, 17D25

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A comparition of finite difference methods in solving Schrodinger equation

Serpil Cıkıt[1], H. Halit Tali [2]

[1] *Halic University, Faculty of Arts and Sciences, 5.Levent Mah. 15 Temmuz Şehitler cad. No:14/12, Eyüpsultan/Istanbul, 34060, Turkey*

serpilcikit@halic.edu.tr

[2] *Halic University, Faculty of Arts and Sciences, 5.Levent Mah. 15 Temmuz Şehitler cad. No:14/12, Eyüpsultan/Istanbul, 34060, Turkey*

hasantali@halic.edu.tr

Abstract: In this study, a one-dimensional time-dependent Schrödinger equation is considered and some known finite difference methods are used to evaluate numerical solutions of Schrödinger equation. The solutions which are obtained by the application of different finite difference methods are found by means of a numerical algorithm. In order to compare the effectiveness of these techniques, results are given in tables and graphs.

Keywords: Schrödinger equation, finite difference methods, partial differential equation

2010 Mathematics Subject Classification: 35Q41, 35Q40, 81Q05

The Integer-antimagic Spectra of Disjoint Union of Hamiltonian Graphs

Uğur Odabaşı

Istanbul University-Cerrahpasa, Istanbul, Turkey

ugur.odabasi@iuc.edu.tr

Abstract: Let A be a non-trivial abelian group. A connected simple graph $G = (V, E)$ is A-*antimagic*, if there exists an edge labeling $f : E(G) \to A\backslash\{0\}$ such that the induced vertex labeling $f^+(v) = \sum_{\{u,v\}\in E(G)} f(\{u, v\})$ is a one-to-one map. The *integer-antimagic spectrum* of a graph G is the set $\mathrm{IAM}(G) = \{k : G \text{ is } {}_k\text{-antimagic and } k \geq 2\}$. In this work, the integer-antimagic spectra for disjoint union of Hamiltonian graphs has been determined.

Keywords: Hamiltonian graphs, Graph labeling, Group-antimagic labeling

2010 Mathematics Subject Classification: 05C78

Dispersion of SH surface waves in an inhomogeneous covered half-space with initial stresses

Mesut Negin[1]

[1] *Department of Civil Engineering, Bahcesehir University, Istanbul, Turkey*

mesut.negin@eng.bau.edu.tr

Abstract: The dispersive behavior of SH surface waves propagating in an elastic inhomogeneous half-space substrate covered by an elastic layer of finite thickness under the effect of initial stresses has been investigated. Density of the material of the half-space are assumed to have linear variation. Classical linearized theory of elastic waves in initially stressed bodies for small deformations is used and the well-known WKB high-frequency asymptotic technique is applied for the theoretical derivations. Numerical results regarding the effect of material inhomogeneity on wave propagation velocity are presented and discussed for a geophysical example. It has been observed that the inhomogeneity play an important role for the propagation of the SH surface wave.

Keywords: surface wave, dispersion, inhomogeneity, initial stresses, WKB

2010 Mathematics Subject Classification: 35L05 , 76B15, 74E05

On Differential Inequalities of Fractional Integro-Differential Equations via Upper and Lower Solutions

Ali Yakar[1], Hadi Kutlay[2]

[1] Department of Mathematics , Tokat Gaziosmanpaşa University, Tokat, Turkey

aliyakar@gmail.com

[2] Department of Mathematics , Tokat Gaziosmanpaşa University, Tokat, Turkey

hkutlay.tokat@gmail.com

Abstract: In this study, we consider the following nonlinear Caputo fractional integro-differential equation

$$(1) \qquad {}^{C}D^{q_1}u(t) = F(t, u(t), I^{q_2}u(t)) + G(t, u(t), I^{q_3}u(t))$$

with boundary condition

$$(2) \qquad g(u(0), u(T)) = 0$$

where $F, G \in C[J \times \mathbb{R} \times \mathbb{R}_+, \mathbb{R}]$, $g \in C[\mathbb{R}^2, \mathbb{R}]$, $u \in C^1[J, \mathbb{R}]$, $J = [0, T]$ and $0 < q_3 < q_2 < q_1 < 1$.

Using the method of lower and upper solutions in terms of specified coupled lower and upper solutions of the Caputo fractional integro-differential equation (1)-(2), we investigate some comparison results. These theorems are required because they provide the framework for improving the monotone iterative technique for solving boundary value differential equations.

Keywords: Fractional Integro-differential equation, Differential inequalities, Upper and Lower solutions, Boundary Value Problem.

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On young-type inequalities of measurable operators

Myrzagali Ospanov [1], Saule Burgumbayeva[1]

[1] *Faculty of Mechanics and Mathematics, L. N. Gumilyov Eurasian National University, Astana, Kazakhstan*

myrzan66@mail.ru, saulenai.ast@gmail.com

Abstract: Let $(,\tau)$ be a semi-finite von Neumann algebra, $L_0()$ be the set of all τ-measurable operators and $\mu_t(x)$ be the generalized singular number of $x \in L_0()$. We proved that if $1 < p,q < \infty$, $\frac{1}{p}+\frac{1}{q}=1$, $r \geq 2\min\{\frac{1}{p},\ \frac{1}{q}\}$ and $x,\ y \in L_0()$, then the Young type inequality $\mu_t(|xy^*|^r) \leq \mu_t(\frac{1}{p}|x|^{pr} + \frac{1}{q}|y|^{qr})$, for all $t > 0$ holds.

In [1], Choudhury and Sivakumar proved that if $k \in \mathbb{N}$ and $p,\ q \notin [2,2k)$, then

$$s_j(|xy^*|)^{\frac{1}{k}} \leq s_j(\frac{1}{p}|x|^{\frac{p}{k}} + \frac{1}{q}|y|^{\frac{q}{k}}), \qquad j=1,2,\cdots,n, \tag{1}$$

where $\mathbb{M}_n$ is the set of all $n\times n$ complex matrices, $s_j(a)$ $(j=1,2,\cdots,n)$ is singular value of $a \in \mathbb{M}_n$. The aim of this talk is further to prove generalizations of (1) in the context of von Neumann algebras. We give that

Theorem 1. Let $x,\ y \in L_0(\mathcal{M})$. Suppose $1 < p,q <$ such that $\frac{1}{p}+\frac{1}{q}=1$. If $r \geq 2\min\{\frac{1}{p},\ \frac{1}{q}\}$, then

$$\mu_t(|xy^*|^r) \leq \mu_t(\frac{1}{p}|x|^{pr} + \frac{1}{q}|y|^{qr}), \quad t > 0.$$

Acknowledgments

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Hermite-Hadamard and Bullen Type Inequalities via Holder-Iscan and Improved Power-Mean Inequality

Fatih Yetgin[1], Yeter Erdas Koc[2]

[1] *Department of Mathematics, Gebze Technical University, Kocaeli, Turkey*

fyetgin@gtu.edu.tr

[2] *Department of Mathematics Ordu Unuversity, Ordu, Turkey*

yeterrerdass@gmail.com

Abstract: This study deal with improvment of some general inequalities which consist of the Hermite-Hadamard and Bullen type. To do so, a general identity for differentiable functions, the Hoder432 Iscan inequality and improved power-mean integral inequality that are given in [1] and [2], respectively, were used. Finally, the obtained results were compared with the previous ones.

Keywords: Convex function, Hermite-Hadamard's inequality, Holder-Iscan inequality, Improved power mean inequality

2010 Mathematics Subject Classification: 26A51, 26D15

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Asymptotic Behavior of Solutions of Sum-Difference Equations

Hakan Adıgüzel[1], Engin Can[1]

[1] *Faculty of Technology, Sakarya University of Applied Sciences, Turkey.*

hadiguzel@subu.edu.tr
ecan@subu.edu.tr

Abstract: In this study, we present an investigation of the asymptotic behavior of solutions of sum-difference equations. Based on beta function and some mathematical inequalities, we have obtained our results. The obtained results can apply to some fractional type difference equations as well. Finally, we present an example to illustrate the validity of the theoretical results.

Keywords: asymptotic behavior, oscillation, nonoscillation, difference equation

2010 Mathematics Subject Classification: 39A05, 39A21

Interactive modeling and visualization of scattered data interpolation surfaces using quartic triangular Bézier patches

Krassimira Vlachkova[1], Krum Radev[1]

[1] *Faculty of Mathematics and Informatics, Sofia University "St. Kliment Ohridski", Sofia, Bulgaria*

krassivl@fmi.uni-sofia.bg
kvradev@uni-sofia.bg

Abstract: We present two new program packages for modeling and visualization of scattered data interpolation surfaces based on quartic interpolation curve networks and quartic triangular Bézier patches. Our work and contributions are in the field of experimental algorithmics and algorithm engineering. We have chosen the open-source data visualization libraries `Plotly.js` and `Three.js` as our main implementation and visualization tools. This choice ensures the platform independency of our packages and their direct use without restrictions. The packages can be used for experiments with user's data sets since they work with the host file system. The latter allows wide testing, modeling, and editing of the resulting interpolation surfaces. We experimented extensively with our packages using data of increasing complexity. The experimental results are presented and analyzed.

Keywords: scattered data interpolation, curve network, minimum norm network, Bézier patch, Plotly.js, Three.js

2020 Mathematics Subject Classification: 65D17, 68U05, 68U07

On the inverse problem for the Burgers equation with integral overdetermination condition

Madi Yergaliyev[1], Muvasharkhan Jenaliyev[2]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan, and al-Farabi KazNU, Kazakhstan*

ergaliev@math.kz

[2] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

muvasharkhan@gmail.com

Abstract: In this work we consider the inverse problem for Burgers equation in the domain $Q_{xt} = \{x, t \mid 0 < x < t,\ t_0 < t < T < \infty,\ t_0 > 0\}$: to find a couple of functions $\{u(x,t),\ \lambda(t)\}$ from the conditions

(1) $$\partial_t u + u\partial_x u - \nu\partial_x^2 u = \lambda(t)f(x),\ \ (x,t) \in Q_{xt},$$

(2) $$\partial_x^j u(0,t) = \partial_x^j u(t,t),\ \ j = 0,\ 1;\ t \in (t_0, T),$$

(3) $$u(x, t_0) = 0,\ \ x \in (0, t_0),$$

(4) $$\int_0^t u(x,t)dx = E(t),\ \ t \in [t_0, T],$$

where $\nu = const > 0$ is a given constant and functions $f(x)$, $E(t)$ satisfy the conditions

(5) $$\begin{cases} f(x) \in L^\infty(t_0, T; L^\infty(0,t)),\ \bar{f}(t) \equiv \int_0^t f(x)dx \neq 0,\ \forall t \in [t_0, T], \\ E(t) \in W^{1,\infty}(t_0, T). \end{cases}$$

Keywords: Inverse problem, Burgers equation, Galerkin method.

2010 Mathematics Subject Classification: 35K55, 35R30, 65M60

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Air flow effects on two bodies lifting off in close ground proximity

Samire Yazar

Department of Mathematics, Gebze Technical University, Gebze/Kocaeli, 41400, Turkey

syazar@gtu.edu.tr

Abstract: In the present setting, we aim to understand the ground effect and lifting off phenomena coupled with the sudden horizontal air flow and free motion of a pile of two bodies or two bodies initially positioned separately. One of the configuration of interest here is near ground in returning case. This case is in the sense of continued departure then returning back to the ground. A case for escaping infinity is also examined depending on the initial configuration as well as an oscillatory behaviour of the body angle that is in contrast to [2]. Ground effects are found dominant during lift-off by declining the lift-off force due to the presented air flow or in effect seem to be declining in escaping infinity and dominant again in returning to the ground. The report presents a survey of the variety of interaction behaviour between air flow and two bodies that is governed by a nonlinear evolutionary system. This evolutionary system has been studied by asymptotic analysis leading on to a comparison with direct numerical work.

Keywords: aerodynamics, take-off, lift-off, multi-body migration, nonlinear dynamical systems, modelling, computation

2010 Mathematics Subject Classification: 35Q35, 70E18, 74F10, 35Q35, 76Bxx

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On the numerical study for third order partial differential equation with nonloclal conditions with nonlocal perturbation of boundary conditions

Allaberen Ashyralyev[1], Kheireddine Belakroum[2]

[1] *Department of Mathematics, Bahcesehir University, 34353, Istanbul, Turkey*
Peoples' Friendship University of Russia (RUDN University)
6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation
Institute of Mathematics and Mathematical Modeling, 050010, Almaty, Kazakhstan
allaberen.ashyralyev@eng.bau.edu.tr
[2] *Department of Mathematics, Brothers Mentouri University, 25000, Constantine, Algeria*

kheireddinebelakroum@gmail.com

Abstract: The current study presents a numerical study for a third order partial differential equation with nonclocal conditions, We present first and second order accuracy differences schemes for nonlocal boundary value problem for a third order partial differential equation. Results of numerical experiments are provided.

Keywords: Numerical solutions,Third order partial differential equation, Difference scheme.

2010 Mathematics Subject Classification: 35G15, 47A62.

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Approximation Solution of pseudoparabolic problem

Dounia Belakroum

Department of Mathematics, Frères Mentouri University,
Constantine, Algeria
belakroum05@yahoo.fr

Abstract: The purpose of this work is to introduce a numerical approach for the approximate solution of semilinear pseudoparabolic problem. The applyed method is based on approximating functions and their derivatives by using the Whittaker cardinal function in order to determine the approximate solutions and consequently the partial differential equation transformed to an algebraic equation system.

Keywords: Whittaker cardinal function, Pseudoparabolic problem, approximate solution.

2010 Mathematics Subject Classification: 35G15, 47A62.

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Two-phase tasks thermal conductivity with boundary conditions of the Sturm type

Mahmud A.Sadybekov[1,2], Umbetkul K.Koilyshov[1,2]

[1] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan.*

[2] *Al-Farabi Kazakh National University, Almaty, Kazakhstan.*

sadybekov@math.kz *koylyshov@mail.ru*

Abstract: An initial-boundary value problem for the heat equation with a piecewise constant coefficient is considered

$$Lu \equiv \left\{ \begin{array}{ll} u_t - k_1^2 u_{xx}, & 0 < x < x_0 \\ u_t - k_2^2 u_{xx}, & x_0 < x < l \end{array} \right\} = f(x,t), \tag{1}$$

in the region $\Omega = \{(x,t) : 0 < x < l, 0 < t < T\}$, with the initial condition

$$u(x,0) = \varphi(x), \quad 0 \le x \le l, \tag{2}$$

boundary conditions of the form

$$\left\{ \begin{array}{l} \alpha_1 u_x(0,t) + \beta_1 u(0,t) = 0, \\ \alpha_2 u_x(l,t) + \beta_2 u(l,t) = 0, \end{array} \right. \quad 0 \le t \le T, \tag{3}$$

and pairing conditions

$$u(x_0 - 0, t) = u(x_0 + 0, t), \quad 0 \le t \le T \tag{4}$$

$$k_1 u_x(x_0 - 0, t) = k_2 u_x(x_0 + 0, t), \quad 0 \le t \le T \tag{5}$$

The coefficients $k_i, \alpha_i, \beta_i, (i = 1,2)$ are real numbers. In addition $|\alpha_1| + |\beta_1| > 0, |\alpha_2| + |\beta_2| > 0$. We use the following notation for individual parts of the region Ω :

$$\Omega_0 = \{(x,t) : 0 < x < x_0, 0 < t < T\}, \quad \Omega_l = \{(x,t) : x_0 < x < l, 0 < t < T\}$$

The following theorems are proved.

Theorem 1. For any function $\varphi(x) \in C[0,l] \cap C^2[0,x_0] \cap C^2[x_0,l]$ and $f(x,t) \in C(\bar{\Omega}) \cap C^{2,1}\left(\overline{\Omega_0}\right) \cap C^{2,1}\left(\overline{\Omega_l}\right)$, satisfying boundary conditions (3) and conjugation conditions (4)-(5), there is a unique classical solution $u(x,t) \in C(\bar{\Omega}) \cap C^{2,1}\left(\overline{\Omega_0}\right) \cap C^{2,1}\left(\overline{\Omega_l}\right)$ of problem (1)-(5).

Theorem 2. For any function $\varphi(x) \in W_2^1(0,l) \cap W_2^2(0,x_0) \cap W_2^2(x_0,l)$, satisfying boundary conditions (3) and conjugation conditions (4)-(5), and any $f(x,t) \in L_2(\Omega)$ there is a unique generalized solution $u(x,t) \in W_2^{2,1}(\Omega)$ of the problem (1)-(5). This solution is a strong solution to problem (1)-(5) and satisfies the estimate

$$\|u\|^2_{L_2(\Omega)} + \|u\|^2_{W_2^{2,1}(\Omega_0)} + \|u\|^2_{W_2^{2,1}(\Omega_l)} \le C\left\{\|f\|^2_{L_2(\Omega)} + \|\varphi\|^2_{W_2^2(0,x_0)} + \|\varphi\|^2_{W_2^2(x_0,l)}\right\}.$$

Keywords: Conjugation problem, heat equations, discontinuous coefficients, Riesz basis, eigenvalues, eigenfunctions.

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On a new difference scheme for a multidimensional inverse source problem backward in time

Ali Ugur Sazaklioglu

University of Turkish Aeronautical Association, 06790, Ankara, Turkey

ausazaklioglu@thk.edu.tr

Abstract: Let $\Omega_n \subset \mathbb{R}^n$, $n \geq 1$, be an open bounded domain such that $\Omega_n = \left\{ x = (x_1, x_2, \cdots, x_n) : 0 < x_j < l,\ j = \overline{1,n} \right\}$,with a smooth boundary $\partial\Omega_n$ resulting that $\overline{\Omega_n} = \Omega_n \cup \partial\Omega_n$. We also denote that $S_n = (0,1) \times \Omega_n$ and $\Gamma_n = [0,1] \times \partial\Omega_n$. In this study, a stable difference scheme is proposed for the numerical solution of the inverse source problem governed by a linear multidimensional parabolic equation backward in time

$$(1) \qquad \begin{cases} u_t\left(t,x\right) + \sum_{r=1}^{\mathfrak{n}} a_r\left(x\right) u_{x_r x_r}\left(t,x\right) - \sigma u\left(t,x\right) & \text{in } S_{\mathfrak{n}}, \\ \qquad = p\left(x\right) + f\left(t,x\right),\ \sigma \geq 0, & \\ u\left(1,x\right) = \psi\left(x\right),\ u\left(t_g,x\right) = \varphi\left(x\right),\ t_g \in [0,1), & x \in \overline{\Omega_{\mathfrak{n}}}, \\ u\left(t,x\right) = 0, & \text{on } \Gamma_{\mathfrak{n}}, \end{cases}$$

where $a_r\left(x\right) > 0$, $f\left(t,x\right)$, $\varphi\left(x\right)$ and $\psi\left(x\right)$ are given sufficiently smooth functions, and $\left(u\left(t,x\right), p\left(x\right)\right)$ is the solution pair to be determined.

