Eighth International Conference on Analysis and Applied Mathematics

ABSTRACT BOOK

of the conference ICAAM 2026

Edited by

Charyyar Ashyralyyev,

and

Mahmud A. Sadybekov

September 26 – October 3, 2026

Bahcesehir University, Türkiye,

Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan

Ghent Analysis & PDE Center, Ghent University, Ghent, Belgium

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Edited by Charyyar Ashyralyyev and Makhmud A. Sadybekov

Eighth International Conference on Analysis and Applied Mathematics. ABSTRACT BOOK of the conference ICAAM 2026

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International Conference on Analysis and Applied Mathematics (ICAAM) aims to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why we planned the conference series to provide a forum for researchers and scientists to communicate their recent developments and present their original results in various fields of analysis and applied mathematics.

The Organizing Committee of ICAAM, Bahcesehir University, Türkiye, Institute of Mathematics and Mathematical Modelling, Kazakhstan, and Analysis PDE Center, Ghent University, Belgium are pleased to invite you to the Seventh International Conference on Analysis and Applied Mathematics, ICAAM 2026. The meeting will be held from September 26 to October, 2026, in Antalya, Türkiye.
The conference is organized biannually.
Previous conferences were held in Gumushane, Türkiye in 2012,
Shymkent, Kazakhstan in 2014,
Almaty, Kazakhstan in 2016,
Lefkoşa (Nicosia), Mersin 10, Türkiye in 2018,
Girne (Kyrenia), Mersin 10, Türkiye in 2020,
Antalya, Türkiye in 2024,
Antalya, Türkiye in 2026.
We are pleased to invite you to the eighth conference which is focused on various topics of analysis and its applications, applied mathematics, and modeling.
The conference will consist of plenary lectures, mini symposiums, and contributed oral presentations.
All proceedings of ICAAM (2012-2022) were published in AIP (American Institute of Physics) Conference Proceedings.
Selected full papers from this conference will be published in peer-reviewed journals.

We are looking forward to welcoming you to Antalya, Türkiye.

Webpage: http://icaam-online.org

Bahcesehir University, Türkiye, Institute of Mathematics and Mathematical Modeling, Kazakhstan, Ghent University, Belgium

Conference Sections and Minisymposiums :

- General Section
- Minisymposium MS1: Functional analysis in interdisciplinary applications
- Minisymposium MS2: Applied Statistics and Data Analysis
- Minisymposium MS3: Biomedical Signal Processing

Invited Speakers

- Prof. Dr. Virginia Kiryakova Institute of Mathematics and Informatics, Bulgaria
  Title:Title: Generalized fractional integrals related to Le Roy-, Feinman integrals- and Rathie I- generalized hypergeometric functions
- Prof. Dr. Ljubisa D.R. Kocinac University of Nis, Serbia
  Title:Title: On sequential Karamata theory
- Prof. Dr. Alexander L. Skubachevskii Peoples' Friendship University of Russia, Russia *Title:Title: Global solutions of the Cauchy problem for the Vlasov-Poisson system and plasma confinement*
- Prof. Dr. Andrey Muravnik Peoples' Friendship University of Russia, Russia *Title:Title: Series expansions of solutions of initial-value problems for hyperbolic differential-difference equations: computational aspects*
- Prof. Alexey Karapetyants, Institute of Mathematics, Mechanic and Com- puter Sciences, Russia Southern Federal University, Russian Federation
  Title: Aggrandization of spaces of holomorphic functions
- Prof. Dr. Alexander Meskhi Kutaisi International University, Georgia; TSU A. Razmadze Mathematical Institute, Georgia *Title:Title: Multilinear Rellich and Hardy inequalities*
- Prof. Dr. Bilal Bilalov Yildiz Technical University, Turkiye; Institute of Mathematics, Ministry of Science and Education of the Republic of Azerbaijan; Azerbaijan University of Architecture and Construction, Azerbaijan *Title:Title: On Applications of the Concept of ⊗-Basis in Approximation Theory and Partial Differential Equations*
- Prof. Dr. Nedyu Popivanov Sofia University, Bulgaria *Title:Title: Pohozhaev Identities and Applications for Mixed Type Equations*

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FOREWORD
AND
KEYNOTE TALKS

Foreword

Allaberen Ashyralyev[1], Michael Ruzhansky[2], Makhmud A. Sadybekov [3]

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Dear Esteemed Guests and Participants,

On behalf of the Organizing Committee for the 7th International Conference on Analysis and Applied Mathematics (ICAAM 2024), we extend a warm welcome to you all. It is a pleasure to have you here in Antalya for this significant event.