In paper [1], a first order of accuracy stable difference scheme was constructed for problem (1), and the proposed difference scheme was analyzed theoretically and numerically. In this study, a more accurate stable difference scheme is proposed for the numerical solution of this problem via the operator approach. Another aim of the present study is to give a mathematical and numerical analysis for the method proposed. Indeed, the proposed method is tested on a model problem and an elaborate numerical analysis is carried out. Note that some techniques used in this study were discussed in paper [2].

Keywords: Inverse source problem, numerical solution, stability

2010 Mathematics Subject Classification: 65J22, 65M32, 65N21

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Inverse problem of restoring the right-hand side of the time-fractional diffusion equation

Murat Sultanov[1], Vladimir Misilov[2], Burkhan Kalimbetov[3],

Rauan Turebekov[3], Yerkebulan Nurlanuly[1]

[1] *Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkestan, Kazakhstan*
murat.sultanov@ayu.edu.kz, yerkebulan.nurlanuly@ayu.edu.kz

[2] *Krasovskii Institute of Mathematics and Mechanics Ural Branch of RAS, Ekaterinburg, Russia, and Ural Federal University, Ekaterinburg, Russia v.e.misilov@urfu.ru*

[3] *Mukhtar Auezov South Kazakhstan University, Shymkent, Kazakhstan*
bkalimbetov@mail.ru, rauan.turebekov@mail.ru

Abstract: The following subdiffusion equation is considered:

$$\frac{\partial^\alpha u}{\partial t^\alpha} + \mathcal{L}u = f(\mathbf{x})q(\mathbf{x},t), \quad \mathbf{x} \in \Omega, 0 < t \le T,$$

$$u(\mathbf{x},t) = 0,\ \mathbf{x} \in \partial\Omega,\ 0 < t \le T, \quad u(\mathbf{x},0) = u_0(\mathbf{x}), \mathbf{x} \in \overline{\Omega}.$$

Here, $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \overline{\Omega} = \prod_{i=1}^{n} [a_i, b_i],\ 0 < \alpha < 1,\ q(x,t) \neq 0$ and $\mathcal{L}$ is an elliptic operator

$$\mathcal{L}u = -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(k_i(\mathbf{x},t)\frac{\partial u}{\partial x_i}\right), x_i \in (a_i, b_i), 0 < t \le T.$$

The fractional Caputo derivative with order α is defined as

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial u(x,s)}{\partial s}(t-s)^{-\alpha}, \mathbf{x} \in \Omega, 0 < t \le T.$$

In the present work, we study the inverse problem of restoring the right-hand part $f(\mathbf{x})$. Additional information for the inverse problem are given in the form $u(\mathbf{x},t) = \varphi(\mathbf{x}), \mathbf{x} \in \overline{\Omega}$. For the numerical solution of the inverse problem, the iterative method of conjugate gradients is used, while at each iteration the direct problem is solved by the method of finite differences using a purely implicit difference scheme [1]. This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan (project AP09258836).

Keywords: inverse problem, fractional derivative, iterative algorithm, implicit difference scheme, conjugate gradient method

2010 Mathematics Subject Classification: 35B65; 35R11

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Numerical solutions of nonlocal problems for inverse hyperbolic-parabolic equation with unknown source

Maksat Ashyraliyev[1], Maral Ashyralyyeva[2]

[1] *Mälardalen University, Sweden*

maksat.ashyralyyev@mdu.se

[2] *Magtymguly Turkmen State University, Turkmenistan*

ashyrmaral2010@mail.ru

Abstract: We present a numerical study of source identification problem for hyperbolic-parabolic equation with nonlocal boundary conditions:

(1)
$$u_{tt}(t,x) - \big(a(x)u_x(t,x)\big)_x + \delta u(t,x) = p(x) + f(t,x),\ 0 < t < 1,\ 0 < x < \ell,$$

(2)
$$u_t(t,x) - \big(a(x)u_x(t,x)\big)_x + \delta u(t,x) = p(x) + g(t,x),\ -1 < t < 0,\ 0 < x < \ell,$$

(3)
$$u(t,0) = u(t,\ell),\ u_x(t,0) = u_x(t,\ell),\ -1 \le t \le 1,$$

(4)
$$u(-1,x) = \varphi(x),\ u(0^+,x) = u(0^-,x),\ u_t(0^+,x) = u_t(0^-,x),\ 0 \le x \le \ell,$$

coupled with one of the following two overdetermination conditions:

(5)
$$u(1,x) = \psi(x),\ 0 \le x \le \ell,$$

(6)
$$\int_0^1 u(\tau,x)d\tau = \psi(x),\ 0 \le x \le \ell,$$

where a, f, g, φ, ψ and given smooth functions, δ is given positive constant and $p(x)$ is an unknown source. Under compatibility conditions and $a(x) \ge a > 0$, $x \in (0,\ell)$, $a(\ell) = a(0)$, source identification problem (1)-(4) coupled with (3) or (6) has a unique solution $\{u,p\}$. We construct the first and second order of accuracy difference schemes for approximate solutions of source identification problems under consideration and discuss a numerical procedure for implementation of these schemes. The problem of nonlocality is resolved by using shooting approach. Numerical examples are included.

Keywords: Source identification problem, hyperbolic-parabolic equation, nonlocal problem, difference scheme

2010 Mathematics Subject Classification: 65M06, 65M32, 35M10, 35R30

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On the second order elliptic system

Kordan Ospanov[1]

[1] *L.N. Gumilyov Eurasian national university, Astana, Kazakhstan*

kordan.ospanov@gmail.com

Abstract: The report discusses sufficient conditions for unique solvability of the elliptic system of two real partial equations

$$\Delta U + A\,(x,y)\,U_x + B\,(x,y)\,U_y + C\,(x,y)\,U = F(x,y), \tag{1}$$

where $A,\ \ B,\ \ C$ are (2×2) matrices, in general, with unbounded smooth elements. Such systems are intensively studied because of their applications in stochastic analysis and financial mathematics. We are interested in the case when the growth of coefficients A and B at infinity are not controlled by the potential C. The issues on regularity of solutions of system (1) and its generalizations, when A and B have linear growth at infinity are investigated by S. Fornaro and L. Lorenzi (2007), M. Hieber and O. Sawada (2005), and when they have growth of order $|x|\ln|x|$ was studied by G. Metafune, D. Pallara and V. Vespri (2005), M. Hieber, L. Lorenzi et al. (2009). The issue remains open: is it possible to find a more general class of correct elliptic systems of the form (1) when the intermediate coefficients A and B have a different order of growth than $|x|\ln|x|$ and are not controlled by the potential C.

The unboundedness of the domain and coefficients, as well as the fundamental difference of system (1) from Schrödinger-type equations, determine the complexity of the studied problem.

We also establish an estimate of the form

$$\|U_{xx}\|_2 + \|U_{yy}\|_2 + \|AU_x\|_2 + \|BU_y\|_2 + \|CU\|_2 \ \le c\|F\|_2$$

for solution U of system (1).

Keywords: elliptic system, unbounded coefficient, generalized solution, coercive solvability

2010 Mathematics Subject Classification: 35B65

Comparison principle for the nonlinear time-space fractional diffusion equation

Meiirkhan Borikhanov

Khoja Akhmet Yassawi International Kazakh–Turkish University, Kazakhstan and

Institute of Mathematics and Mathematical Modeling, Kazakhstan

meiirkhan.borikhanov@ayu.edu.kz, borikhanov@math.kz

Abstract: In this paper, we study the Cauchy-Dirichlet problem to the nonlocal nonlinear diffusion equation with polynomial nonlinearities

$$(1) \quad \begin{cases} \mathcal{D}^{\alpha}_{0|t}u + (-\Delta)^s_p u = \gamma|u|^{m-1}u + \mu|u|^{q-2}u, \ \ (x,t) \in \Omega \times (0,T), \\ u(x,t) = 0, \, x \in \mathbb{R}^N \setminus \Omega, \, t \in (0,T), \\ u(x,0) = u_0(x), \, x \in \Omega, \end{cases}$$

involving time-fractional Caputo derivative of order $\alpha \in (0,1)$ (see [1, p. 91])

$$\mathcal{D}^{\alpha}_{0|t}u(t) = I^{1-\alpha}_{0|t}\frac{d}{dt}u(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha}u'(s)\,ds, \ \ \forall t \in (0,T]$$

and space-fractional p-Laplacian operator (see [2, Lemma 5.1])

$$(-\Delta)^s_p u(x) = C_{N,s,p}\,\mathrm{P.V.}\int_{\mathbb{R}^N}\frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+sp}}dy,$$

where $s \in (0,1), p \geq 2$ and

$$C_{N,s,p} = \frac{sp2^{2s-2}}{\pi^{\frac{N-1}{2}}}\frac{\Gamma(\frac{N+sp}{2})}{\Gamma(\frac{p+1}{2})\Gamma(1-s)}$$

is a normalization constant and "P.V." is an abbreviation for "in the principal value sense".

For various sets of γ, μ, m and q, we present a simple proof of the comparison principle for the problem under consideration using just algebraic relations. The comparison principle is used to classify the blow-up phenomenona, and the existence of global weak solutions.

Keywords: quasilinear parabolic equation, time-space fractional derivative, comparison principle, blow-up and global solution

2010 Mathematics Subject Classification: 35R11, 35A01, 35B51, 35K55

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On Fredgholm property and on the index of the generalized Neumann problem

Bakytbek Koshanov[1], Arai Kuntuarova[2]

[1] *Al-Farabi Kazakh National University , Kazakhstan*

koshanov@math.kz

[2] *Abai Kazakh National Pedagogical University, Kazakhstan*

araika.14.89@mail.ru

Abstract: In simply connected region D in the plane bounded by the simple smooth contour Γ, we consider the elliptic equation
(1)

$$\sum_{r=0}^{2l} a_r \frac{\partial^{2l} u}{\partial x^{2l-r} \partial y^r} + \sum_{0 \le r \le k \le 2l-1} a_{rk}(x,y) \frac{\partial^k u}{\partial x^{k-r} \partial y^r} = f(x,y), (x,y) \in D$$

with real coefficients $a_r \in \mathbb{R}$ and $a_{rk} \in C^\mu(\overline{D})$, $\Gamma = \partial D \in C^{2l,\mu}$, $0 < \mu < 1$.
Problem S. The generalized Neumann problem consists in finding the solution $u(x,y)$ of equation (1) in the domain D by boundary conditions

(2) $$\left.\frac{\partial^{k_j - 1} u}{\partial n^{k_j - 1}}\right|_\Gamma = g_j, \quad j = 1, \ldots, l,$$

where $1 \le k_1 < k_2 < \ldots < k_l \le 2l$ and $n = n_1 + in_2 -$ the unit external normal. In [1], problem (1), (2) was investigated for $a_{kr} \neq 0$ and $f \neq 0$ in the space of functions $C_a^{2l-1,\mu}(\overline{D})$.

The report established: a sufficient condition for the Fredholm property of problem (1), (2) in the space $C^{2l,\mu}(\overline{D})$; equivalence of the Fredholm condition of the problem to the complementarity condition (or Shapiro–Lopatinsky) [1]. A formula for the index of the problem ind S is calculated.

Keywords: high order elliptic equations, boundary value problem, normal derivatives, Fredholm solvability of the problem, formula for problem index

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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The beginning of the economic and mathematical modeling in Central Asia

Meyram Issin[1], Anar Romazanova[2]

[1] *L.N. Gumilyov Eurasian National University, Kazakhstan*
issin.meyram@gmail.com
[2] *L.N. Gumilyov Eurasian National University, Kazakhstan*
romazanova7070@mail.ru

Abstract: The article studies the problem of the interaction of mathematical methods and problems with market content in the works of scientists of medieval Central Asia. The purpose of the article was to find out where the economic and mathematical methods of applied mathematics begin in Central Asia and what the periods of their development are [1].

Historical and narrative methods were used to study the problem. Scientists of medieval Central Asia used to solve market problems: the “triple rule” method, the “rule of five quantities” method, linear equation, geometric progression, a system of two linear equations with two unknowns, etc [2]. These methods, together with the methods of operations research, belong to economic and mathematical methods.

Keywords: economic and mathematical methods, medieval Central Asia, mathematicians, methods in operations research, Al-Khwarizmi’s treatise on algebra

2020 Mathematics Subject Classification: 91-10

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Maximal regularity and two-sided estimates for the approximation numbers of solutions of the nonlinear Sturm-Liouville equation with rapidly oscillating coefficients in $L_2(R)$

Mussakan Muratbekov[1], Madi Muratbekov[2]

[1] *M.Kh. Dulaty Taraz Regional University, Kazakhstan*

musahan_ m@mail.ru

[2] *Esil University, Kazakhstan*

mmuratbekov@kuef.kz

Abstract: This report is devoted to a theorem on the maximum regularity of solutions of the nonlinear Sturm-Liouville equation with greatly growing and rapidly oscillating potential in the space $L_2(\mathbb{R})\,(\mathbb{R} = (-\infty, \infty))$ and two-sided estimates of the Kolmogorov widths of the sets associated with solutions of the nonlinear Sturm-Liouville equation. As is known, the obtained estimates given the opportunity to choose approximation apparatus that guarantees the maximum possible error.

Throughout this note we mainly use techniques from our work [1].

Keywords: approximation numbers, Sturm-Liouville theory, s-numbers, Kolmogorov numbers, oscillating coefficients, maximal regularity

2010 Mathematics Subject Classification: 34B24, 34L30, 35B65, 47B06

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Boundedness of one class of integral operators

Askar Baiarystanov[1], Ryskul Oinarov[2]

[1,2] *L.N. Gumilyov Eurasian National University*

[1] *oskar_62@mail.ru*
[2] *o_ryskul@mail.ru*

Abstract: This report is devoted to establishing the boundedness from L_p to L_q for $1 < q < p < \infty$ of the integral operators

$$\mathcal{K}^+ f(x) = \int_0^x u(x)K(x,s)v(s)f(s)ds, \quad x > 0, \tag{1}$$

$$\mathcal{K}^- g(s) = \int_s^\infty v(s)K(x,s)u(x)g(x)dx, \quad s > 0, \tag{2}$$

with kernel $K(x,s) \geq 0$ for $x \geq s > 0$, when the kernel belongs to a class wider than the class $\mathcal{O}$ satisfying the condition: there exists a number $h \geq 1$ and $h^{-1}\big(K(x,t) + K(t,s)\big) \leq K(x,s) \leq \big(K(x,t) + K(t,s)\big)$ for $x \geq t \geq s > 0$. The problem of boundedness of the operators (1) and (2) with a kernel from the class $\mathcal{O}$ can be found in [1]. At present, criteria of the boundedness of the operators (1) and (2) with a kernel from the class $\mathcal{O}$ are applied in numerous works.

Keywords: integral operator, weight function, kernel, bounded.

2010 Mathematics Subject Classification: 26D10, 26D15.

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Generalized Gaussian function as a kernel for the support vector machine algorithm

Betul Hicdurmaz[1], Aysenur Erdem[2]

[1,2] *Department of Mathematics, Istanbul Medeniyet University, Istanbul, Turkey*

[1] *betulhicdurmaz@gmail.comn*

[2] *aysenurerdm@gmail.com*

Abstract: Design of a new kernel for Support Vector Machine (SVM) is studied in literature by many authors. For a comprehensive review of studies on SVM with new kernel design or kernel comparison, one can refer to [1].

In the present paper, a new generalization of the Gaussian function is presented and applied on SVM algorithm. Theoretical basis for being a kernel function is also investigated for the new generalized Gaussian function. Since Gaussian function is the most common kernel in SVM algorithms, an extended version of the classical Gaussian function can be preferred due to its better performance in many different cases.

The extended Gaussian function is analyzed numerically for its behaviour near zero point and infinity to prove that the extended Gaussian function satisfies the soft conditions for a SVM kernel. In the experimental analysis part, performance of the extended Gaussian function is analyzed on classical datasets as a SVM kernel for classification. High performing new established kernels are also considered for comparison with the new extended Gaussian kernel.

Keywords: Support vector machine, classification, Gaussian function, data science.

2010 Mathematics Subject Classification: 26A48, 62P69, 46F30

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Artificial intelligence training in applied mathematics

Maral Ashyralyyeva[1], Sulgun Garajayeva[1]
and Berdymyrat Myratlyyev[2]

[1] *Department of Applied Mathematics and Informatics Magtymguly Turkmen State University*

[2] *Department of Information Systems and Technology Magtymguly Turkmen State University*

ashyrmaral2010@mail.ru, sgarajaeva@mail.ru, Myratly@gmail.com

Abstract: Artificial intelligence (AI) is the science and technology of creating intelligent machines, especially intelligent computer programs; the property of intelligent systems to perform creative functions that are traditionally considered the prerogative of people.

AI is related to the similar task of using computers to understand human intelligence, but is not necessarily limited to biologically plausible methods.

One of the private definitions of intelligence, common to a person and a «machine», can be formulated as follows: «Intelligence is the ability of a system to create programs during self-learning to solve problems of a certain complexity class and solve these problems».

The logical approach to building AI systems is based on reasoning modeling. The theoretical basis is logic. The logical approach can be illustrated by the use of the Prolog logical programming language and system for these purposes. Programs written in the Prolog language represent sets of facts and inference rules without a rigid specification of the algorithm as a sequence of actions leading to the desired result.

For the specialization of applied mathematics and computer science, a short course on artificial intelligence was developed, as well as laboratory work using the Prolog programming language.

Keywords: artificial intelligence,computer programs, prolog

Stability of Second Order Difference Scheme for the Time Delay Telegraph Equation

Allaberen Ashyralyev[1], Deniz Agirseven[2], Koray Turk[2]

[1] *Deparment of Mathematics, Bahçeşehir University, Turkey, and*

Peoples' Friendship University of Russia (RUDN University), Russian Federation, and

Institute of Mathematics and Mathematical Modeling, Kazakhstan

allaberen.ashyralyev@eng.bau.edu.tr

[2] *Deparment of Mathematics, Trakya University, Turkey*

denizagirseven@gmail.com, korayturk@trakya.edu.tr

Abstract: In this study, we investigate the second order of accuracy absolutely stable difference scheme for the approximate solution of the initial value problem for the time delay telegraph equation in a Hilbert space with self-adjoint positive definite operator. We prove the main theorem on stability of this difference scheme. In practice, we present absolutely stable difference schemes for the approximate solution of two initial value problems. Finally, to support the theoretical result, numerical example of the initial-boundary value problem for two dimensional delay telegraph equation with Dirichlet condition presented.

Keywords: Delay telegraph equation, difference scheme, stability

2010 Mathematics Subject Classification: 65M06, 65M12, 35G10

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Recognition of erythrocytes in SEM images

Oleg A. Semenov[1]

[1] *Ammosov North-East Federal University, Yakutsk, Russia*

semoleg1986@gmail.com

Abstract: The aim of the study is to develop a technique for machine recognition of erythrocyte shapes obtained using a scanning electron microscope (SEM). In the article Study of the morphology of erythrocytes in patients with cervical cancer: a technique for machine recognition of the shapes and sizes of erythrocytes SEM images, we described how to recognize erythrocytes using the OpenCV program. But due to the presence of strong noise in the images, it is necessary to use neural networks. Therefore, cellpose segmentation algorithms were used. Followed by obtaining the contours of objects in the form of vectors.1 Thus, the area and diameters of each erythrocyte.

With help 102 SEM images were processed with the help and characteristics of 29985 pieces of erythrocyte programs were determined. For the control group, the average diameter of erythrocytes in men is 7.309 μm, in women 7.380 μm, the detection of two-dimensional surface areas of erythrocytes in men and women, 41.260 μm2 and 41.150 μm2, respectively. It was determined that in women the range of the range of erythrocyte area is 28% than in men, and the range of the range of diameter is 11.30%. The difference between the medians of the diameters of the constituted is less than 2%. A comparative analysis by age and nationality was carried out, where differences in the diameter and area of erythrocytes were established. Erythrocytes of dysmorphic forms were found in the experimental groups.

The proposed processing of the results and their analysis using a computer image recognition program can be effectively used in the analysis of images of erythrocytes obtained by scanning electron microscopy. The presented automated program can recognize erythrocytes, calculate their number, diameters, areas, taking into account changes in morphology both in normal and in pathology.

Keywords: erythrocyte shapes, computer image recognition, erythrocytes

Numerical solution of the transfer equation by the discontinuous Galerkin method with Legendre multiwavelets

Dinara Tamabay[1], Nurlan Temirbekov[2], Bakytzhan Zhumagulov[2]

[1] *Al-Farabi Kazakh National University*

dtamabay@gmail.com

[2] *National Engineering Academy of the Republic of Kazakhstan*

temirbekov@rambler.ru

Abstract: Consider the model transfer equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0, \tag{1}$$

$$u\mid_{t=0} = u^0(x), u(t,0) = u_1(t), u(t,1) = u_2(t), \tag{2}$$

in the region $\Omega = [a,b]$, $v = const > 0$. We will look for an approximate solution of system (1) in the form $z_n(x,t) = \sum_{j=1}^{n} c_j(t)\psi_j(x)$, where $\psi_j(x)$ forms an orthonormal system in $X_n \in X$, where X is a Banach space.

After substituting approximate solution into (1), and integrating by parts, the orthogonality condition according to discontinuous Galerkin method will be presented as

$$\int_\Omega \frac{\partial z_n}{\partial t}\psi_k dx = \int_\Omega v div(z_n)\psi_k dx - \oint_{\partial\Omega} v\widehat{z}_n\psi_k n_x dl, k = 1, ..., n. \tag{3}$$

Denoting the mass matrix as B, for $M(Z_n)$ - take the right part, we get $B\frac{\partial z_n}{\partial t} = M(Z_n)$. A comparative analysis of the convergence and accuracy of the developed discontinuous Galerkin method and the finite difference method will be carried out.

The work was supported by grant funding from the Ministry of Education and Science of RK (IRN AR0856012).

Keywords: Transfer equation, projection methods, Legendre multiwavelets, numerical solution, discontinuous Galerkin method

2010 Mathematics Subject Classification: 65L60, 65M60, 65T60

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Galerkin-Wavelets Chebyshev to Solve Nonlinear Fredholm Integro-Differential Equations

Youcef Henka[1], Samir Lemita[2]

[1] *Laboratoire de Mathématiques Appliquées et de Modélisation. Université 8 Mai 1945 Guelma, B.P. 401 Guelma 24000, Algérie.*

henka.youcef.henka@gmail.com

[2] *Ecole Normale Supérieure de Ouargla. Cité Ennacer, Ouargla 30000, Algeria.*

lem.samir@gmail.com

Abstract: we focuse on a numerical study for nonlinear Fredholm integro-differential equations with initial conditions which have the following position:

$$\begin{cases} u(t) = f(t) + \displaystyle\int_0^1 K(t,s,u(s),u'(s),u''(s))ds, \\ u(0) = \alpha_1, \quad u'(0) = \alpha_2, \end{cases} \tag{1}$$

where $u(t), f(t) \in H^2([0,1])$, $K, \partial_t K, \partial_t^2 K \in C([0,1]^2 \times \mathbb{R}^3)$, and $\alpha_1, \alpha_2 \in \mathbb{R}$.

We apply the Galerkin method using Chebyshev wavelets to approximate the exact solution. This numerical method gives us a nonlinear algebraic system which would be solved by using the successive approximations method. Furthermore, we show the validity of the proposed method through some illustrative examples.

Keywords: Fredholm integro-differential equation, Nonlinear equation, Galerkin method, Chebyshev wavelets.

2010 Mathematics Subject Classification: 45J05, 65T60, 65N30, 34A34

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On the solvability of the problem of synthesizing distributed and boundary controls in the optimization of heat processes

Akylbek Kerimbekov

Kyrgyz-Russian Slavic University, Bishkek, Kyrgyz Republic

akl7@rambler.ru

Abstract: We study the solvability of the problem of synthesis of distributed and boundary controls in the optimization of heat processes described by partial integro-differential equations with the Fredholm integral operator. Functions of external and boundary actions are nonlinear with respect to the controls. For the Bellman functional, an integro-differential equation of a specific form is obtained and the structure of its solution is found, which allows this equation to be represented as a system of two equations of a simpler form. An algorithm for constructing a solution to the problem of synthesizing distributed and boundary controls is described, and a procedure for finding the controls as a function (functional) of the state of the process is described.

Synthesis problem: Consider a controlled thermal process described by the boundary value problem

(1)

$$v_t - Av = \lambda \int_0^T K(t,\tau)v(\tau,x)d\tau + f[t,x,u(t,x)], \quad x \in Q,$$

$$0 < t < T,$$

$$v(0,x) = \psi_1(x), \quad v_t(0,x) = \psi_2(x), \quad x \in Q,$$

$$\Gamma v(t,x) \equiv \sum_{i,k=1}^{n} a_{ik}(x) v_{x_k}(t,x) \cos(\nu, x_i) + a(x)v(t,x) = p[t,x,\vartheta(t,x)],$$

$$x \in \gamma, \quad 0 < t < T$$

Here A is an elliptic operator. $f[t,x,u(t,x)] \in H(Q_T)\,\forall$ distributed control $u(t,x) \in H(Q_T)$, $p[t,x,\vartheta(t,x)] \in H(\gamma_T)\,\forall$ bounded control $\vartheta(t,x) \in H(\gamma_T)$, $\gamma_T = \gamma \times (0,T)$. In the synthesis problem, it is required to find such controls $u^0(t,x) \in H(Q_T)$ and $\vartheta^0(t,x) \in H(\gamma_T)$, which minimize the integral quadratic functional

(2)

$$J[u(t,x),\vartheta(t,x)] = \int_Q \left[(v(T,x) - \xi_1(x))^2 + (v_t(T,x) - \xi_2(x))^2 \right] dx$$

$$+ \int_0^T \left(\alpha \int_Q M^2[t,x,u(t,x)]dx + \beta \int_\gamma N^2[t,x,\vartheta(t,x)]dx \right) dt, \quad \alpha,\beta > 0,$$

defined on the set of generalized solutions of the boundary value problem (1.1)-(1.5).Here $\xi_1(x) \in H(Q); \xi_2(x) \in H(Q); M[t,x,u(t,x)] \in H\left(Q_T\right) \forall u(t,x) \in H\left(Q_T\right), N[t,x,\vartheta(t,x)] \in H\left(\gamma_T\right) \forall \vartheta(t,x) \in H\left(\gamma_T\right)$ - given functions. In this case, the required controls $u^0(t,x)$ and $\vartheta^0(t,x)$ should be found as a function (functional) of the state of the controlled process, i.e., in the form

$$
\begin{aligned}
u^0(t,x) &= u\left[t,x,v(t,x),v_t(t,x)\right], \quad (t,x) \in Q_T, \\
\vartheta^0(t,x) &= \vartheta\left[t,x,v(t,x),v_t(t,x)\right], \quad (t,x) \in \gamma_T.
\end{aligned} \tag{3}
$$

Keywords: integro-differential equation, Fredholm operator, generalized solution, Bellman functional, Frechet differential, optimal control synthesis

2010 Mathematics Subject Classification: 49K20

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Study of the morphology of erythrocytes in patients with cervical cancer: a technique for machine recognition of the shapes and sizes of erythrocytes SEM images

S. N. Mamaeva[1], O. A. Semenov[1], I. V. Kononova[2], A. R. Ermolaev[3], V. V. Gerasimova[3], G. V.Maksimov[4,5]

[1] *Ammosov North-East Federal University, Yakutsk, Russia*

[2] *Yakut scientific center of complex medical problems, Yakutsk, Russia*

[3] *State Budgetary Institution of the Republic of Sakha, Yakutia, «Blood Transfusion Station» Yakutsk, Russia*

[4] *Lomonosov Moscow State University,Moscow, Russia*

[5] *Federal State Autonomous Educational Institution of Higher Education "National Research Technological University" MISiS " (Moscow, Russia)*

semoleg1986@gmail.com

Abstract: The aim of the study is to develop a technique for machine recognition of the shapes and sizes of scanning electron microscope (SEM) images of erythrocytes of patients with cervical cancer (CC) obtained on a SEM during radiation therapy (RT). The article considers the study of the morphology of erythrocytes of patients with CC who were underwent RT using machine image recognition methods based on images of erythrocytes obtained by SEM. It is shown that the use of this method already at the beginning of RT allows us to observe significant changes in the morphology of red blood cells of patients with cervical cancer, while the use the method of JMicroVision v1.2.7 states a slight difference of the diameters of red blood cells in the normal and pathological on the basis of the same experimental data. This paper presents for the first time a method for studying the morphology of erythrocytes in patients with cancer based on SEM images of erythrocytes. The presented automated program can recognize erythrocytes, calculate their number, diameters, areas, taking into account changes in morphology both in normal and pathological conditions, can work with a large amount of data with a significant acceleration of the calculation of parameters and with a greater approximation of these data to real ones, which significantly increases the efficiency of analysis in research, in the diagnosis and monitoring of therapy for CC and other types of diseases. In addition, these developments can help to increase the scope of scanning electron microscopes for solving a wide range of problems in medicine related to images in the field of diagnostics – medical introscopy.

Keywords: cervical cancer, erythrocytes, scanning electron microscope, machine recognition, radiation therapy.

On eigenvalues of the perturbed differentiation operator on a segment

Nurlan S. Imanbaev
Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
South Kazakhstan State Pedagogical University, Shymkent, Kazakhstan,
imanbaevnur@mail.ru

Abstract

In the functional space $W_2[-1,1]$, we consider the eigenvalue problem of the loaded differential operator

$$L_1 y = y'(t) + \lambda y(-1)\Phi(t) = \lambda y(t), \quad -1 \le t \le 1 \tag{1}$$

with the boundary value condition

$$y(-1) = y(1), \tag{2}$$

where Φ is a function with bounded variation and $\Phi(-1) = \Phi(1) = 1$, $\lambda \in \mathbb{C}$ is a spectral parameter.

It is required to find the complex values λ for which the operator equation (1) has non-zero solutions.

One of features of the considered problem, adjoint to (1)-(2), is the spectral problem with occurrence of the spectral parameter $\overline{\lambda}$ into the boundary value condition with the integral perturbation:

$$L_1^* v = v'(t) = \overline{\lambda} v(t), \quad -1 \le t \le 1 \tag{3}$$

with the boundary value condition

$$v(-1) - v(1) = -\overline{\lambda} \cdot \int_1^1 v(t)\Phi(t)dt, \tag{4}$$

where Φ is a function with bounded variation and $\Phi(-1) = \Phi(1) = 1$, $\overline{\lambda} \in \mathbb{C}$ is a spectral parameter.

Lemma 1. The characteristic determinant $\Delta_1(\lambda)$ of the spectral problems (1)-(2) and (3)-(4) is represented as follows

$$\Delta_1(\lambda) = e^{-\lambda} - e^{\lambda} + \lambda \cdot \int_{-1}^{1} e^{\lambda t}\Phi(t)dt \qquad (5).$$

Due to the formula (5), conclusions about eigenvalues of the first-order differential operators L_1 and L_1^* are established. We get the following result.

Theorem 1. If Φ is a function of bounded variation and $\Phi(-1) = \Phi(1) = 1$, then all zeros of the entire function $\Delta_1(\lambda)$, that is, all eigenvalues of differentiation operators L_1 and L_1^* belong to the strip $|Re\lambda| = |x| < k$, for some k, where $\lambda = x + iy$, and also form a countable set and have asymptotics $\lambda_n^1 = in\pi + O(1)$ as $n \to \infty$.

Keywords: Loaded differential operator, eigenvalue, boundary value condition.
2020 Mathematics Subject Classification Numbers: 34B09, 15A18, 34L20.

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On the solvability of the boundary problem in the optimization of the oscillational process

I. F. Abdyldaeva[1], G. M. Kenenbaeva[2], A Kerimbekov[3]

[1] *Kyrgyz-Turkish Manas University, Bishkek, Kyrgyz Republic*

efa-69@mail.ru, elmira.abdyldaeva@manas.edu.kg

[2] *Kyrgyz-Russian Academy of Education, Bishkek, Kyrgyz Republic*

gkenenbaeva@mail.ru

[3] *Kyrgyz-Russian Slavic University, Bishkek, Kyrgyz Republic*

akl7@rambler.ru

Abstract:: The problem of transferring a controlled oscillatory process from the initial state to the final state in a given time is researched. In the case when the function of the external source is non-linearly dependent on the control function. It is established that the desired control is defined as a solution to systems of nonlinear Fredholm I order integral equations. A sufficient condition for the solvability of a nonlinear optimization problem is found.

The article deals with the problem of nonlinear optimization, where the controlled process is described by the Cauchy problem:

$$\begin{gathered} {}_t + aV_x = g(x)p[u(t)], x \in (x_1, x_2), t \in [0, T] \\ V(0, x) = \varphi(x), x \in (x_1, x_2) \end{gathered} \tag{1}$$

and it is required to find the control function $u(t)$ so that the controlled process passes from the initial state $\varphi(x)$ to the final state $\xi(x)$ in a finite time T. Here a is a constant $g(x)$ and $p[u(t)]$ are given functions from the Hilbert space, so where the function $p[u(t)]$ is assumed to be monotonic with respect to the control function $u(t)$. It is established that the desired control is defined as a solution to systems of nonlinear Fredholm integral equations of the first kind. A sufficient condition for the solvability of a nonlinear optimization problem is found.

Keywords: oscillatory process, nonlinear optimization, nonlinear Fredholm integral equations of the first kind, boundary value problem, control

2010 Mathematics Subject Classification: 49K20

R-modified Crank-Nicolson difference schemes for the delay Schrodinger type partial differential equation

Allaberen Ashyralyev[1], Deniz Agirseven[2]

[1] Deparment of Mathematics, Bahçeşehir University, Turkey, and

Peoples' Friendship University of Russia (RUDN University), Russian Federation, and

Institute of Mathematics and Mathematical Modeling, Kazakhstan

allaberen.ashyralyev@eng.bau.edu.tr

[2] *Deparment of Mathematics, Trakya University, Turkey*

denizagirseven@trakya.edu.tr

Abstract: In present work, the stable difference schemes for the linear Schrödinger problem in a Hilbert space are studied. The second order of accuracy r-modified Crank- Nicolson difference schemes for the approximate solutions of this problem are presented. The stability of these difference schemes is established. In practice, the stability inequalities for the solutions of difference schemes for the three types of Schrödinger equations are obtained. A numerical method is proposed for solving one and two-dimensional delay Schrödinger equations.

Keywords: Delay Schrödinger equations, difference scheme, stability

2010 Mathematics Subject Classification: 39A27, 65M06, 65M12, 65N06

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Solving Volterre-Fredholm integral equations by natural cubic spline function

Sarfraz Salim[1], Karwan Jwamer[2], Rostam Saeed[3]

[1]*Department of Mathematics, College of Basic Education, University of Sulaimani, Sulaymaniyah, Iraq*

sarfraz.salim@univsul.edu.iq

[2] *Department of Mathematics, College of Science, University of Sulaimani, Sulaymaniyah, Iraq*

karwan.jwamer@univsul.edu.iq

[3]*Department of Mathematics, College of Science, Salahaddin University-Erbil, Iraq*

rostam.saeed@su.edu.krd

Abstract: This study determines the numerical solution of linear mixed Volterra-Fredholm integral equations of the second kind using the natural cubic spline function. The proposed method is based on employing the natural cubic spline function of the unknown function at an arbitrary point and using the integration method to turn the Volterra-Fredholm integral equation into a system of linear equations concerning to the unknown function. An approximate solution can be easily established by solving the given system. This is accomplished with the help of a computer program that runs on Python 3.9.