We sincerely appreciate the diverse group of scholars and practitioners who have traveled from various countries and cities to join us. Your interest and participation are vital to the success of this conference.

The ICAAM conference serves as a premier platform for mathematicians engaged in analysis and applied mathematics to converge, exchange insights, and explore new trends. The interplay between applied mathematics and analysis fosters innovative research and opens new avenues for exploration. This conference aims to facilitate the exchange of recent advancements and original contributions across these dynamic fields.

Our previous ICAAM conferences have been hosted in Turkiye (2012, 2022,2024), Kazakhstan (2014, 2016), and Northern Cyprus (2018, 2020). We are thrilled to welcome you back to Turkiye for this year's event. This year, we are honored to host 214 mathematicians from 21 different countries. The proceedings from past ICAAM conferences (2012-2024) were published in seven volumes by the American Institute of Physics (AIP) Conference Proceedings and are indexed in the Scopus database. Selected papers have also been featured in special issues of renowned journals such as Filomat, Boundary Value Problems, Numerical Functional Analysis and Optimization, International Journal of Applied Mathematics, and the Bulletin of the Karaganda University-Mathematics. ICAAM 2026 is organized by:

- Bahçeşehir University, Turkiye
- Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
- Analysis PDE Center, Ghent University, Belgium

We are delighted to continue our collaboration with these esteemed institutions. Our joint efforts include:

- Weekly "Analysis and Applied Mathematics" seminars, both in-person and online.
- The international journal "e-Journal of Analysis and Applied Mathematics."

These seminars have fostered global collaboration and knowledge sharing, with 97 successful sessions held since their inception in 2022-2026. Most of these sessions are available on YouTube, and a recent Springer publication features extended abstracts from these seminars, highlighting contributions from over 20 countries.

This five-day conference features 7 plenary lectures and 80 presentations, covering a broad spectrum of topics in analysis and applied mathematics. Highlights of ICAAM 2026 include:

- 227 participants from 20 countries.
- An Abstract Book for ICAAM 2026.
- Extended abstracts to be published in AIP Conference Proceedings.
- Selected full papers to be featured in special issues of journals such as Bulletin of the Karaganda University-Mathematics, Journal of Mathematical Sciences (SERIES A), and the e-Journal of Analysis and Applied Mathematics.

We believe your participation will greatly enhance the value of this conference, and we are committed to making your experience enriching and enjoyable. Should you need any assistance, please do not hesitate to contact us. Our heartfelt thanks go to: The Scientific Committee of ICAAM 2026 for their invaluable support. The distinguished speakers for their contributions and keynote addresses. Special acknowledgment to Abdullah S. Erdogan, Charyyar Ashyralyyev, Deniz Agirseven, Maksat Ashyraliyev, Yıldırım Özdemir, Özgür Yıldırım, Kheireddine Belakroum, Ramil Salimov, and Akmuhammet Ashyralyyev for their invaluable assistance. We also extend our gratitude to the leadership of Bahçeşehir University, Istanbul; Institute of Mathematics and Mathematical Modeling, Almaty; and Analysis PDE Center, Ghent University for their support and sponsorship. Thank you once again for joining us and contributing to the success of ICAAM 2026. We look forward to a stimulating and productive conference.

Warm regards,

CHAIRS

Generalized fractional integrals related to Le Roy-, Feinman integrals- and Rathie I-generalized hypergeometric functions

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Abstract: Till recently, the operators of Generalized Fractional Calculus (GFC) ([1]) with the Fox H- and Meijer G-functions as singular kernels have been considered as the most general case of integrations and differentiations of fractional orders and multi-orders. The H-function (as an extension of the G-function) is a generalized hypergeometric function including practically almost all Special Functions (SF) related to classical Calculus and to Fractional Calculus (FC). However, it happened that still there are important classes of SF that cannot be represented in terms of the H-functions, like the polylogarithms, Riemann Zeta-function, Feynman integrals, Le Roy function and their recent multi-index versions, appearing in models in statistical physics and probability, see [2]. That is why, the not popular I-function of Rathie has attracted our attention as a far extension of the H-function, since it encompasses also these mentioned classes of SF. Now, we introduce GFC operators with I-functions as singular kernels: analogues of the Erdélyi-Kober fractional integrals [3], and then generalized fractional integrals [4] extending these with H- and G-functions from [1]. It is proved that all the basic axioms of FC are satisfied, and important examples and relations to new classes of SF follow, some of them appearing as eigenfunctions for the new GFC operators.