Keywords: Volterra Integral Equation, Fredholm Integral Equation, natural cubic spline

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Stability inequalities and numerical solution for neutral Volterra delay integro-differential equation

Hulya Acar[1], Ilhame Amirali[2]

[1] *Department of Mathematics, Faculty of Art and Sciences, Duzce University, 81620, Duzce, Turkey*

hulyaacar98@gmail.com

[2] *Department of Mathematics, Faculty of Art and Sciences, Duzce University, 81620, Duzce, Turkey*

ailhame@gmail.com

Abstract: The aim of the present paper is to introduce a new numerical method for solving neutral Volterra delay integro-differential equations (VDIDEs). We consider neutral VDIDEs of the following form:

$$u'(t) + a(t)u'(t-r) + b(t)u(t) + \int_{t-r}^{t} K(t,s)u(s)ds = f(t), \tag{1}$$

$$u(t) = \varphi(t), \quad -r \le t \le 0, \tag{2}$$

where $I = (0,T] = \cup_{p=1}^{m} I_p,\ I_p = \{t : r_{p-1} < t \le r_p\},\ 1 \le p \le m$ and $r_s = sr$, for $0 \le s \le m,\ \bar{I} = [0,T]$, $I_0 = [-r,0]$. $b(t) \ge 0,\ f(t),\ a(t)\ (t \in \bar{I}),\ \varphi(t)\ (t \in I_0)$ and $K(t,s)\ ((t,s) \in \bar{I} \times \bar{I})$ are given functions, r is a constant delay. Moreover, we will assume that $a,b,f \in C(\bar{I}),\ \varphi \in C^2(I_0)$ and $\frac{\partial^2 K}{\partial s^2} \in C(\bar{I}^2)$ $(s = 0,1,2)$.

Using the numerical quadrature formula, we create a finite difference scheme on a uniform mesh for the problem (1)-(2).The presented numerical method obtains a second-order convergence in discrete maximum norm. Furthermore, we illustrate the efficacy of the proposed method by constructing examples.

Keywords: Volterra delay integro-differential equation, finite difference scheme, uniform convergence, error estimate

2010 Mathematics Subject Classification: 45J05, 45D05, 65L20, 65L70, 65R20

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Weighted finite element method for one problem of the fracture mechanics

Viktor A. Rukavishnikov, Elena I. Rukavishnikova

Computing Center, Far-Eastern Branch,

Russian Academy of Sciences, Russian Federation

vark0102@mail.ru, rukavishnikova-55@mail.ru

Abstract: Mathematical models with a singularity play a decisive role in predicting the development of processes in fracture mechanics. Computational methods for problems of second-order linear elasticity equations with a singularity caused by the presence of re-entrant corners at the domain boundary are widely studied in the engineering literature. We have constructed a weighted finite element method (WFEM) based on the introduction of the definition of an R_ν-generalized solution for the system of Lamé equations [1-3]. This method allows finding a solution with high accuracy compared to the classical finite element method. At the same time, it retains the simplicity of the structure of the stiffness matrix, unlike other numerical methods of increased accuracy. In this contribution, we proved an estimate for the rate of convergence of an approximate solution by a weighted finite element method to an R_ν -generalized solution with a convergence rate of $O(h)$, i.e., without loss of precision. For effective use of WFEM, it is necessary to correctly set the control parameters for the calculations. An algorithm has been developed for determining the optimal WFEM parameters for finding an approximate solution to the Lamé system in domains with a boundary containing re-entrant corners α ranging from π to 2π. The general body of optimal parameters (BOP) for WFEM is determined, which can be used for computational domains with a re-entrant corner of any value from the specified range. At the same time, it is shown that the error in finding an approximate solution using parameters from the BOP deviates from the error for the best approximation by a small amount. This opens up possibilities for creating industrial codes based on WFEM.

Keywords: Boundary value problems with a singularity, weighted finite element method

2010 Mathematics Subject Classification: 65N30, 35Q30

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Unweighted FEM for solving Navier-Stokes squations in rotation form in domain with a reentrant corner

Viktor A. Rukavishnikov, Alexey V. Rukavishnikov

[1] *Computing Center of the Far Eastern Branch of the Russian Academy of Sciences, Khabarovsk, Russia*

vark0102@mail.ru
78321a@mail.ru

Abstract:

The L-stable second order single diagonal implicit Runge-Kutta method for finding a solution of nonlinear Navier-Stokes equations in rotation form:

$$\frac{\partial \mathbf{u}}{\partial t} - \mu\triangle\mathbf{u} + (\text{ curl }\mathbf{u})\times\mathbf{u} + \nabla P = \mathbf{f} \text{ in } \Omega\times(0,T), \quad \text{div }\mathbf{u} = 0 \text{ in } \Omega\times(0,T),$$

$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0 \quad \text{in} \quad \Omega, \quad \mathbf{u} = \mathbf{g} \quad \text{on} \quad \partial\Omega\times(0,T)$$

is applied.

A feature of the numerical solving of the problem, will be that the domain Ω is L-shaped — it has a reentrant corner on $\partial\Omega$ equal to $\frac{3\pi}{2}$ with a vertex O that coincides with the origin $(0,0)$. The error of the classical FEM is $O(h^{0.54})$ in the norm of a space $\mathbf{W}_2^1(\Omega)$, which is almost twice as poor in order in comparison with a one in a convex domain. We define an R_ν-generalized solution of a problem in weighted sets in each time step, construct a numerical method and established the first order of convergence with respect to the grid step h at each time $t_n = n\triangle t$ in the norm of a weighted space $\mathbf{W}_{2,\nu}^1(\Omega)$. The result is achieved without refining the mesh in the vicinity of the singularity point.

Keywords: Navier-Stokes equations, singularity, finite element method

2010 Mathematics Subject Classification: 35Q30; 35A20

Noncommutative symmetric space associated with a weight

Turdebek N. Bekjan

Department of Computational Mathematics and Database Analysis, Astana IT University, Astana 010000, Kazakhstan

t.nurlybekuly@astanait.edu.kz

Abstract: We define noncommutative symmetric spaces associated with a weight and study their properties. We also introduce noncommutative symmetric Hardy spaces and give their properties.

The aim of this talk is to define noncommutative quasi symmetric spaces and noncommutative quasi symmetric Hardy spaces associated with a weight, to extend the results in [2] to the case that E is a separable p-convex symmetric quasi Banach function space on $[0, a)$ for some $0 < p <$.

Keywords: faithful normal locally finite weight, noncommutative symmetric space, noncommutative symmetric Hardy space, semifinite von Neumann algebra.

2010 Mathematics Subject Classification: 46L52; 47L05

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Solving of system partial operator equations of the first order by the method of additional argument

Aijarkyn Ashirbaeva [1], **Gulkhan Sadykova** [2]

[1] Osh Technological University named after M.Adyshev, Osh, Kyrgyzstan
aijarkyn.osh@mail.ru

[2] Osh Technological University named after M.Adyshev, Osh, Kyrgyzstan
Gsadykova.osh@gmail.com

Abstract: Consider the system of partial operator equations

$$D[a_1(t,x,u_1,...,u_n)]u_1(t,x) = a_1(t,x,u_1,...,u_n) + F_1(t;u_1),$$

$$D[a_2(t,x,u_1,u_2)]u_2(t,x) = F_2(t,x;u_1,u_2),$$

(1) $$D[a_3(t,x,u_1,u_2,u_3)]u_3(t,x) = F_3(t,x;u_1,u_2,u_3),$$

$$... \quad ... \quad ... \quad ... \quad ... \quad ...$$

$$D[a_n(t,x,u_1,...,u_n)]u_1(t,x) = F_n(t,x;u_1...,u_n),$$

$(t,x) \in [0,T] \times [0,X]$ with initial conditions

$$u_1(0,x) = x,$$

(2) $$u_k(0,x) = \phi_{k-1}(x), x \in [0,X], k = 2..n,$$

where the given functions $a_i, i = 1..n$ are continuous and bounded with their derivatives; the differential operator $D[w] := \partial/\partial t + w\partial/\partial x$;

$W(t,x,...;u_1...,u_n)$ is an operator transforming functions $u_1...,u_n$ as whole to a function with arguments $t,x,....$

To solve the task the method of additional argument is applied.

(The task (1)-(2) by means of introduction of additional variable is reduced to a system of integral equations suitable for investigation).

This method was also used in [1], [2], [3] and other publications.

Keywords: partial operator equations, differential operator, methods of additional argument, a system of integral equations.

2010 Mathematics Subject Classification: 35F20, 35F25

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Local linear estimators of the conditional mode function under random right censoring

Oussama Bouanani[1],

[1] *University of Abdlhamid Ibn Badis Mostaganem, Algeria. Laboratory of Stochastic Models, Statistic and Applications, Dr. Taher Moulay University of Saida, Algeria*

oussama.bouanani@univ-mosta.dz

Abstract: In this paper, we present an alternative statistical approach to kernel estimation (L.C.E) which is the local linear estimation (L.L.E) of the conditional mode function, when the response variable is randomly censored and the explanatory variable takes value in an infinite-dimensional space. The main purpose of this work is to establish the asymptotic normality with explicit rates of the constructed estimator. Furthermore, the effectiveness of our results is illustrated by a simulated study.

Theorem 1. Under assumptions (H1)-(H6), we have

$$\left(\frac{n\phi_x(h_K)\,h_H^3(f^{x(2)}(\theta(x)))^2}{\sigma^2(x,\theta(x))}\right)^{1/2}\left(\widehat{\theta}(x)-\theta(x)\right)\xrightarrow{D}\mathcal{N}(0,1), \tag{1}$$

where

$$\sigma^2_\theta(x,\theta x))=\frac{M_2\,f^x(\theta(x))}{M_1^2\overline{G}(\theta(x))}\int(H^{(2)}(t))^2dt,$$

withe $M_j=K^j(1)-\int_{-1}^{1}(K^j(u))'\Psi_{(x)}(u)du,$ for $j=1,2.$

$\xrightarrow{D}$ denoting the convergence in distribution.

Keywords: Censored data; Local linear estimation; Functional data; Asymptotic normality; Conditional mode.

2010 Mathematics Subject Classification: 62G05, 62G20, 62G30, 62P30, ·62M10.

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Oscillation and comparison properties of non-classical boundary value problems

Oktay Sh. Mukhtarov[1,2], Kadriye Aydemir[3], Merve Yücel [4]

[1] *Department of Mathematics, Faculty of Arts and Science, Tokat Gaziosmanpaşa University, Turkey, omukhtarov@yahoo.com*

[2] *Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Azerbaijan, omukhtarov@yahoo.com*

[3] *Department of Mathematics, Faculty of Arts and Science, Amasya University, Turkey, kadriyeaydemr@gmail.com*

[4] *Department of Mathematics, Çorum Hitit University, Çorum, Turkey, merve.yucel@outlook.com.tr*

Abstract: The number and the location of the zeros of the solutions of linear and nonlinear differential equations are of utmost importance in the qualitative analysis of differential equations. In the recent years, there is growing interest in deriving a new oscillation and comparison results for various type non-classical differential eqautions. Non-classical boundary value problems for Sturm-Liouville type differential equations, which are defined on two and more disjoint intervals under additional interaction conditions at the common ends of these intervals, the so-called transmission problems appear frequently in solving of many actual problems in mathematical physics, such as in vibrating string, in vibrating folded membranes, in electromagnetic processes in ferromagnetic media with different dielectric properties, in hydraulic fracturing, in elastic multi-structures, in heat and mass transfer etc. In this study we will extend and generalize some classical oscillation and comparison results to two-interval Sturm-Liouville problems which consist of two interval differential equation, boundary conditions at the end points of the considered intervals and additional interaction conditions at the common end of these intervals.

Keywords: Non-classical boundary value problems, oscillation and comparison theorems.

2010 Mathematics Subject Classification: 34B24, 34L10, 34L20

Oscillation and comparison properties of non-classical boundary value problems

Oktay Sh. Mukhtarov[1,2], Kadriye Aydemir[3], Merve Yücel [4]

[1] *Department of Mathematics, Faculty of Arts and Science, Tokat Gaziosmanpaşa University, Turkey, omukhtarov@yahoo.com*

[2] *Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Azerbaijan, omukhtarov@yahoo.com*

[3] *Department of Mathematics, Faculty of Arts and Science, Amasya University, Turkey, kadriyeaydemr@gmail.com*

[4] *Department of Mathematics, Çorum Hitit University, Çorum, Turkey, merve.yucel@outlook.com.tr*

Abstract: The number and the location of the zeros of the solutions of linear and nonlinear differential equations are of utmost importance in the qualitative analysis of differential equations. In the recent years, there is growing interest in deriving a new oscillation and comparison results for various type non-classical differential eqautions. Non-classical boundary value problems for Sturm-Liouville type differential equations, which are defined on two and more disjoint intervals under additional interaction conditions at the common ends of these intervals, the so-called transmission problems appear frequently in solving of many actual problems in mathematical physics, such as in vibrating string, in vibrating folded membranes, in electromagnetic processes in ferromagnetic media with different dielectric properties, in hydraulic fracturing, in elastic multi-structures, in heat and mass transfer etc. In this study we will extend and generalize some classical oscillation and comparison results to two-interval Sturm-Liouville problems which consist of two interval differential equation, boundary conditions at the end points of the considered intervals and additional interaction conditions at the common end of these intervals.

Keywords: Non-classical boundary value problems, oscillation and comparison theorems.

2010 Mathematics Subject Classification: 34B24, 34L10, 34L20

Regularization of solutions of Volterra integral equations of the third kind inthe space of continuous functions

Obodoeva G.S.[1], **Toigonbaeva A.K.** [2]

[1] Osh State University, Osh, Kyrgyzstan
obodoevag@mail.ru

[2] Osh State University, Osh, Kyrgyzstan
atoigonnaeva@mail.ru

Abstract: Considered the Volterra integral equation of the third kind and a perturbed equation by a small parameter. Conditions to provide tending of solution of the second equation to solution of the first one as the small parameter tends to zero are found:

Consider the equations

(1) $$a(t)u(t) + \int_0^t k(t,s)u(s)ds = f(t), t \in [0,T], T > 0,$$

(2) $$(\varepsilon + a(t))v(t,\varepsilon) + \int_0^t k(t,s)v(s,\varepsilon))ds = f(t)$$

where $k(t,s)$, $f(t)$ and $a(t)$ are given functions, $0 < \varepsilon$ is a small parameter, $b(t)$ a non-decreasing continuous function; $b(t) \leq a(t) \leq pb(t), p > 1$ is a constant; $u(t)$ and $v(t,\varepsilon)$ are unknown functions.

The problem is to find conditions providing that the solution $v(t,\varepsilon)$ of equation (2) tends to the solution $u(t)$ of equation (1) as $\varepsilon \to 0$.

1) For any fixed $t \in [0,T]$: $k(t,s) \in L^q(0,T), q \geq 1; k(t,t) \in L^1(0,T)$; $(\forall t \in [0,T])(k(t,t) \geq 0)$.

2) $(\forall(0 \leq \eta \leq \tau \leq T))(\forall s \in [0,T])(|k(\tau,s) - k(\eta,s)| \leq q(s)(\int_\eta^\tau k(\zeta,\zeta)d\zeta + b(\tau) - b(\eta))$

where $(\forall t \in [0,T])(q(t) \geq 0); q(t) \in L^q(0,T)$.

Denoted: $C[0,T]$ is the space of continuous functions with the norm $\|u(t)\|_C = \max \|u(t)\|$; $C^\gamma_\varphi[0,T]$ is the space of functions $u(t)$ defined on $[0,T]$ and satisfying

(3) $$|u(t) - u(s)| \leq C|\varphi(t) - \varphi(s)|^\gamma, \varphi(t) = \int_0^t k(s,s)ds + a(t), t \in [0,T],$$

C is positive constant independent of $u(t)$ but not on t and s.

Keywords: Volterra integral equation, integral equation of the third kind regularzation.

2010 Mathematics Subject Classification: 45A05, 45D05, 47A52

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Splitting method for stability of delay-differential equations under perturbations

Zhumagul Zheentaeva[1], Sanavar Zhakieva[2]

[1] *Kyrgyz-Uzbek International University named after B. Sydykov*

jjk_kuu@mail.ru

[2] *Kyrgyz-Uzbek International University named after B. Sydykov*

jakieva84@mail.ru

Abstract: The problem to stabilize an object ($u(t)$) under permanently acting perturbations ($f(t)$) by means of feedback ($-pu(t)$) is considered. It is known that rocking of $u(t)$ for too large value of p instead of stabilization takes place because of irremediable delay (h) of control. The product $\Delta :=$ ph is an absolute constant. To detect boundaries of such constants Lyapunov functions were to be used earlier. The method of splitting the space of solutions [1] is proposed here.

Theorem 1. If $f(t) \in C(R_+), |f(t)| \leq f_0, \Delta < 1$ then a solution of the initial value problem

$$u'(t) = -pu(t-h) + f(t), t \in R_+; u(0) = 0 \tag{4}$$

is bounded: $|u(t)| \leq (1+2\Delta)f_0/p/(1-\Delta), t \in R_+)$.

Remark. $(\sin t)' \equiv -sin(t-\pi/2)$; the upper boundary for Δ is $\pi/2 =$ $1.57...$.

Proof (briefly). Prove by induction by steps of length h in $n \in N_0$:

$|u(n)| \leq f_0/p/(1-\Delta); |u(n) - u(s)| \leq 2(n-s)f_0/(1-\Delta), n-h \leq s \leq n.$

The shift operator: $Su(\cdot)(t) := u(0) + \int_{-h}^{t}(-pu(s) + f(s))ds, -h \leq t \leq 0.$

$|Su(\cdot)(0)| \leq |u(0)|(1-\Delta) + \int_{-h}^{0}(p|(u(s) - u(0)| + |f(s)|)ds \leq$

$\leq (1 - \Delta + \Delta^2 + (1-\Delta))f_0/p/(1-\Delta) = f_0/p/(1-\Delta);$

$|Su(\cdot)(t) - Su(\cdot)(0)| \leq p|u(0)| \cdot |t| + \int_0^t(p|(u(s) - u(0)| + |f(s)|)|ds| \leq$

$\leq (1/(1-\Delta) + ph/(1-\Delta) + 1)f_0|t| = 2f_0|t|/(1-\Delta)$

$\leq 2f_0h/(1-\Delta) = 2f_0\Delta/p/(1-\Delta).$

Theorem is proven.

Keywords: delay-differential equation, solution, stability, perturbation

2010 Mathematics Subject Classification: 34K20, 34K27, 34K35

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Boundary control of rod temperature field with a selected point

Baltabek Kanguzhin[1,2], Kanzharbek Imanberdiyev[1,2], Akerke Serik[1]

[1] *Al-Farabi Kazakh National University, Kazakhstan*
[2] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

kanguzhin53@gmail.com, kanzharbek75ikb@gmail.com

Abstract: In this work, we study the issue of boundary control of rod temperature field with a selected point x_0.

(1) $$u_t(x,t) - u_{xx}(x,t) + \alpha u(x_0,t) = f(x,t), \quad (x,t) \in Q,$$

where $Q = \{(x,t):\ 0 < x < b,\ 0 < t < T < +\infty\}$.

It is assumed that at the initial moment $t = 0$ the temperature along the rod of length b is given by law $u(x,0) = u_0(x)$, $0 < x < b$, where $u_0(x)$ is a twice continuously differentiable function. At the moment of time $t = T$ the temperature of the rod is equal to $u(x,T) = \gamma(x)$, $0 < x < b$, where $\gamma(x)$ is also a twice continuously differentiable function. The main purpose of the work is to clarify the conditions for the existence of the boundary control $u(0,t) = \mu(t)$, $u(b,t) = \eta(t)$, which ensures the transition of the temperature field from the state $\{u(x,0) = u_0(x)\}$ to the state $\{u(x,T) = \gamma(x)\}$. Similar problems were considered in [1, 2].

According to the optimization method, we choose the following functional

$$\mathcal{J}[\mu,\eta] = \|u(\cdot,T;\mu,\eta) - \gamma(\cdot)\|^2_{W_2^1(0,b)} + \beta_1 \int_0^T |\mu(t)|^2 dt + \beta_2 \int_0^T |\eta(t)|^2 dt,$$

where β_1, β_2 are positive numbers, γ is a given function from class $W_2^1(0,b)$.

The boundary control problem is as follows: it is required to find boundary controls $(\mu(t), \eta(t))$ and the corresponding solution $u(x,t)$, that satisfies equation (1) with initial boundary controls

(2) $$u(0,t) = \mu(t), \quad u(b,t) = \eta(t), \quad 0 < t < T,$$

(3) $$u(x,0) = u_0(x), \quad 0 < x < b,$$

and minimizes functional $\mathcal{J}[\mu,\eta]$.

Keywords: initial-boundary value problem, heat equation, boundary control, Green's function, Fredholm integral equation of the second kind, spectral properties, eigenfunction, eigenvalues

2010 Mathematics Subject Classification: 35A23, 35K05, 35P05

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The weak solutions of a boundary value interface problem

Hayati Olğar[1], Oktay Sh. Mukhtarov[1,2], Fahreddin S. Muhtarov[3]

[1] *Department of Mathematics, Faculty of Arts and Science, Tokat Gaziosmanpaşa University, Turkey, hayatiolgar@gmail.com*

[2] *Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Azerbaijan, omukhtarov@yahoo.com*

[3] *Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences, Azerbaijan, fahreddinmuhtarov@gmail.com*

Abstract: In recent years, more and more researchers are interested in the discontinuous differential operators for its wide application in physics and engineering. Such problems are connected with discontinuous material properties, such as heat and mass transfer which can be found in [3], vibrating string problems when the string loaded additionally with point masses, the heat transfer problems of the laminated plate of membrane, etc. (see [1, 2, 4]).

In this study, some spectral properties of a boundary value problem for a second order differential equation together with additional interface conditions is investigated. First of all, the weak solution of the boundary value interface problem is defined. Secondly, some new linear operators associated with the boundary-value-interface problem is defined in an appropriate Hilbert space. Thirdly, the boundary-value-interface problem is reduced to the polynomial operator equation. Finally, we proved that this polynomial operator equation is self-adjoint.

Keywords: Spectral problem, boundary and interface conditions, self-adjoint, polynomial operator

2010 Mathematics Subject Classification: 34B24, 34L10, 34L20

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Boundary value problems for fractional PDES with the Liouville fractional derivative

Arsen Pskhu

Institute of Applied Mathematics and Automation KBSC RAS, Russia

pskhu@list.ru

Abstract: The report discusses the solvability and properties of solutions to boundary value problems in unbounded domains for the equation

$$\left(\frac{\partial}{\partial x}+\frac{\partial^\alpha}{\partial y^\alpha}\right)u(x,y)=f(x,y), \tag{1}$$

and their application to the fractional diffusion-wave equation

$$\left(\frac{\partial^{2\alpha}}{\partial y^{2\alpha}}-\frac{\partial^2}{\partial x^2}\right)u(x,y)=f(x,y). \tag{2}$$

Here $\frac{\partial^\alpha}{\partial y^\alpha}$ denotes the Liouville fractional derivative of order α, $\alpha\in(0,1)$, with respect to y with origin at the point $y=-\infty$[1]:

$$\frac{\partial^\alpha}{\partial y^\alpha}u(x,y)=\frac{1}{\Gamma(1-\alpha)}\frac{\partial}{\partial y}\int_{-\infty}^{y}u(x,t)(y-t)^{-\alpha}dt \tag{3}$$

As is known (see, e.g., [2-4]), equations with operators of the form (3) induce problems without initial conditions (asymptotic problems). Here we consider just such problems. In addition, some properties of solutions of equations (1) and (2) are discussed in comparison with solutions of the corresponding equations with fractional derivatives that have origins at finite points and equations of integer order.

Keywords: Liouville fractional derivative, first order fractional PDE, diffusion-wave equation, asymptotic problem

2010 Mathematics Subject Classification: 35F15, 35R11

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Generalization of Tikhonov's theorem and extracting information on objects in metrical spaces

Adaham Zhoraev[1], Kadyrbek Aknazar uulu[2]

[1] *Kyrgyz-Uzbek International University named after B. Sydykov*

adaham_67@mail.ru

[2] *Kyrgyz-Uzbek International University named after B. Sydykov*

kadyrbek_6621@mail.ru

Abstract: Tikhonov's theorem [1] provides conditions to restore (hidden) objects by their (observable) bijective images. On the one hand, some images are not bijective (for instance, projections), on the other hand we sometimes need not the whole object but any information on it (especially, if the object moves in a kinematical space). Let X be a compact space of "objects Y be a space of their observable images, Z be a space of "information $P : X \to Z$ be continuous.

Theorem. If 1) $f : X \to Y$ is surjective and continuous; 2) $(f(x_1) = f(x_2)) \Rightarrow (P(x_1) = P(x_2))$ then the assertion $(\exists x \in X)((y = f(x)) \wedge (z = P(x))$ defines a continuous function $g : Y \to Z$.

Proof. Let $y_0 \in Y$. Due to 1) $(\exists x_0 \in X)(y_0 = f(x_0))$ and $(\exists z_0 \in Z)(z_0 = P(x_0))$. If $((y_0 = f(x_1)) \wedge (y_0 = f(x_2))$ then by 2) $P(x_1) = P(x_2)$. Hence, the function g is defined uniquely. Suppose that (*) there exists such sequence $\{x_k | k \in N\} \subset X$ that $\{y_k := f(x_k) | k \in N\}$ converges to y_0 but $\{z_k := P(x_k) | k \in N\}$ does not converge to z_0. Then there exists such $\varepsilon > 0$ that any infinite subset $Z_1 \subset \{z_k\}$ is out of $E_0 := (\varepsilon-$neighborhood of $z_0)$. Consider the corresponding subset $X_1 \subset \{x_k\}$. By compactness there exists a subset $X_2 \subset X_1$ converging to any $x' \in X$. Then the corresponding subset $Z_2 \subset Z_1$ converges to $z' := P(x') \notin E_0$. Hence, $P(x') \neq P(x_0)$. But $f(x') = f(x_0)$; the assumption (*) has implied a contradiction. Hence the function g is continuous. Theorem is proven.

Example. Let X be the set of segments in R^n_+ with Hausdorff metric, $Y := R^n_+, Z := R_+$. Denote $f(x) := \{projection\ of\ x\ onto\ Ox_j\text{-}axis | j = 1..n\}$, $P(x)$ is the length of x. Then, by Theorem, a continuous function $g : Y \to Z$ exists. Hence, the length of x can be found by its projections although x itself cannot be found.

Keywords: topological space, information, projection, motion

2010 Mathematics Subject Classification: 54D05

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Existence results for nonlinear Hadamard type fractional boundary value problems

Tugba Senlik Cerdik

Department of Mathematics, Beykent University

tugbacerdik@beykent.edu.tr

Abstract:

In this work, we investigate the fractional boundary value problem by the Hadamard fractional derivative:

(1)

$$\begin{cases} {}^{H}D^{\sigma}_{1^+}w(t) + f(t, w(t)) = 0, \quad n-1 < \sigma \leq n, \quad t \in (1, e), \\ w^{(k)}(1) = 0, 0 \leq k \leq n-2, \quad w(e) = \displaystyle\sum_{i=1}^{k} \tau_i w(\eta_i) + \int_1^e k(t)w(t)\frac{dt}{t}, \end{cases}$$

where ${}^{H}D^{\sigma}_{1^+}$ is the Hadamard-type fractional derivative of order σ, $n \in \mathbb{N}, n \geq 3$, $f \in \mathcal{C}([1, e] \times [0, \infty), (0, \infty))$, $k \in \mathcal{C}([1, e], [0, \infty))$ and $\tau_i \geq 0$, $(i = 1, 2, ..., k)$, $1 < \eta_1 < \eta_2 < ... < \eta_k < e$.

Using the properties of Green's function and a fixed point theorem in cone, existence of positive solutions for Hadamard fractional boundary value problem is obtained. Some fundamental concepts of Hadamard type fractional calculus are given in [1] and [2].

Keywords: Hadamard fractional differential equation, Green's function, Positive solution

2010 Mathematics Subject Classification: 34B10, 34B14

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The uniqueness result for Hadamard type fractional boundary value problems with mixed boundary conditions

Fulya Yoruk Deren, Tugba Senlik Cerdik

Department of Mathematics, Ege University

Department of Mathematics, Beykent University

fulya.yoruk@ege.edu.tr, tugbacerdik@beykent.edu.tr

Abstract:

Fractional calculus is a branch of mathematics which deals with the characteristic of the differentiation and integral of any order. (See [1,2] and references therein) Nowadays, study of fractional boundary value problems gained increasing interest among scientists, since differential equations of fractional order model many real world problems more accurate and realistic than ordinary differential equations. Also, the fractional boundary value problems have wide application areas such as physics, biology, control theory, heat transfer, analytical and numerical methods, and economics and allows many investigations. Considering these, in this talk, it is aimed to consider Hadamard type fractional boundary value problem and develop the uniqueness results of solutions of the Hadamard fractional differential equation with mixed boundary conditions on a semi-infinite interval. Some of the techniques to be used within the scope of this study are the Green's function method and the Banach fixed point theorem.

Keywords: Hadamard fractional boundary value problem, Banach fixed point theorem

2010 Mathematics Subject Classification: 34B10, 34B14

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Analysis of functional relationships of particulates thermal conductivity

Jumamurad Uvdiev[1], Leonid Marushin[1], Leonid Marushin[1]

[1] Moscow Polytechnic University, Moscow, Russian Federation

pte@mospolytech.ru

Abstract: The study of porous materials thermal characteristics can be carried out by application of functional simulation methods with due account for real structure of heterogeneous system (number of components, porosity, particle sizes and ways of their interaction with each other, etc.) with subsequent calculation of thermal conductivity for in-situ conditions determined by the system gas pressure, temperature, presence of convectional and radiation components of thermal conductivity, etc.

An effective method of theoretical study of heterogeneous systems thermal conductivity stipulates application of general conductivity principle based on an analogy between differential equations of steady heat flow, electric current, electric and magnetic induction, mass flow. This analogy allows us to use basic relations of electrostatics and electrodynamics for calculation of the system thermal conductivity.

The article discusses experimental values of thermal conductivity of different oxides depending on the degree of porosity and temperature.

Analysis of thermal conductivity of beryllium oxide, titanium dioxide, calcium oxide with the given porosity was carried out pursuant to the quoted method for dense beryllium oxide.

The results of theoretical study of mechanisms of heat carrier dissipation in the tested samples are presented as an exponential functional relationship [1-8] $\lambda = AT^x$.

Keywords: Translation models, heterogeneous systems, thermal conductivity models, porosity, structure

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On the stability of solution of the telegraph equation with involution

Allaberen Ashyralyev[1], Maral Ashyralyyeva[2], Ogulbabek Batyrova[3]

[1] *Department of Mathematics, Bahcesehir University, Turkey,*
Peoples Friendship University Russia, Ul Miklukho Maklaya 6, Moscow 117198, Russian Federation
Institute of Mathematics and Mathematical Modeling, 050010, Almaty, Kazakhstan,

aallaberen@gmail.com

[2] *Department of Applied of Mathematics and Informatics,*

Magtymguly Turkmen State University, Ashgabat, Turkmenistan

ashyrmaral2010@mail.ru

[3] *Department of Mathematics, Near East University, Nicosia, TRNC, Mersin 10, Turkey,*
Oguz Han Eng. and Technology University of Turkmenistan,Ashgabat, Turkmenistan

ogulbabek93@gmail.com

Abstract: In the present paper, the initial value problem for the second order partial differential equation with dumping term and involution is investigated. We establish equivalent initial value problem for the fourth order partial differential equations to the initial value problem for second order linear partial differential equations with dumping term and involution. Theorem on stability estimates for the solution of the initial value problem for the second order partial differential equation with dumping term and involution is proved.

Keywords: Involution, boundedness, stability.

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27.

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Existence and uniqueness solution of wave equations with involution

E. Mussirepova, A.A. Sarsenbi, A.M. Sarsenbi,

M. Auezov South Kazakhstan University, Kazakhstan, Shymkent

elmira.musirepova01@auezov.edu.kz; abdisalam. sarsenbi @ auezov.edu.kz; abzhahan@gmail.com;

Abstract: Consider the equation

(1) $$u_{tt}(x,t) = u_{xx}(x,t) - \alpha u_{xx}(-x,t) - q(x)\,u(x,t)\,, \ \ (x,t) \in \Omega,$$

with initial date

(2) $$u(x,0) = \varphi(x)\,, \ \ u_t(x,0) = \psi(x)\,, \ \ -1 \le x \le 1,$$

and boundary conditions

(3) $$u(-1,t) = u(1,t) = 0, \ \ 2) \ \ u_x(-1,t) = u_x(1,t) = 0,$$

where $\Omega = \{-1 < x < 1, \ t > 0, \ -1 < \alpha < 1, \ q(x)$ is a complex-valued function from the class $[-1,1]$. Equation (1) with periodic and antiperiodic boundary conditions was studied in the work [1].

Theorem. Let 1) all eigenvalues of the spectral problem

(4) $$L_\alpha X(x) \equiv -X''(x) + \alpha X''(-x) + q(x)\,X(x) = \lambda X(x)$$

with conditions (3) are simple, and the number $\lambda = 0$ is not an eigenvalue; 2) $q(x) \in C^2[-1,1]$; 3) $\varphi(x) \in C^4[-1,1]$ and functions $\varphi(x)\,, \ L_\alpha \varphi$ satisfy the conditions (3), (4); 3) $\psi() \in {}^2[-1,1]$ satisfy the boundary conditions (3). Then the mixed problem (1), (2), (3) has a unique solution of the type

$$u(x,t) = \sum_{k=1}^{\infty} \left(a_k \cos\sqrt{\lambda_k}t + b_k \sin\sqrt{\lambda_k}t \right) X_k(x),$$

where $\{X_k(x)\}$ is the system of eigenfunctions of problem (4), (3) .

The work was supported by a grant from the Ministry of Science and Education of the Republic of Kazakhstan (grant no.AP08855792)

Keywords: wave equation, eigenfunction, involution perturbation, eigenvalue problem, basis.

2010 Mathematics Subject Classification: 35L05, 35L20, 34B05

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House Price Prediction Using Machine Learning Methods

Zeki Kaan Akkartal[1], Süreyya Akyüz[2]

[1] *Big Data Analytics and Management, Bahcesehir University, Turkey*

kakkartal@hotmail.com

[2] *Department of Mathematics, Faculty of Engineering and Natural Sciences, Bahcesehir University, Turkey*

sureyya.akyuz@eng.bau.edu.tr

Abstract: With the rise in housing demands, the real estate industry is expanding and changing at a quick pace, and new luxury home projects come up on a daily basis. The accuracy of pricing determines the success of residential market activities. The purpose of this study is to predict house prices of Istanbul and Izmir for the available dataset, which is obtained from a real estate company, by using popular machine learning methods and a hybrid method. Extreme Gradient Boosting (XGB), Light Gradient Boosting Machines (LGBM), Neural Networks (NN) and Deep Neural Networks (DNN) are the stand-alone models, whereas Random Forest (RF), Voting and Stacking Generalized Regression are the ensemble models that are used to predict the house prices in this project. The hybrid method [2] showed the best performance with 92,1% r-squared score. The parameters of LGBM [3] and NN are tuned by using cross validation and Optuna [4]. Voting, which showed the best performance by reaching 90,0% of r-squared score, has the base estimators RF [1], XGB and tuned LGBM. Stacking, which performed the second best with 89,9% r-squared score, has the base estimators RF, XGB and tuned LGBM, whereas the meta learner is tuned LGBM, which showed the third best performance as a stand alone boosting algorithm. NN and DNN have lower model metrics than tree based boosting/ensemble methods.