Keywords: generalized fractional calculus, Erdélyi-Kober fractional integrals, special functions: I -, H-, G-, Mittag-Leffler and Le Roy type-functions

2020 Mathematics Subject Classification: 26A33, 33E20, 33E12, 30D20, 30D15, 34L10

References:

1. V. Kiryakova, Generalized Fractional Calculus and Applications, Longman-J. Wiley, Harlow-New York, 1994.
2. V. Kiryakova, J. Paneva-Konovska, Going Next after "A guide to special functions in fractional calculus": A discussion survey, Mathematics, vol.12, Art. 319, 2024.
3. V. Kiryakova, J. Paneva-Konovska, The generalized Fox–Wright function: the Laplace transform, the Erdélyi–Kober fractional integral and its role in fractional calculus, Mathematics, vol.12, Art. 1918, 2024.
4. V. Kiryakova, Generalized fractional integrals based on the Rathie I-function as a kernel, Nonlinear Dyn., vol. 113, 34423–34437, 2025.

On sequential Karamata theory

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Abstract: We present some recent results on Karamata theory of regular variation of sequences of positive real numbers. The results concern the so-called index function and its relation to fixed point theory [1], and connections between sequential Karamata theory and statistical convergence [2].

Keywords: Regular variation, translational regular variation, index function, statistical convergence

2020 Mathematics Subject Classification: 26A12, 40A05, 40A35, 40G15

References:

[1] D. Fatić, D. Djurčić, Lj. D. R. Kočinac, Translational regular variability and the index function, Axioms, vol. 13, no 3, 189, 2024.

[2] K. Demirci, F. Dirik, Lj. D. R. Kočinac, S. Yildiz, Regular variation and almost A-statistical convergence, Proc. Inst. Math. Mech., in oress.

A priori estimates of solutions for the Vlasov–Poisson system and plasma confinement

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Abstract: This lecture is devoted to the second mixed problem for the Vlasov–Poisson system with external magnetic fielf in a half–space. The above problem models the kinetics of the high temperature plasma in a fusion reactor. If plasma reaches a boundary of the domain, it can lead to the destruction of the reactor. Therefore we study solutions of the above mentioned mixed problem such that the supports of density distribution functions for charged particles are located strictly inside domain. For this purpose we use an external magnetic field. We prove existence of global classical solution and obtain a priori estimate for electric field strength. This estimate allows to obtain sufficient conditions for external magnetic field in algebraic form, which garantee that the supports of density distribution functions are lying at a given distance from a boundary [1, 2].

This research was sponsored by the Ministry of Science and Higher Education of the Russian Federation (Megagrant, No. 075-15-2022-1115).

Keywords: a priori estimates, Vlasov–Poisson system, plasma confinement

2020 Mathematics Subject Classification: 35Q83, 35M32

References:

[1] A. L. Skubachevskii, Vlasov–Poisson equations for a two component plasma in a homogeneous magnetic field, Russian Mathematical Surveys, vol. 69, no 2, 291–330, 2014.

[2] A. L. Skubachevskii, On the Existence of Global Solutions for the Vlasov–Poisson System in a Half-space and Plasma Confinement, Lobachevskii Journal of Mathematics, vol. 45, no 2, 280–292, 2024.

Hyperbolic equations with differential-difference operators with respect to spatial variables: case of symbols with alternating real parts

Andrey Muravnik

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Abstract: This report is devoted to multidimensional hyperbolic differential-difference equations of the kind

$$\frac{\partial^2 u}{\partial t^2} = \Delta u + au(x+h,t),$$

where a is a real parameter and $h := (h_1, \dots, h_n)$ is a vector parameter.

We explicitly construct a three-parameter family of its infinitely smooth global solutions.

The fundamental novelty (cf. [1] and references therein) is that the real part of the symbol of the differential-difference operator (with respect to the spatial independent variables) of the investigated equation is allowed to change its sign.

This research was carried out at the expense of a grant of the Russian Science Foundation No. 24-11-00073.