Keywords: Housing Pricing, Random Forest, Neural Networks, Regression

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Construction of solutions of non-linear integro-differential equations of the second order with derivative under the integral sign

Elmira Mamaziaeva [1], **Zhoomart Mambetov** [2]

[1] Osh State University, Osh, Kyrgyzstan
mamaziaeva67@mail.ru

[2] Osh Technological University named after M.Adyshev, Osh, Kyrgyzstan
Zhoomart_mambetov@mail.ru

Abstract: Consider the following nonlinear partial integro-differential equation of the second order

$$\frac{\partial^2 u(t,x)}{\partial t^2} = u(t,x)\frac{\partial}{\partial x}\left[u(t,x)\frac{\partial u(t,x)}{\partial x}\right] - \frac{\partial u(t,x)}{\partial x}\frac{\partial u(t,x)}{\partial t}+$$

$$+\int_0^t\int_{-\infty}^{\infty} K(t,s,\xi)\frac{\partial u(s,\xi)}{\partial \xi}d\xi ds + g(t), (t,x) \in R_1^2 = R_+ \times R, \tag{1}$$

with the initial conditions

$$\frac{\partial^k u(t,x)}{\partial t^k}\Big|_{t=0} = (-1)^k x, k = 0,1, x \in R. \tag{2}$$

Theorem. If

$$+\int_0^{\infty}\int_{-\infty}^{\infty} |K(t,s,\xi)| d\xi ds \leq \gamma = const$$

then the initial value problem (1)-(2) has a unique solution in the space $CB^{(2)}(R_1^2)$ of continuous and bounded up to derivatives of k-th order functions.

It can be represented as follows:

$$u(t,x) = \frac{x}{1+t} - \frac{1}{1+t}\int_0^t\int_0^{\eta}(\eta-\rho)\left[\int_0^{\rho}\int_{-\infty}^{\infty} K(\rho,s,\xi)\frac{1}{1+s}d\xi ds + g(\rho)\right]d\rho +$$

$$+\int_0^t (t-\rho)\left[\int_0^{\rho}\int_{-\infty}^{\infty} K(\rho,s,\xi)\frac{1}{1+s}d\xi + g(\rho)\right]d\rho.$$

In proof of Theorem, the scheme to apply the method of additional argument to non-linear partial integro-differential equations of higher order given in [1] and results of [2] were used.

Keywords: partial integro-differential equation, non-lineal equation, methods of additional argument.

2010 Mathematics Subject Classification: 35B30, 35C05, 35E15

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Numerical modeling of variations of the Earth's electromagnetic field

Nurlan Temirbekov[1], Laura Temirbekova[2], Zhanar Demeubayeva[3]

[1] *National Engineering Academy of the Republic of Kazakhstan*
temirbekov@rambler.ru
[2] *Abai Kazakh National Pedagogical University,* [3] *D. Serikbayev East Kazakhstan technical university*

Abstract: In this paper, the natural electromagnetic fields of the Earth are numerically modeled.

In the Tikhonov-Kanyar model, the most convenient characteristic of the medium and the field is the impedance, which is the ratio of mutually perpendicular components of the electric and magnetic field strengths [1, 2]

$$z = \frac{E_x}{H_y} \tag{1}$$

Where E_x is the northern component of currents, H_y is the eastern component of magnetic field variations. Relationship between magnetic variations and Earth currents is described as follows

$$\frac{\partial E_x}{\partial H_y} = i\omega\mu_0 H_y, \frac{\partial H_y}{\partial H_y} = E_x \tag{2}$$

where $\rho = \frac{1}{\sigma_n}$ is the one-dimensional distribution of specific electrical resistances in the section, σ_n is the conductivity of the environment. The solution of the direct MTS problem is solved using the Lanskaya formula. A variational method is used to numerically solve the inverse MTS problem. The conductivity coefficient of the medium is determined by the gradient method.

The work was supported by grant funding from the Ministry of Education and Science of RK (IRN AR0856012).

Keywords: Numerical method, magnetotelluric sounding, inverse problem

2010 Mathematics Subject Classification: 65Z05

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Inverse problem for the Oskolkov equation

Shakir Aidos

Al-Farabi Kazakh National University, Kazakhstan

ajdossakir@gmail.com

,5cm **Abstract:** Let Ω be a bounded domain in $\mathbb{R}^d$, $d \geq 2$, with a smooth boundary $\partial\Omega$, and $Q_T = \{(x,t) : \; x \in \Omega, \; 0 < t < T\}$ is a cylinder with lateral Γ_T. In this work, we consider the following inverse problem determining the triple of functions $(\mathbf{u}(x,t), \nabla\mathbf{p}(x,t), \mathbf{f}(t))$, which satisfy the following system equations:

(1)

$$\mathbf{u}_t - \varkappa\Delta\mathbf{u}_t - \nu\Delta\mathbf{u} - \int_0^t K(t-s)\Delta\mathbf{u}(\mathbf{x},s)ds - \nabla\mathbf{p} = f(t)\mathbf{g}(\mathbf{x},t) \text{ in } Q_T,$$

(2) $$\text{div } \mathbf{u}(\mathbf{x},t) = 0 \text{ in } Q_T,$$

(3) $$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}) \text{ in } \Omega,$$

(4) $$\mathbf{u}(\mathbf{x},t) = 0 \quad \text{on} \quad \Gamma_T,$$

(5) $$\int_\Omega \mathbf{u}\cdot\boldsymbol{\omega}(\mathbf{x})d\mathbf{x} = e(t) \quad \text{on} \quad t \in [0,T].$$

Here $\mathbf{u}(\mathbf{x},t)$ is the velocity field, $\mathbf{p}(\mathbf{x},t)$ is the pressure, the vector-valued functions $\mathbf{u}_0(\mathbf{x})$ and $\mathbf{g}(\mathbf{x},t)$, scalar functions $K(t)$ and $e(t)$ are given. The constant ν accounts for the dynamic viscosity, $\varkappa$ is a relaxation coefficient.

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Optimized road lane detection through a combined Canny edge detection, Hough transform, and scaleable region masking toward autonomous driving

Samane Monfared[1], Lavdie Rada[1]

[1] *Bahcesehir University, Istanbul,Turkey*

samane.sharify@gmail.com,lavdie.rada@eng.bau.edu.tr

Abstract: Nowadays, autonomous vehicles are developing rapidly toward facilitating human car driving. One of the main issues is road lane detection for a suitable guidance direction and car accident prevention. This paper aims to improve and optimize road line detection based on a combination of camera calibration, the Hough transform, and Canny edge detection. The video processing is implemented using the Open CV library with the novelty of having a scale able region masking. The aim of the study is to introduce automatic road lane detection techniques with the user's minimum manual intervention.

Keywords: Hough transform, lane detection, Canny edge detection, optimization, camera calibration, image processing, video processing, real-time lane detection, Hough probabilistic transform, autonomous driving

Mathematical modeling for energy efficiency of buildings

Mesut Urun

Near East University Department of Mathematics, Nicosia, TRNC, Mersin 10, Turkey,

Bahçeşehir University Department of Mathematics, Istanbul, Turkey,

Galatasaray University Department of Marine Engines Operation, Istanbul mesuturun@gmail.com

Abstract: Energy is one of the most important issues in the world. Scientists have predicted that the existing energy resources will decrease very seriously since the middle of the 21st century, and the concept of energy efficiency has started to be discussed at the beginning of this century. The idea of producing more work with less energy has been a subject that has been appreciated and researched all over the world. This study gives a mathematical model of the nonstationary energy consumption calculation problem

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} - \frac{\partial}{\partial x}\left(a\left(x\right)\frac{\partial u(t,x)}{\partial x}\right) + \delta u(t,x) = f\left(t,x\right),\ t \in \left(0,T\right),\ x \in \left(0,l\right), \\ u\left(0,x\right) = u\left(\lambda,x\right) + \varphi\left(x\right),\ x \in \left[0,l\right], \lambda \in \left(0,T\right], \\ u\left(t,0\right) - \psi(t) = bu_x\left(t,0\right), -u\left(t,l\right) - \mu(t) = cu_x\left(t,l\right),\ t \in \left[0,T\right] \end{cases}$$

for the one-dimensional heat equation with Robin conditions.

The model is well-posedness in Hölder spaces of the mixed one-dimensional nonlocal problem with Robin conditions. In this study, an effective numerical method is also developed for energy consumption calculation which is related to this mathematical model. The three case problems are taken to test this numerical method. The study also aims to develop a mathematical model in which the result can be found at any time.

Keywords: mathematical modeling, heat equation, non-local problem, difference scheme, stability.

2010 Mathematics Subject Classification: 35K90, 58J35, 35K61, 35K20

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Blow up of solutions for a nonlinear stochastic wave equation with a time-dependent damped term

Hatice Taskesen[1], Necat Polat[2]

[1] *Department of Mathematics, Van Yuzuncu Yil University, Van, Turkey*

haticetaskesen@yyu.edu.tr

[2] *Department of Mathematics, Dicle University, Diyarbakir, Turkey*

npolat@dicle.edu.tr

Abstract: The wave equation is one of the fundamental partial differential equation which arises in different fields such as electromagnetic, traffic flows, fluid dynamics, general relativity, acoustics, atmosphere and ocean dynamics, chemical reactions and biological sciences. By adding a noise term to deterministic equations one can incorporate neglected degrees of freedom, or can involve fluctuations of exterior fields that describes the media. By taking this effects with a space-time white noise into account, the following equation

(1)
$$du_t + [\alpha(t)\, u_t - \Delta u]\, dt = f(u)\, dt + \sigma(u, u_t, \nabla u)\, dW(x,t), \quad x \in \Omega, t > 0$$

with initial and boundary conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \tag{2}$$

$$u(x,t) = 0, \quad x \in \partial\Omega, \quad t > 0 \tag{3}$$

is studied in this work. Here $\Omega \subset \mathbb{R}^n$, $d \geq 1$ is a bounded domain with smooth boundary, $W(x,t)$ is a Wiener process, $\alpha(t) : [0,\infty) \to (0,\infty)$ is a nondecreasing, bounded differentiable function. A random exponential attractor was constructed for initial-boundary value problem of (1)-(3) with an additional $g(x,t)$ term in [1]. In this work, we obtain a local existence result, and then we give the conditions that ensure the blow-up of solutions.

Keywords: Time-dependent damping,stochastic wave equation, initial-boundary condition

2010 Mathematics Subject Classification: 60H15;35A01;35B44

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On the Cauchy problem for a generalized two-component shallow water wave system

Nurhan Dündar[1], Necat Polat[2]

[1] *The Ministry of National Education, Diyarbakir, Turkey*

dundarnurhan@gmail.com

[2] *Department of Mathematics, Dicle University, Diyarbakir, Turkey*

npolat@dicle.edu.tr

Abstract: Shallow water waves and model equations are very important to mathematical and physical theory. Furthermore, water wave modeling is a complex process and often leads to models that are difficult to mathematically analyze and solve numerically. In this study, a generalization of the Camassa-Holm equation, a model for shallow water waves, is investigated:

(1)
$$\begin{cases} u_t - u_{xxt} + ku_x + [h(u)]_x - 2u_x u_{xx} - uu_{xxx} + \theta\theta_x = 0, & t>0,\ x \in \mathbb{R}, \\ \theta_t + (\theta u)_x = 0, & t>0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \\ \theta(0,x) = \theta_0(x), & x \in \mathbb{R}, \end{cases}$$

where $h(u) \in C^\infty(\mathbb{R})$ is the given nonlinear function. This system turns into an integrable two-component Camassa-Holm shallow water system for $h(u) = \frac{3}{2}u^2$ [1]. In this work, we establish the local well-posedness and blow-up scenarios for system (1).

Keywords: Shallow water wave, generalized Camassa-Holm system, local well-posedness, blow-up

2010 Mathematics Subject Classification: 35Q35, 35G25,35L05, 35B44

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Some new Cesàro sequence spaces of order α

Medine Yeşilkayagil Savaşcı[1], Feyzi Başar[2]

[1] *Faculty of Applied Sciences, Uşak University, 1 Eylül Campus, 64200 - Uşak, Türkiye*

medine.yesilkayagil@usak.edu.tr

[2] *Department of Primary Mathematics Teacher Education, İnönü University, 44280 – Malatya, Türkiye*

feyzibasar@gmail.com, feyzi.basar@inonu.edu.tr

Abstract. Let $\alpha \in \mathbb{R}$ with $\alpha > -1$ such that $-\alpha \notin \mathbb{N}$. The inverse $C_\alpha^{-1} = \left(\tilde{c}_{nk}^{(\alpha)}\right)$ of the *Cesàro matrix* C_α *of order* α with $\alpha \in \mathbb{N}$ is determined by

$$\tilde{c}_{nk}^{(\alpha)} = \begin{cases} \binom{n-k-\alpha-1}{n-k}\binom{k+\alpha}{k} & , \quad \max\{0, n-\alpha\} \leq k \leq n, \\ 0 & , \quad k > n \end{cases}$$

for all $k, n \in \mathbb{N}$, [1] and [2]. Using this matrix we introduce the spaces $\ell_\infty(C_\alpha)$, $f(C_\alpha)$ and $f_0(C_\alpha)$ of Cesàro bounded, Cesàro almost convergent and Cesàro almost null sequences, of order α, respectively.

Our main results are:

Theorem 1.2. *The sequence spaces* $f_0(C_\alpha)$ *and* $f(C_\alpha)$ *are* BK*-spaces.*

Theorem 1.3. *The sequence spaces* $f_0(C_\alpha)$ *and* $f(C_\alpha)$ *are norm isomorphic to the spaces* f_0 *and* f*, respectively, i.e.,* $f_0(C_\alpha) \cong f_0$ *and* $f(C_\alpha) \cong f$*.*

Theorem 1.4. *Let* $\alpha \geq 0$*. Then, the Cesàro matrix of order* α *is strongly regular.*

Theorem 1.5. *Let* $\alpha \geq 0$*. Then, the inclusions* $f_0 \subset f_0(C_\alpha)$ *and* $f \subset f(C_\alpha)$ *strictly hold.*

Theorem 1.6. *The inclusion* $f_0(C_\alpha) \subset f(C_\alpha)$ *strictly holds.*

Theorem 1.7. *The* α*-dual of the space* $f(C_\alpha)$ *is the set* d *defined by*

$$d := \left\{ a = (a_k) \in \omega : \sup_{K \in \mathcal{N}} \sum_n \left| \sum_{k \in K} \binom{n-k-\alpha-1}{n-k}\binom{k+\alpha}{k} a_n \right| < \infty \right\}.$$

Keywords: Normed sequence space, α-, β- and γ-duals and matrix mappings

2010 Mathematics Subject Classification: 46A45, 40C05

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Optimal boundary control problem for the oscillation process described by integro-differential equation with fredholm integral operator

Elmira Abdyldaeva
Kyrgyz-Turkish Manas University, Kyrgyzstan
elmira.abdyldaeva@manas.edu.kg
efa-69@mail.ru

Abstract: In the paper, the solvability of the nonlinear boundary optimization problem have investigated the for the oscillation processes, described by the integro-differential equation in partial derivatives with the Fredholm integral operator. It has been established that the system of nonlinear integral equations obtained with respect to the components of the vector boundary optimal control has the property of equal relations, which makes it possible to simplify the procedure for constructing a solution to the nonlinear optimization problem. An algorithm has been developed for constructing solutions of the nonlinear optimization problem

Consider the following nonlinear optimization problem:

$$J[u_1(t,x),u_2(t,x),...,u_m(t,x),]=\int_Q [V(T,x)-\xi_1(x)]^2+[V_t(T,x)-\xi_2(x)]^2dx+$$

$$+\beta\int_0^T\int_Q\sum_1^m|u_k(t,x)|dxdt\to min,\quad \beta>0$$

$$V_{tt}-AV=\lambda\int_0^T K(t,\tau)V(\tau,x)d\tau, x\in Q\subset R^n, 0<t\leq T,$$

$$V(0,x)=\psi_1(x), V_t(0,x)=\psi_2(x),\quad x\in Q,$$

$$\Gamma V(t,x)\equiv\sum_{i,j=1,n}^n a_{ij}(x)V_{x_j}(t,x)cos(\delta,x_i)+a(x)V(t,x)=$$

$$=f[t,x,u_1(t,x),...,u_m(t,x)], x\in\gamma, 0<t\leq T$$

Here A is the elliptic operator, δ is a normal vector, emanating from the point $x\in\gamma$; $K(t,\tau)$ is a given function of H(D), $D=[0\leq t\leq 1, 0\leq\tau\leq 1)]$, $\psi_1(x)\in H_1(Q)$, $\psi_2(x)\in H(Q)$,$\xi_1(x)\in H(Q)$,$\xi_2(x)\in H(Q)$, are given functions; $f(t,x,u_1(t,x),u_m(t,x))\in H(Q_T)$ is a boundary source function

$f_{u_i}(t,x,u_1(t,x),u_m(t,x)) \neq 0, \forall t \in (0,T); (u_i(t,x), i = 1,...,m) \in H(Q_T)$ is a control function, λ is a parameter, T is a fixed moment of time and $\alpha > 0$ is a constant.

The research was conducted using the methodologies that were developed by professor A.I.Egorov, and professor A.Kerimbekov on the basis of the maximum principle [1-4]. Sufficient conditions were found for the existence of a solution and an algorithm is developed for solving this problem in the form of a triple $(u_1^0(t,x),...,u_m^0(t,x)), V^0(t,x), J[(u_1^0(t),...,u_m^0(t))])$, where $(u_1^0(t,x),...,u_m^0(t,x))$ is the desired optimal vector control, $V^0(t,x)$ is the optimal process, $J[(u_1^0(t),...,u_m^0(t))]$ is the minimum value of the functional.

Keywords: Optimal boundary control problem, Boundary value problem, generalized solution, functional, maximum principle, optimality condition, nonlinear integral equation.

2010 Mathematics Subject Classification: 49K20

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Methodological approaches to assesing the effectiveness of innovative projects

Sulgun Myradova[1], Rahim Myradov[1]

[1] *Turkmen State Institute of Architecture and Construction, Turkmenistan*

s.i.myradowa@gmail.com

Abstract: The effectiveness of an innovative project is determined by comparing the useful result obtained during its implementation and the investment costs that led to it. As incentive for the introduction of advanced technologies and the development of innovative products, a useful result is an increase in income, a reduction in current production costs, an increase in the profit of the enterprise, a decrease in energy consumption and resource intensity of products, etc. Investment costs include the costs of carrying out feasibility studies of investment opportunities, developing a business plan for the implementation of an investment project, for research and development work, development of design and estimate documentation, implementation of design and survey work, purchase of equipment, construction and installation work, etc. To assess the effectiveness of an innovative project in market conditions, investment performance indicators are used, such as net discounted income, internal rate of return, payback period, profitability index. This is explained by the fact that modern innovation projects require significant initial investments, and the effect of their implementation is stretched over a long period of time. At the same time, when assessing the effectiveness of innovative projects, it is necessary to consider certain features of their implementation. The purpose of the implementation of innovative projects is the reproduction of production potential on the basis of advanced and progressive technologies or the release of an innovative product. Innovative equipment and technologies are aimed at obtaining additional advantages over competitors by improving the use of production resources. The duration of the life cycle of an innovative project is large, as it is a cycle during which the idea is transformed into an innovation that can satisfy the newly emerging and already existing requirements of consumers. At the same time, there are additional time costs for the development of innovations, their development and promotion to the market. The price of innovative products should find recognition in the market. Prices for traditional products have received their confirmation in the market and over time tend to decrease due to the action of objective economic laws. The number of parameters in determining the effectiveness of innovations is greater in comparison with traditional equipment and technology. Performance indicators should consider not only the total value of the useful result from the implementation of innovations, which can be obtained for the entire useful life, but also its increase in comparison with analogues. Thus, in addition to the indicators of absolute efficiency recommended for assessing the effectiveness of investment projects, it is necessary to use indicators of comparative efficiency, such as the payback period of additional investments, the comparative value of the integral effect, the costs listed, including the cost of the life cycle.

Currently, many companies use the integral effect or net discounted income, internal rate of return, and payback period as the main indicators for assessing the effectiveness of innovative projects. There are some additional indicators, such as profitability index, return on invested capital, and life cycle cost. Indicators for assessing the effectiveness of innovative projects are interpreted as follows: project is considered effective if its net discounted income is positive, and inefficient if net discounted income is negative or zero; the greater the net discounted income, the more efficient the project; project is considered effective if the net discounted income becomes positive during the regulatory payback period, and inefficient if the net discounted income becomes positive during the accounting period, but after the end of the regulatory payback period; of several alternative projects (project options), the best option is the one with the largest net discounted income and the lowest payback period within the standard; project is recognized as effective if the internal rate of return exceeds the discount rate; when choosing options for scientific and technical projects, preference is given to a project with a large value of the internal rate of return; project is recognized as effective if the profitability index exceeds one, when choosing options for scientific and technical projects, preference is given to a project with a large profitability index; project is recognized as effective with a positive value of the profitability of the invested capital; project is recognized as effective in comparison with others at a minimum cost of the life cycle. At the same time, the fulfillment of the basic parameters of the life cycle must be ensured. Based on the calculation of net discounted income, the payback period of a scientific and technical project is determined. The payback period for individual innovative projects may exceed the regulatory period based on individual decisions of senior management. Such projects may include scientific and technical projects implemented within the framework of international cooperation or innovative projects that solve the most important and strategic tasks. Thus, the use of the described approaches to assessing the effectiveness of innovative projects allows us to form a system of criteria for making decisions on the feasibility of their implementation, to justify strategic decisions on the innovative development of the company, as well as to identify the economic advantages of innovative projects in comparison with traditional equipment and technologies.

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A New Numerical Method for Solving a Hydrodynamics Problem in a Class of Unsmooth Functions

Bahaddin Sinsoysal[1], Mahir Rasulov[2], Revane Iskenderova[2]

[1] *Istanbul Gedik University, Faculty of Engineering, Department of Computer Engineering, Istanbul, Turkey*

bsinsoysal@gmail.com

[2] *Institute of Oil and Gas of ANAS, Department of Numerical Modeling of Intralayer Dynamic Processes, Baku, Azerbaijan*

mresulov@gmail.com, ravana.iskandarova@bk.ru

Abstract: We consider the following problem in the upper half of the Euclidean $R^2_+(x,t)$ space

$$\frac{\partial u(x,t)}{\partial t}+\frac{\partial \varphi(u(x,t))}{\partial x}-\psi(u(x,t))=0, \tag{1}$$

$$u(x,0)=u_0(x), x\geq 0, \tag{2}$$

where $u_0(x)$ is given function and, $\varphi(u)$ and $\psi(u)$ are known functions, and have the following properties:

- $\varphi(u)$, $\psi(u)$ and $\varphi'(u)$, $\psi'(u)$ are continuous functions, and they are bounded for bounded u, and $\varphi''(u)$ does not change its sign,
- $\varphi(u)\geq 0$ and $\varphi'(u)\geq 0$ for $u\geq 0$, and the argument u has values such that the function $\psi(u)$ becomes zero at these points,
- $\psi'(u)$ is bounded function for $u\geq 0$.

In this article, in order to show what behaviors are expected from the process, problem in (1),(2) is handled only mathematically, with respect to wave propagation, without considering the mechanism of any chemical reaction. In general, soft solutions found by the characteristics method do not enable us to explore the dynamics from the beginning of the process to the end. To check the effectiveness of the proposed method, and to find a clear expression of the analytical solution, instead of equation (1), the following equation is considered

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}-u(1-u)=0. \tag{3}$$

Keywords: Buckley-Leverett's problem, weak solution, numerical solution in a class of unsmooth functions

2010 Mathematics Subject Classification: 35J66, 65M06

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Investigation of optical soliton solutions of higher order nonlinear Schrödinger equations with Kudryashov nonlinear refractive index

Melih Cinar[1], Ismail Onder[1], Neslihan Ozdemir[2], Handenur Esen[1], Muslum Ozisik[1], Aydin Secer[3] and Mustafa Bayram[3,*]

İstanbul Gelisim University, Turkey
[3]Computer Engineering Dept. Biruni University, Turkey [1] *Mathematical Engineering Dept. Yildiz Technical University, Turkey*
[2] *Computer Engineering Dept.*

İstanbul Gelisim University, Turkey
[3] *Computer Engineering Dept. Biruni University, Turkey** *mustafabayram@biruni.edu.tr*

Abstract: In this paper, we have investigated optical soliton solutions of higher order nonlinear Schrödinger equations with Kudryashov nonlinear refractive index using some analytical methods. various kinds of solutions have been successfully obtained and some graphs of obtained solutions have been illustrated in two and three dimensional using Matlab. The used approaches are effective and strong tools that may be used for diverse traveling wave solutions for various nonlinear physical models. The obtained solutions might be useful for future works in various areas.
Keywords: Optical solitons, Schrodinger equations, Kudryashov refractive index.

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Optical soliton with Schrödinger-Hirota equation having parabolic law by generalized Kudryashov scheme

Hasan Cakicioglu[1], Muslum Ozisik[2], Aydin Secer[2,3] and Mustafa Bayram[3,*]

[1] *DBasic Sciences Dept. National Defence University,Turkey*
[2] Mathematical Engineering Dept. Yildiz Technical University, Turkey
[3]Computer Engineering Dept. Biruni University, Turkey*mustafabayram@biruni.edu.tr

Abstract: In this study, we investigated optical soliton solutions of Schrödinger-Hirota equation with parabolic law nonlinearity (SHE-PL) which is an important model to describe optical pulse propagation in nonlinear optics and dispersive optical fiber, via generalized Kudryashov method (GKM). We successfully applied GKM to SHE-PL for the first time and we acquired optical soliton solutions. We gained singular, kink, dark and bright soliton solutions and simulated the obtained solutions graphically. We presented 3D, contour and 2D portraits, analyzed physical properties of soliton solutions for some suitable special parameter values. The results, comments and necessary explanations that were not reported in previous studies are presented in detail in the relevant sections.
Keywords: Parabolic law; Schrödinger-Hirota equation; Generalized Kudryashov metod; Soliton solution.
Mathematics Subject Classification: 35QXX, 35C08, 35Q55, 35Q60.

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Soliton solutions for a nonlinear dynamical system in conformable sense

Aydın Secer[1,2], Mustafa Bayram[2], Neslihan Ozdemir[3], Melih Cinar[1], Handenur Esen[1], Ismail Onder[1]

[1] *Department of Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey*
[2] *Department of Computer Engineering, Biruni University, Istanbul, Turkey*
[3] *Software Engineering, Gelisim University, Istanbul, Turkey*

asecer@yildiz.edu.tr
mustafabayram@biruni.edu.tr
neozdemir@gelisim.edu.tr
mcinar@yildiz.edu.tr
handenur@yildiz.edu.tr
ionder@yildiz.edu.tr

Abstract: We consider nonlinear complex generalized Zakharov dynamical system with conformable derivative in this study. The proposed model is utilized in plasma physics. Conformable fractional derivative is used to acquire more comprehensive analytical solutions. Analytical solutions for the presented equation are produced with the help of the Sardar sub-equation approach which is an effective and useful method. We depict various 3D and 2D graphs in order to investigate the behavior of the obtained solitons. Especially, to examine the effect of the unknown parameters to behavior, 2D plots are represented for different values of parameters.
Keywords: Soliton solution, Zakharov dynamical system, conformable derivative,

Sardar sub-equation method

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On the optical soliton solutions of a couple of equations with Kerr law nonlinearity

Ismail Onder[1], Melih Cinar[1], Handenur Esen[1], Neslihan Ozdemir[2], Muslum Ozisik[1], Aydin Secer[3] and Mustafa Bayram[3,*]

[1] *Mathematical Engineering Dept. Yildiz Technical University,TURKEY*
[2] *Computer Engineering Dept. İstanbul Gelisim University, TURKEY*
[3] *Computer Engineering Dept. Biruni University, TURKEY*
[*] *mustafabayram@biruni.edu.tr*

Abstract: In this study, we examined Schrödinger-Hirota (SHE) and Cubic Quartic Fokas-Lenells equations (CQ-FLE) with Kerr law nonlinearity via analytical method. The SHE and CQ-FLE have a great importance in nonlinear optics and they are frequently used by the researchers. In today's communication world, the transmission of optical waves over long distances without losing their shape and power is of great importance. While some equations model the signal pulse in fiber optic cables, some models are developed to eliminate the problems encountered during signal propagation. We used an analytical technique namely the unified Riccati equation expansion method (UREEM) to solve the investigated equations. Firstly, nonlinear partial differential equations (NLPDEs) are converted to nonlinear ordinary differential equation (NODE) form by using the complex wave transform. Then by utilizing the UREEM optical soliton solutions are derived and the resultant equations are depicted with 2D and 3D simulations. The necessary explanations and interpretations are presented in the relevant sections.

Keywords: Optical solitons, Schrodinger-Hirota equation, Fokas-Lenells equation, Unified Riccati equation expansion method.

Mathematics Subject Classification: 35QXX, 35C08, 35Q55, 35Q60.

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Analysis of optical solitons of the perturbed Fokas-Lenells equation by the generalized projective Riccati equations method

Aydin Secer[1,2], Mustafa Bayram [2], Neslihan Ozdemir[3]

[1] *Yildiz Technical University, Department of Mathematical Engineering, Istanbul, Turkey*

asecer@biruni.edu.tr

[2] *Biruni University, Department of Computer Engineering, Istanbul, Turkey*

mustafabayram@biruni.edu.tr

[3] *Istanbul Gelisim University, Department of Software Engineering, Istanbul, Turkey*

neozdemir@gelisim.edu.tr

Abstract: In this study, we applied the generalized projective Riccati equations method to the perturbed Fokas-Lenells equation, which models the light propagation inside optical fibers. This analytical scheme provides several optical soliton solutions of the considered model. As a result, various graphs of the solutions obtained by choosing the proper parameters for the physical interpretation of the nonlinear model are presented. The performance of the method is concise, efficacious, and reliable for examining nonlinear partial differential equations.

Keywords: Fokas-Lenells equation, the generalized projective Riccati equations method, optical fiber

2010 Mathematics Subject Classification: 35C08, 35Q51

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The effect of customer satisfaction on repeat visit intention of hotel guests

Tulen Saner[1], Halima İsa[2], Serkan İlseven[3]

[1] *Faculty of Tourism, Near East University, Lefkosa, North Cyprus (via Mersin 10-Turkey)*

tulen.saner@neu.edu.tr

[2] *Kaduna Polytechnic, Northern Nigeria.*

hallynur76@gmail.com

[3] *Faculty of Arts and Sciences, Near East University, Lefkosa, North Cyprus (via Mersin 10-Turkey).*

serkan.ilseven@neu.edu.tr

Abstract: Customer satisfaction is one of the most popular concepts in the hospitality industry, maximizing customer happiness, which is generally recognized as a core factor contributing to hotel business growth. The research investigated the impact of customer satisfaction on the repeat visits of hotel guests. A structured questionnaire was conducted face-to-face with 253 hotel guests.

SPSS version 25 is used for analysis. Findings indicate that there is a significant relationship between hotel guests' satisfaction and the intention to visit. Findings also indicate that there is a significant relationship between gender and revisit intention. Both female and male customers have the intention to visit again, whereby the male hotel guests' response is higher than the female hotel guests' intention to revisit the hotel.

The hospitality industry is a highly competitive industry. Providing an excellent customer experience is or should be the central focus of every hospitality business. In this paper, it was recommended to the management that hotel guest satisfaction should be the priority. There is a high possibility that satisfied hotel guests will revisit and also recommend the hotel.

Keywords : Customer satisfaction, recommendation, and intention to revisit

Stochastic optical solitons of the (2+1)-dispersive nonlinear Schr446dinger equation having multiplicative white noise via Ito sense

Selvi Altun[1], Muslum Ozisik[2], Aydın Secer[3], Mustafa Bayram[3]

[1] *Suleymaniye Mh. Destan Sk. Inegol, Bursa, Turkey*

[2] Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey

[3] Computer Engineering, Biruni University, Istanbul, Turkey

Abstract: This study aims to examine the stochastic optical soliton solutions of the nonlinear (2+1)-dimensional nonlinear Schr446dinger equation (NLSE) with Kerr law nonlinearity by multiplicative noise in It444 sense and the behavioral changes on soliton dynamics by using the generalized Kudryashov method (GKM). Making use of wave transform, the stochastic (2+1) dimensional NLSE with the Kerr law nonlinearity has been turned into a nonlinear ordinary differential equation (NODE) and the constraint relations have been obtained. First, the generalized Kudryashov method is applied over the NODE form, then, in compliance with the properties of the proposed method the soliton functions and the soliton sets have been obtained. As the next stage, the obtained solutions are checked whether they provide the nonlinear partial differential equation (NLPDE) or not. It is also shown with graphic presentations that the solutions obtained produce the basic soliton shapes. This examination of the Kerr law nonlinearity form of the stochastic (2+1)-dimensional NLSE having multiplicative white noise via It444 sense has been introduced for the first time in this study.

Keywords: Noise strength; stochastic optical soliton; the generalized Kudryashov scheme; Kerr law.

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Study of Dynamic Behavior of Pressure Taking into Consideration the Stopog Effect in a Vertical Well

Mahir Rasulov[1], Bahaddin Sinsoysal[2], Huseynova Fizze[1], Jalalov Garib[3],

[1] *Institute of Oil and Gas of ANAS, Department of Numerical Modeling of Intralayer Dynamic Processes, Baku, Azerbaijan*

mresulov@gmail.com

[2] *Istanbul Gedik University, Faculty of Engineering, Department of Computer Engineering, Istanbul, Turkey*

bsinsoysal@gmail.com

[3] *Institute of Oil and Gas of ANAS, Department of Hydrogasodynamics of the Layer Systems, Baku, Azerbaijan*

Abstract: It is known that the information necessary for the determination of some hydrodynamic indicators of oil fields can be obtained only when the well is suddenly closed and opened [1]. In this case, the time derivative is included in the boundary condition, which causes certain difficulties in the application of classical solution methods. In this article, an algorithm for finding of an approximate solution of the following dimensionless problem by the finite difference method is proposed

$$\frac{\partial u(\xi,\tau)}{\partial \tau} = \frac{\partial^2 u(\xi,\tau)}{\partial \xi^2} + f(\xi,\tau), \quad 0 \le \xi \le 1,$$

$$u(\xi,0) = 1, \delta_0 \frac{\partial u(0,\tau)}{\partial \tau} + \frac{\partial u(0,\tau)}{\partial \xi} = 0, u(1,\tau) = 1.$$

For the purpose of evaluating the approximate solution, an expression for the analytical solution of problem is obtained in the form of a series of rapidly convergent residuals

$$u(\xi,\tau) = \frac{-1}{2\pi\sqrt{-1}} \sum_{\nu} \int_{C_\nu} \lambda e^{\lambda^2 \tau} \int_0^1 G(\xi,\eta,\lambda) \Big[u_0(\xi) + \int_0^\tau e^{\lambda^2(\tau-\theta)} f(\eta,\theta) d\theta \Big] d\eta d\lambda$$

$$+ \frac{-1}{2\pi\sqrt{-1}} \sum_{\nu} \int_{C_\nu} \lambda_\nu \frac{Y(\xi,\lambda,h)}{\Delta(\lambda)} d\lambda.$$

Here, C_ν is a closed contour including only one pole of the subintegral function corresponds to $G(\xi,\eta,\lambda)$.

Keywords: Residue method, residue representation of the solution, expansion formula **2010 Mathematics Subject Classification:** 35K10, 35P10

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Basicity of eigenfunctions of second order differential operators with involution

Abdisalam A. Sarsenbi, Abzhahan M. Sarsenbi

M. Auezov South Kazakhstan University, Kazakhstan, Shymkent

abdisalam. sarsenbi @ auezov.edu.kz; abzhahan@gmail.com;

Abstract: Consider the second order nonselfadioint differential operator

$$L_{\alpha q} : D\left(L_{\alpha q}\right) \subset L_2\left(-1,1\right) \to L_2\left(-1,1\right)$$

by formula $L_{\alpha q}y = -y''\left(x\right) + \alpha y''\left(-x\right) + q\left(x\right)y\left(x\right)$ with a complex-valued coefficient $q\left(x\right) = q_1\left(x\right) + iq_2\left(x\right)$. and with domain

$D\left(L_{\alpha q}\right) = \{y\left(x\right) \in W_2^2\left[-1,1\right] : U_i\left(y\right) = a_{i1}y'\left(-1\right) + a_{i2}y'\left(1\right) + a_{i3}y\left(-1\right) + a_{i4}y\left(1\right) = 0, ,$

where a_{ij} are given complex numbers, $i = 1,2$. Linear forms $U_1\left(u\right),\ U_2\left(u\right)$ will be considered linearly independent.