Keywords: Hyperbolic equations, differential-difference operators, smooth solutions, global solutions

2020 Mathematics Subject Classification: 35L10, 35R10

References:

[1] A.B. Muravnik, On hyperbolic equations with arbitrarily directed translations of potentials, Mathematical Notes, vol. 115, no 5, 772–778, 2024.

Pohozhaev identities and applications for mixed type equations

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Joint work with Daniela Lupo (Italy), Kevin Payne (USA, Italy), Lubomir Dechevski (Norway) and Evgeny Moiseev (Russia)

Abstract: Beginning from the seminal Pohodzaev paper (1965), the Pohodzaev Identities, first developed for the Dirichlet problem for elliptic quasilinear equations, have been extended to study many cases for different PDE classes of equations, even for some fractional PDE's. One of reasons is that, for example, from the Pohodzaev identities in the case of supercritical power follows very fast the nonexistence of nontrivial solutions in so called "starshaped domains". Also, in the opposite case of subcritical power (especially for elliptic equations) there exist some nontrivial solutions, look for example the remarkable results of Brezis-Nirenberg (1983) (for the history of the area see the very new book [1]).

The present talk will focus on the extension of Pohodzaev approach in the last 25 years to the study its applications for mixed elliptic-hyperbolic quasilinear equations. This study began from the series of results of Lupo - Payne, where using some symmetries in the linear homogeneous case, the dilation invariance law is strongly related to certain Pohodzaev type identities that give examples of critical exponent phenomena. Let mention here some BVP, in the linear case with Tricomi equation, Gellerstedt equation, Grushin equation, or in the last few years with the operator $L := y^{m_1}\partial^2_{xx} + x^{m_2}\partial^2_{yy}$, in some appropriated "starshaped domain", connecting with the linear case statements.

In the present talk we will fix and discuss two possible interesting questions:

(1) What happen with "existence of nontrivial solutions" in this area? First of all, as a big difference with the elliptic case, we must look in this situation not in the class of classical smooth solutions, but in some special functional weight class of "generalized solutions". And the first question is to find such class of "generalized solutions", connecting the cases of existence and nonexistence in both respectively super and sub power situations (for example see our paper [2] in the 2D case of Goursat problem for a degenerated hyperbolic equation).

(2) This is the reason why in the present talk we will give some examples of nonexistence of nontrivial solutions in supercritical and critical cases not only for the classical smooth solutions, but in the class of generalized solutions also. Some open problems will be formulated and discussed.

(3) Also, to fix how different the situation sometimes is, we explain a very old classical Guderley-Morawetz problem for mixed-type hyperbolic-elliptic equations on the plane that models transonic flows in fluid dynamics. It has been studied by Morawetz, Lax and Phillips and going later to some multidimensional BVP, namely Protter-Morawetz problems, where the situation is quite different than on the plane. In the last case using the Pohodzaev identities we proved nonexistence results (in the supercritical and critical cases) but for classical solutions only. Let mention also that in the linear situation, the Protter-Morawetz problem is strongly overdetermined for the classical solvability. Alternatively, the notion of generalized solutions that may have singularities was introduced. Looking for some appropriated class of generalized singular solutions, we found (see [3]) that even for the usual wave equation in the Protter-Morawetz domain (the Protter statement is including this case also) we constructed an infinity smooth right-hand side function, for which the uniquely determined generalized solution has an exponential growth at the only one point of singularity. The singularity is isolated at the vertex of the boundary characteristic light cone and does not propagate along the cone.

Keywords: Pohozhaev identities, power nonlinearity, a semi-Fredholm operator, critical exponent, supercritical cases, nonexistence of nontrivial solutions

2020 Mathematics Subject Classification: 35B33, 35L05, 35M12

References:

[1] V. A. Galaktionov and E. Mitidieri, Pokhozhaev's Legacy - The Art of Nonexistence and Blow-up in Nonlinear PDEs and Inequalities, Trieste-Bath-Lucinico - 2014/2026, 1–370 pages.

[2] D. Lupo D., K.R. Payne, N. Popivanov, On the degenerate hyperbolic Goursat problem for linear and nonlinear equations of Tricomi type, Nonlinear Analysis: Theory, Methods & Applications, Elsevier, 108, 2014, pp. 29–56, https://doi.org/10.1016/j.na.2014.05.009

[3] N. Popivanov, T. Popov, I. Witt, Solutions with exponential singularity for (3+1)-D Protter problems. Journal of Hyperbolic Differential Equations, 20, N2, World Scientific Publicaton, 2023, https://doi.org/10.1142/S0219891623500145, 475–498.

General Section

A class of polynomial differential systems with an unique algebraic limit cycle

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Abstract: In this report we consider a multi-parameter family of polynomial differential systems of the form

$$
(1)\qquad \begin{cases} x' = \beta^4 x + \left(4\beta^3\lambda x^3 + bx^2y + 4\beta^3\lambda xy^2 + ay^3\right) + \\ 6\beta^2 x\left(\lambda x^2 + \lambda y^2\right)^2 + 4\beta x\left(\lambda x^2 + \lambda y^2\right)^3 + x\left(\lambda x^2 + \lambda y^2\right)^4, \\ y' = \beta^4 y + \left(-bx^3 + 4\beta^3\lambda x^2y - axy^2 + 4\beta^3\lambda y^3\right) + \\ 6\beta^2 y\left(\lambda x^2 + \lambda y^2\right)^2 + 4\beta y\left(\lambda x^2 + \lambda y^2\right)^3 + y\left(\lambda x^2 + \lambda y^2\right)^4, \end{cases}
$$

where a, b, β, λ are real constants. We prove that these systems are integrable and determine the explicit expression of their first integral. Moreover, we explicitly determine an algebraic limit cycle of the system, concrete examples are presented to illustrate the applicability of our results.

Keywords: Planar polynomial differential system, First integral, Algebraic limit cycle.

2020 Mathematics Subject Classification: 34A05, 34C05, 34CO7, 34C25.

References:

[1] R. Benterki, J. Llibre, *Polynomial differential systems with explicit non-algebraic limit cycles,* EJDE, no 78 (2012), 1-6.

[2] R. Boukoucha, *On the dynamics of a family of planar differential systems with two cycles explicity given*, Rocky moutain journal of mathematics, 53 (2023), No. 6,1709-1719.

[3] R. Boukoucha, *Explicit limit cycles of a family of polynomial differential systems*, EJDE, Vol. 2017 (2017), No. 217, pp. 1-7.

[4] F. Dumortier, J. Llibre, J. Artés, *Qualitative Theory of Planar Differential Systems,* (Universitex) Berlin, Springer (2006).

[5] J. Chavarriga, H. Giacomini, J. Giné, J. Llibre, *Darboux integrability and the inverse integrating factor*, J. Differential Equations 194 (2003), no. 1, 116-139.

[6] C. Christopher, J. Llibre, *Integrability via invariant algebraic curves for planar polynomial differential systems*, Ann. Differential Equations 16. (200) 5-19.

[7] A. Gasull, H. Giacomini, J. Torregrosa, *Explicit non-algebraic limit cycles for polynomial systems,* J. Comput. Appl. Math. 200 (2007) 448-457.

Modeling Tumor-Immune Dynamics: Synergistic Combination Treatment and the Role of Dynamic PD-L1 Regulation

Bruce Pell [1], Aigerim Kalizhanova[2], Aisha Tursynkozha[3], Denise Dengi[4], Ardak Kashkynbayev[2], Yang Kuang [5]

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Abstract: The efficacy of cancer immunotherapies, particularly immune checkpoint inhibitors (ICIs), is often hindered by adaptive immune resistance mediated by the PD-1/PD-L1 pathway. Combination therapies, such as pairing an ICI with a potent immunostimulant, represent a promising strategy to overcome this challenge. Foundational models of single-agent and combination (Avelumab/NHS-muIL12) immunotherapies, while capturing key dynamics, fail to reproduce complex preclinical data like non-monotonic tumor responses.

The main focus of this talk is the introduction of a refined model that addresses this gap by treating PD-L1 expression as a dynamic variable. This variable accounts for basal production, activation by immunostimulants, and suppression by anti-PD-L1 drugs. We will show how this refined model, with a single set of parameters, successfully reproduces the experimental data across all six treatment arms including monotherapies and combinations. This approach provides a robust, mechanistic explanation for both dose-dependent treatment failure (driven by adaptive PD-L1 upregulation) and therapeutic synergy. We will discuss the systematic development of the model and simulation results, alongside a rigorous mathematical analysis of the generic and full models focusing on the existence, uniqueness, and local asymptotic stability of tumor-free and tumorous equilibria [1].

Keywords: cancer immunotherapy; differential equations; PD-L1; adaptive immune response; mathematical modeling; immunostimulant.