Consider the differential operator $L_{\alpha q}$ whose domain is generated by one of the following four types of boundary conditions:

$$U_1\left(y\right) = y\left(-1\right) = 0,\ \ U_2\left(y\right) = y\left(1\right) = 0; (D)$$

$$U_1\left(y\right) = y'\left(-1\right) = 0,\ \ U_2\left(y\right) = y'\left(1\right) = 0(N);$$

$$U_1\left(y\right) = y\left(-1\right) - y\left(1\right) = 0,\ \ U_2\left(y\right) = y'\left(-1\right) - y'\left(1\right) = 0; (P)$$

$$U_1\left(y\right) = y\left(-1\right) + y\left(1\right) = 0,\ \ U_2\left(y\right) = y'\left(-1\right) + y'\left(1\right) = 0.(AP)$$

Theorem. Let the following three conditions be satisfied: 1) all eigenvalues of the operators $L_{\alpha q}$ are simple; 2) the complex-valued coefficient $q\left(x\right)$ belongs to the class $L_1\left(-1,1\right)$, and in the case of problems (P) and (AP) we additionally demand that $\alpha \neq 0$; 3) $\left|Im\lambda_k\right| \leq const$. Then the system of eigenfunctions of operator $L_{\alpha q}$ forms the Riesz basis in $L_2\left(-1,1\right)$.

The work was supported by a grant from the Ministry of Science and Education of the Republic of Kazakhstan (grant no.AP08855792)

Keywords: wave equation, eigenfunction, involution perturbation, eigenvalue problem, basis.

2010 Mathematics Subject Classification: 35L05, 35L20, 34B05

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MS1: Functional analysis in interdisciplinary applications

Critical exponents to the semilinear pseudo-parabolic equations

Berikbol T. Torebek [1]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Ghent University, Belgium*

torebek@math.kz

Abstract: In the present paper, we study an inhomogeneous pseudo-parabolic equation with nonlocal nonlinearity. Based on the test function method, we have proved the blow-up result for the critical case (without nonlocality), which answers an open question posed by Zhou in [1], and in particular case, it improves the result obtained in [2]. An interesting fact is that in nonlocal nonlinearity case, the problem does not admit any global solutions.

Keywords: semilinear pseudo-parabolic equation, critical exponent, nonexistence of global solution

2010 Mathematics Subject Classification: 35K70, 35A01, 35B44

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Initial inverse problem for the time-fractional wave equation

Niyaz Tokmagambetov[1]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan*

tokmagambetov@math.kz

Abstract: In this report we consider a time-fractional wave equation for positive operators, including the classical Laplacian with the Dirichlet boundary condition. Determinations of initial velocity and perturbation are investigated. It is also shown that these initial inverse problems of determining the initial data are ill-posed. Moreover, under some conditions well-posedness properties of the inverse problems are proved. As an appendix, we also provide some proof of the direct problem. Here, we develop the theoretical part of the initial inverse problems of finding the initial data for the time-fractional wave equations, preceding the study of numerical algorithms for solving these problems.

Keywords: time-fractional wave equation, initial data, ill-posed problem

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

Blow-up solutions of damped Klein-Gordon equation on the Heisenberg groups

Bolys Sabitbek

Queen Mary University of London, UK

b.sabitbek@qmul.ac.uk

Abstract: In this note, we prove the blow-up of solutions of the semilinear damped Klein-Gordon equation in a finite time for arbitrary positive initial energy on the Heisenberg group. This work complements the paper [1] by the first author and Tokmagambetov, where the global in time well-posedness was proved for the small energy solutions.

Keywords: Blow-up, sub-Laplacian, Heisenberg group, damped Klein-Gordon equation

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Recent progress on Hardy type inequalities

Durvudkhan Suragan [1]

[1] *Nazarbayev University, Kazakhstan, and*

Institute of Mathematics and Mathematical Modeling, Kazakhstan

durvudkhan.suragan@nu.edu.kz

Abstract: In this talk, we will discuss a new improvement of the classical L^p-Hardy inequality on the multidimensional Euclidean space. Recently, in [1], there has been a new kind of development of the one-dimensional Hardy inequality. Using some radialisation techniques of functions and then exploiting symmetric decreasing rearrangement arguments on the real line, the new multidimensional version of the Hardy inequality will be presented. Some consequences and generalizations will be also discussed. This talk is mainly based on [2]-[3].

Keywords: Hardy inequality, Sharp Constant, Symmetric rearrangement, Uncertainty principle.

2010 Mathematics Subject Classification: 26D10; 35A23; 46E35

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Analogue of Tricomi problem for mixed parabolic-hyperbolic equation with third-order boundary condition

Bauyrzhan Derbissaly[1,2]

[1] *Al-Farabi Kazakh National University, Almaty, Kazakhstan*

[2] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*

derbissaly@math.kz

Abstract: Consider the problem

$$Lu = \begin{cases} u_t - u_{xx}, \ t > 0, \\ u_{tt} - u_{xx}, \ t < 0, \end{cases} = f(x,t), \ (x,t) \in \Omega, \tag{1}$$

$$(b_0 u(x,t) - d_0 u_x(x,t))_{|AA_0} = 0, \tag{2}$$

$$(c_1 u_{xxx}(x,t) - a_1 u_{xx}(x,t) - d_1 u_x(x,t) + b_1 u(x,t))_{|BB_0} = 0, \tag{3}$$

$$u(x,t)_{|AC} = 0, \tag{4}$$

where Ω is a domain independent variables x and t, bounded by $t > 0$ with segments $AA_0, \ BB_0, \ A_0B_0$, where $A(0,0), \ B(1,0), \ A_0(0,1), \ B_0(1,1)$ and by $t < 0$ with characteristics $AC : x+t = 0, \ BC : x-t = 1$ of the equation (1), $a_1, b_0, b_1, c_1, d_0, d_1$ are given numbers, f is given function.

The problem (1)-(4) is called an analogue of the Tricomi problem, because the boundary condition (4) coincides with the classical condition of the Tricomi problem (see, for example [1]).

Theorem 2.1. *Let $f \in C^1(\overline{\Omega})$ then the classical solution of the problem* (1)-(4) *exists, unique, belongs to $C^1(\overline{\Omega}) \cap C^{3,1}_{x,t}(\overline{\Omega}_1) \cap C^2(\overline{\Omega}_2)$.*

Keywords: parabolic-hyperbolic type equation, Tricomi problem, problem with a spectral parameter in a boundary condition, Riesz basis.

Funding. The research is financially supported by a grant from the Ministry of Science and Education of the Republic of Kazakhstan (No. AP08855352).

2010 Mathematics Subject Classification: 35A09, 35L05, 35M13

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Inverse source problem for the Dunkl-heat equation

Bayan Bekbolat

Al-Farabi Kazakh National University, Kazakhstan
Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium
Institute of Mathematics and Mathematical Modeling, Kazakhstan
Suleyman Demirel University, Kazakhstan

bekbolat@math.kz

Abstract: In this work we study non–local in time evolution type equations generated by the Dunkl operator. Direct and inverse problems are investigated to the Caputo time-fractional heat equation with the parameter $0 < \gamma \leq 1$. In particular, well-posedness properties are established for the forward problem. To adopt techniques of the harmonic analysis we solve the problems in the Sobolev type spaces associated with the Dunkl operator. Our special interest is an inverse source problem for the Caputo-Dunkl heat equation. As additional data the final time measurement is taken. Since our inverse source problem is ill–posed we also show the stability result. Moreover, as an advantage of our calculus used here, we derive explicit formulas for the solutions of the direct and inverse problems.

Joint work with Daurenbek Serikbaev and Niyaz Tokmagambetov.

Keywords: Dunkl operator, heat equation, inverse problem, direct problem, Cauchy problem, Dunkl transform, inverse Dunkl transform.

2020 Mathematics Subject Classification: Primary 35R30; Secondary 35R11, 35C15.

References:

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Numerical simulation and parallel computing of the wave equation with a singular coefficient

Arshyn Altybay

Institute of Mathematics and Mathematical Modeling, Kazakhstan, and

al-Farabi Kazakh National University, Kazakhstan

arshyn.altybay@gmail.com

Abstract: We consider the numerical simulation of the wave equation with singular coefficients. Numerical experiments are done for the families of regularised problems in one- and two-dimensional cases. In particular, the appearance of a substantial second wave is observed, travelling in the opposite direction from the point/line of singularity. In addition, we develop GPU-based parallel computing algorithms for the two-dimensional wave equation with a singular coefficient to reduce the computational time. In this work we consider the Cauchy problem for the wave equation with singular coefficients. Namely, for $T > 0$, we study the Cauchy problem

$$(1) \quad \begin{cases} u_{tt}(t,x) - \sum_{j=1}^{d} \partial_{x_j} \left(h_j(x) \partial_{x_j} u(t,x) \right) = 0, & (t,x) \in [0,T] \times \mathbb{R}^d, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^d, \end{cases}$$

where $\boldsymbol{h} : \mathbb{R}^d \to \mathbb{R}^d$, $x \mapsto \boldsymbol{h}(x) = (h_1(x), ..., h_d(x))^T$ is a vector valued function. Our model is a general case of a well known physical model when $h = h_j,\ j = 1, \ldots, d$, is real valued. In this particular case, h denotes the water depth and u represents the free surface displacement. The singularity of h can be interpreted as sudden changes in the water depth caused by the interaction of the wave with complicated topographies of the sea floor such as bays and harbors.

Throughout this note we mainly use techniques from our works [1-2].

Keywords: wave equation, numerical Simulation, singular coefficient, regularisation, very weak solution, numerical analysis, parallel computing.

2010 Mathematics Subject Classification: 35L81, 35L05, 35D30, 35A35.

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Inverse problem of determining the source function in a time-fractional pseudo-parabolic equation

Daurenbek Serikbaev

Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Ghent University, Belgium

daurenbek.serikbaev@ugent.be

Abstract: Let $\mathcal{H}$ be a separable Hilbert space and let $\mathcal{L}$, $\mathcal{M}$ be operators with a discrete spectrum on $\mathcal{H}$. For

$$\mathcal{D}_t^{\alpha}[u(t) + \mathcal{L}u(t)] + \mathcal{M}u(t) = f \text{ in } \mathcal{H},\ 0 < t < T, \tag{1}$$

$$u(0) = \varphi \text{ in } \mathcal{H}, \tag{2}$$

we study

Inverse source problem. Let φ and $u(T)$ are given. Find a pair of functions $(u(t), f)$.

As for this kind of inverse problem for parabolic equation, see Ruzhansky et al [1] for example. Here we prove existence and uniqueness of the solution in the abstract setting of Hilbert spaces.

Keywords: Pseudo-parabolic equation, Caputo fractional derivative, weak solution, inverse problem

2020 Mathematics Subject Classification: 35R30, 35G15, 45K05

References:

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On the reverse Stein-Weiss inequality

Aidyn Kassymov[1,2],

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Ghent University, Ghent, Belgium*

kassymov@math.kz

Abstract: In this talk, we give a reverse version of the integral Hardy inequality on multi-dimensional Euclidean space in the case $0 < q \leq p < 1$. Also, as for applications we show the reverse Hardy-Littlewood-Sobolev and the Stein-Weiss inequalities in the case $0 < q \leq p < 1$ in the Euclidean space. **Keywords:** Reverse Hardy inequality, metric measure space, Reverse Hardy-Littlewood-Sobolev inequality, Reverse Stein-Weiss inequality

2010 Mathematics Subject Classification: 22E30, 43A80

Geometric and hypoelliptic functional inequalities

Nurgissa Yessirkegenov

Institute of Mathematics and Mathematical Modeling, Kazakhstan, and Suleyman Demirel University, Kaskelen, Kazakhstan

nurgissa.yessirkegenov@gmail.com

Abstract: In this talk we will discuss hypoelliptic extensions of Hardy, Sobolev, Rellich, Gagliardi-Nirenberg, Caffarelli-Kohn-Nirenberg, Hardy-Little- wood-Sobolev, Trudinger-Moser, and other inequalities on nilpotent Lie groups. We will then concentrate also on discussing their best constants, ground states for higher order hypoelliptic Schrodinger type equations, and the solutions to the corresponding variational problems. If time permits, we will discuss versions of the above inequalities in the settings of general (non-unimodular) Lie groups.

This talk is based on several joint works with Michael Ruzhansky (Queen Mary University of London and Ghent University).

Keywords: Hardy inequality, Sobolev inequality, Rellich inequality, Hardy-Littlewood-Sobolev inequality, Caffarelli-Kohn-Nirenberg inequality, noncompact Lie group, non-unimodular Lie group

2010 Mathematics Subject Classification: 46E35, 22E30, 43A15

On generalized singular number of positive matrix of τ measurable operators

Maktagul Alday [1], Madi Raikhan[2]

[1] *L. N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan*

maktagul@mail.ru

[2] *Astana IT University, Nur-Sultan, Kazakhstan*

madi.raikhan@astanait.edu.kz

Abstract: Let $(, \tau)$ be a semi-finite von Neumann algebra, $L_0()$ be the set of all τ-measurable operators, $\mu_t(x)$ be the generalized singular number of $x \in L_0()$. We extend the related inequalities of 2×2 positive semi-definite block matrices to 2×2 positive matrices of $-measurableoperators$.

Throughout this note we mainly use techniques from works [1-3].

Keywords: generalized singular number; τ-measurable operator; semifinite von Neumann algebra

2010 Mathematics Subject Classification: 46L52; 47L05

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Error analysis of rotating quarter-wave plate based Mueller Matrix polarimeter

Ömer Polat[1], Murat Kazancı[2], Abdülbaki Belet[2], Kaan Hacıosmanoğlu[2]

[1] *Faculty of Engineering , Bahcesehir University*
omer.polat@eng.bau.edu.tr

[2] *Nanoscience and Nanoengineering Program, Graduate School, Istanbul Medeniyet University, 34700 Istanbul, Turkey*

murat.kazanci@medeniyet.edu.tr

Abstract: Muller-Matrix polarimetry (MMP) technique is an effective tool used for the optical characterization of samples that is sensitive to polarized light[1]. With this technique, six different optical parameters of the sample can be investigated[2]. In this study, an error analysis of a quarter wave plate (QWP)-based Mueller Matrix polarimeter was done. Two different sources of errors, misalignment of the polarizers and the shift in retardance value, because of non-ideal QWPs, were considered. The equation of the light signal that passes through the polarimeter was modified by adding new error terms. Our results showed that, with help of the modified equation, errors in measurement arising due to optical elements of MMP can be theoretically predicted before the experiment. This leads to the measurement of the optical parameters of the samples in a more accurate way.

Keywords: Muller matrix polarimetry, light modulation, signal processing, Fourier series

2010 Mathematics Subject Classification: 78A10,78A40,78A55,42B05

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Minisymposium MS2: Fractional Chaotic Systems: Singular and Non-Singular Kernels

A local meshless RBF method for solving fractional integral equations

Mehdi Radmanesh[1], M. J. Ebadi[2,3,*]

[1] *Department of Mathematics, Graduate University of Advanced Technology of Kerman, Kerman, Iran*
m.radmenesh@gmail.com

[2] *Department of Mathematics, Chabahar Maritime University, Chabahar, Iran*

[3] *Section of Mathematics, International Telematic University Uninettuno, Corso Vittorio Emanuele II, 39, 00186 Roma, Italy*
ebadi@cmu.ac.ir

Abstract: This paper proposes a localized radial basis functions collocation method(LRBFCM) for the numerical solutions of one and two dimensional fractional integral equations (2D-FIEs). This method reduces the main problem into several local sub-problems with small sizes; Therefore, it reduces the ill-conditioning of the problem. Since the collocation approach and strong form of equation are used and that the inversion of matrices with small sizes are only needed for the matrix operations, the proposed method becomes efficient. Test problems of linear, nonlinear, Volterra and Fredholm types are presented and the efficiency of the method is shown according to the numerical results.

Keywords: Fractional calculus, Local meshless methods, Fractional integral equations(FIEs), Collocation methods.

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

References:

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q- differential equations with interval uncertainty

Mahmut Taner , N. Kuruoglu

Istanbul Gelisim University, Istanbul, Turkey
e-mail: mtaner@gelisim.edu.tr

Abstract: This presentation is devoted to obtain solutions of q-differential equations with interval uncertainty. Indeed, based on generalized Hukuhara difference, we are going to interpret any interval q-system with two related deterministic q-systems that involving fractional Caputo derivative. To do this, we provide some existence and uniqueness results of solutions of such systems, then some illustrative will be solved in detail to show the role of different type of differentiability and considering the systems with uncertainty.

Keywords: Generalized Hukuhara difference, Interval differentiability, Caputo fractional derivative, q- systems.

Fractal-Fractional Dynamic Systems: New findings

Soheil Salahshour[1,*], Ali Ahmadian[2]

[1] *Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey*
soheil.salahshour@eng.bau.edu.tr
[2] *Decision Lab, Mediterranea University of Reggio Calabria, Reggio Calabria, Italy*

Abstract: In this talk, we are going to discuss solutions of dynamic systems involving fractal-fractional derivatives. For this purpose, we firstly considered Caputo derivative and then we obtained some new findings that improved some newly published papers. Indeed, there is no direct way to cover well-known Caputo derivative using proportional Caputo derivative. In this talk, we will response completely to this deficiency.

Keywords: Fractal-Fractional Derivative, Proportional derivative, Caputo derivative.

Expected Value of Supremum of Some Fractional Gaussian Processes

Hossein Jafari[1,*], M. J. Ebadi[1,2]

[1] *Department of Mathematics, Chabahar Maritime University, Chabahar, Iran*
jafari@cmu.ac.ir
[2] *Section of Mathematics, International Telematic University Uninettuno, Corso Vittorio Emanuele II, 39, 00186 Roma, Italy*
ebadi@cmu.ac.ir

Abstract: There has been considerable interest in studying the Gaussian fractional processes due to their applications in various scientific areas, including queueing systems, telecommunications, image processing, and finance. We obtain closed-form lower and upper bounds for the expectation of the supremum of some fractional Gaussian processes, sub-fractional, bi-fractional, and multi-fractional Brownian motions which are non-stationary Gaussian processes. This expected supremum value is important in applications, such as finance and queueing systems. We apply the covariance functions, the decomposition of the processes, and probability inequalities to find bounds. Malliavin calculus techniques are applied in some cases to find bounds for the density of the supremum and the expected value of the supremum of these processes.

Keywords: Gaussian fractional processes, Lower and upper bounds, Queueing systems, Malliavin calculus, Multi-fractional Brownian motions, Fractional calculus.

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