2020 Mathematics Subject Classification: 92C50, 92C60, 34D23, 34C60

References:

[1] Pell, B., Kalizhanova, A., Tursynkozha, A., Dengi, D., Kashkynbayev, A., & Kuang, Y. (2025). Dynamic PD-L1 regulation shapes tumor immune escape and response to immunotherapy. Cancers, 17(23), 3803. https://doi.org/10.3390/cancers17233803

Mathematical modeling of heat transfer in hollow building blocks with internal partitions

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Abstract: The work examines mathematical modeling of heat transfer in hollow building blocks with internal partitions. The installation of internal partitions suppresses large-scale convective flows and reduces radiative transfer, resulting in a reduction in overall heat flux by tens of percent.

The authors' own numerical results obtained using computational fluid dynamics methods for various block configurations, including those with vertical and inclined partitions and low-emission coatings, will be presented. The influence of geometric parameters (height, orientation, number of partitions) on the flow structure, temperature fields and effective thermal conductivity is shown. Furthermore, the work presents a critical review of current research in this area, including experimental and numerical studies. Special attention is given to a comparative analysis of various methods for increasing the thermal resistance of blocks and identifying the most promising configurations [1-4]. The results can be used in the design of energy-efficient wall materials.

This work was supported by the Russian Science Foundation (Project No. 25-79-10293).

Keywords: Mathematical modeling, hollow building blocks, boundary conditions, convection, radiation, conduction

2020 Mathematics Subject Classification: 76D05, 80A19, 80A21

References:

[1] N. Bondareva, I. Miroshnichenko, V. Simonova, M. Sheremet, Incorporation of Bio-Based Infills into Hollow Building Blocks: A Comprehensive Review, Energies, vol. 19, 1965, 2026.

[2] Y. Chihab, M. Garoun, N. Laroussi, Dynamic thermal performance of multilayer hollow clay walls filled with insulation ma-terials: Toward energy saving in hot climates, Energy and Built Environment, vol. 5, 70–80, 2024.

[3] J. M. Blanco, Y. García Frontera, M. Madrid, J. Cuadrado, Thermal Performance Assessment of Walls Made of Three Types of Sustainable Concrete Blocks by Means of FEM and Validated through an Extensive Measurement Campaign, Sustainability, vol. 13, 396, 2021.

[4] S. Summa, G. Remia, C. Di Perna, F. Stazi, Experimental and numerical study on a new thermal masonry block by comparison with traditional walls, Energy and Buildings, vol. 292, 113125, 2023.

A deterministic approach to global optimization based on an auxiliary function

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Abstract: This report is devoted to a deterministic method for the global optimization of a multimodal objective function in many variables, based on a special auxiliary function.

The auxiliary function is constructed via an integral transformation of the objective function $F(x)$ and is a function of a single variable $\alpha \in \mathbb{R}$:

$$g_m(F,\alpha) = \int_E (|F(x)-\alpha| - (F(x)-\alpha))^m \, dx, \qquad m \in \mathbb{N},\ m > 1.$$

The function $g_m(F,\alpha)$ possesses several important properties, including continuity, differentiability, convexity, monotonicity, and others. These properties, together with its dependence on only one variable, make the auxiliary function an effective and unique tool for solving global optimization problems.

In this work, based on the properties of the function $g_m(F,\alpha)$ previously established by the authors [1-2], necessary and sufficient conditions for the existence of a global minimum of the function $F(x)$ are obtained. It is shown that the value of the global minimum of the objective function coincides with the upper bound of the zeros of the auxiliary function. Thus, the problem of finding the global minimum of the original objective function is reduced to the problem of finding the largest zero of the auxiliary function.

In addition to various multimodal functions of many variables, the objective function may also represent loss functions arising in machine learning problems, as well as functions describing economic, physical, and other processes. The main requirement imposed on the objective function is continuity.

Using the proposed method, the problem of minimizing the loss function of a nonlinear regression model and recovering its parameters is solved. A complete algorithm for finding the global minimum of the loss function is proposed, and its numerical implementation is carried out with recovery of the model parameters to a prescribed accuracy.

Keywords: Global optimization, nonlinear regression, loss function, auxiliary function, deterministic methods.

2020 Mathematics Subject Classification: 90C26, 62F30, 65K05 **References:**

[1] Zh. S. Tutkusheva, G. N. Kazbekova, R. B. Seilkhanova, A. K. Kairakbaev, Wegstein's method for calculating the global extremum, Mathematical Modelling of Engineering Problems, vol. 9, no 2, 405–410, 2022.

[2] Zh. S. Tutkusheva, K. T. Otarov, Application of the auxiliary function method to the search for the global minimum of functions of many variables, Mathematics and Mechanics of Engineering Problems, vol. 11, no 5, 1323–1329, 2024.

Very weak well-posedness of parabolic equations with singular coefficients and inhomogeneous Dirichlet data

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Abstract: We study the very weak well-posedness of the initial–boundary value problem for the parabolic equation

$$\begin{cases} \partial_t u(t,x) - \nabla \cdot \big(A(t,x)\nabla u(t,x)\big) + b(t,x)\cdot \nabla u(t,x) + V(x)u(t,x) = f(t,x), \\ \hfill (t,x) \in (0,T)\times\Omega, \\ u(0,x) = u_0(x), \quad x \in \Omega, \\ u(t,x) = g(t,x), \quad (t,x) \in (0,T)\times\partial\Omega, \end{cases}$$

where the diffusion matrix A, drift term b, potential V, source term f, initial datum u_0, and boundary datum g may have singularities of distributional type. In the regular case, we first establish existence, uniqueness, and a priori estimates for weak solutions under non-homogeneous Dirichlet boundary conditions. We then introduce a very weak solution framework based on regularisation of the coefficients and data. Under suitable distributional ellipticity of A and non-negativity of V, we prove existence and uniqueness of very weak solutions. Finally, we show consistency with the classical theory: when the coefficients and data are regular, representatives of the very weak solution converge to the unique weak solution in the natural energy norm.

Keywords: parabolic equation, divergence form, shift term, energy methods, Galerkin approximation, singular coefficients, regularisation, very weak solution

2020 Mathematics Subject Classification: 35K20, 35D30, 35K67

Solution of a nonlocal problem for the heat equation using the potential method

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Abstract: One nonlocal problem for the heat equation is considered

$$L(u) \equiv u_t - u_{xx} = 0 \quad \textit{in domain} \quad \Omega = \{(x,t) : 0 < x < l, \quad 0 < t < T\} \tag{1}$$

with initial conditions

$$u(x,0) = f(x), \quad 0 \le x \le l \tag{2}$$

boundary conditions of the form

$$\begin{cases} a_1 u(0,t) + a_2 u(l,t) = \varphi(t), \\ b_1 u_x(0,t) + b_2 u_x(l,t) = \psi(t), \end{cases} \qquad 0 \le t \le T \tag{3}$$

the coefficients $a_i, b_i (i = 1, 2)-$ real numbers.

Using the potential method, problem (1)-(3) is reduced to Volterra integral equations. The solution of the Volterra integral equation was found and the solvability of the stated problem (1)-(3) was investigated.

Acknowledgments: This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. BR31714735)

Keywords: heat equation, nonlocal problems, potential method, integral equations, solvability conditions

2020 Mathematics Subject Classification: 35A09, 35L20, 35M13

Solution of forward and inverse problems for the convection-diffusion-reaction equation using neural networks

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Abstract: This paper considers forward and inverse problems for a two-dimensional convection–diffusion–reaction equation describing the propagation of harmful aerosol substances in the atmosphere. The mathematical model takes into account diffusion processes, convective transport by airflow, chemical reactions, and localized pollution sources represented by delta functions. The Physics-Informed Neural Networks (PINN) approach is applied for solving the considered problems.

The computational domain is defined as

$$\Omega = (0,1) \times (0,1) \times (0,T), \tag{1}$$

and the governing equation is given by

$$\frac{\partial \varphi_q}{\partial t} + \nabla \cdot (\mathbf{w}\varphi_q) = \Delta\varphi_q + \alpha_q\varphi_q + \beta_q + f_q(x,y), \tag{2}$$

where $\mathbf{w} = (u, \vartheta)$ is the airflow velocity vector, $\varphi_q(x,y,t)$ denotes the aerosol concentration, α_q and β_q are reaction coefficients, and $f_q(x,y)$ describes pollution sources.

The pollution sources are represented as a superposition of delta functions:

$$f_q = \sum_{j=1}^{m} Q_j \delta(\mathbf{r} - \mathbf{r}_j), \tag{3}$$

where Q_j is the source intensity and $\mathbf{r}_j$ denotes the source location.

The initial concentration distribution is defined by

$$\varphi_0(x,y) = \frac{Q_j}{2\sqrt{\mu\sigma}} \exp\left(-\sqrt{\frac{(x-x_j)^2 + (y-y_j)^2}{\mu}}\right). \tag{4}$$

In the forward problem, the concentration distribution is determined from known initial and boundary conditions. In the inverse problem, unknown model parameters such as source intensities, diffusion coefficients, or reaction parameters are reconstructed using observational concentration data.

Within the PINN framework, the solution is approximated by a neural network $\varphi_\theta(x,y,t)$, where θ denotes trainable parameters. The loss functional is defined as

$$\mathcal{L} = \mathcal{L}_{PDE} + \mathcal{L}_{IC} + \mathcal{L}_{BC} + \mathcal{L}_{data}. \tag{5}$$

Here $\mathcal{L}_{PDE}$ enforces the governing equation, $\mathcal{L}_{IC}$ corresponds to the initial condition, $\mathcal{L}_{BC}$ represents the boundary conditions, and $\mathcal{L}_{data}$ minimizes the discrepancy between observational data and neural network predictions.

Numerical experiments demonstrate that the PINN method provides stable and accurate approximations without constructing computational meshes. The proposed approach effectively solves both forward and inverse problems and can be applied to atmospheric pollution modeling and other multidimensional problems of mathematical physics.

Keywords: convection–diffusion equation, PINN, neural networks, forward problem, inverse problem, atmospheric pollution, deep learning.

2020 Mathematics Subject Classification: 35K57, 35R30, 65M99, 68T07.

References:

[1] M. Raissi, P. Perdikaris, G.E. Karniadakis, Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, *Journal of Computational Physics*, 378 (2019), 686–707.

[2] G.E. Karniadakis, I.G. Kevrekidis, L. Lu, et al., Physics-informed machine learning, *Nature Reviews Physics*, 3 (2021), 422–440.

I-lacunary statistical convergence in partial metric spaces

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Abstract:

In this paper, we make a new framework for statistical convergence by using ideals and introduce new notions, namely, I-statistical convergence and I-lacunary statistical convergence in Partial Metric spaces and mainly investigate their relationship and also make some observations about these classes

Keywords: Ideal, filter, I-statistical convergence, I-lacunary statistical convergence, partial metric space.

2020 Mathematics Subject Classification: 40A05, 40D25

References:

[1] S.G. Matthews,Partial metric topology, Ann. New York Acad. Sci.,728(1994), 183197.

[2] Pratulananda Das, E. Savas, S. K. Ghosal, On generalizations of certain summability methods using ideals, Appl. Math. Lett. 24 (2011) 1509 - 1514.

[3] Erdal Bayram and Cigdem A. Bektas, Lacunary statistical convergence of orde α in Partial metric spaces, Facta Universitatis (Nis) Ser. Math. Inform. 38(4) (2023), 761769

Parametric nonlocal initial-boundary value problems for a degenerate hyperbolic equation

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Abstract: Let $\Omega = \{(x, y) :\ 0 < x < 1,\ 0 < y < T\}$ be a rectangular domain. Consider the following degenerate hyperbolic equation:

$$Lu \equiv y^m u_{xx} - u_{yy} - b^2 y^m u = 0, \tag{1}$$

where $m > 0$, $b \geq 0$, and $T > 0$ are given real parameters.

In this work, we study two problems for equation (1) with the initial conditions and different parameter-dependent nonlocal boundary conditions.

Problem P_α. *Find a function $u(x, y) \in C^1\left(\overline{\Omega}\right) \cap C^2(\Omega)$, which is a solution of equation* (1) *in the domain Ω, satisfies the classical initial conditions*

$$u(x, 0) = \tau(x), \quad u_y(x, 0) = \nu(x), \quad 0 \leq x \leq 1, \tag{2}$$

and the following nonlocal boundary conditions:

$$u_x(0, y) = \alpha\, u_x(1, y), \quad 0 \leq y \leq T, \tag{3}$$

$$u(0, y) = u(1, y), \quad 0 \leq y \leq T, \tag{4}$$

where α is a given real parameter, and $\tau(x)$ and $\nu(x)$ are sufficiently smooth given functions.

Problem P_β. *Find a function $u(x, y) \in C^1\left(\overline{\Omega}\right) \cap C^2(\Omega)$, which is a solution of equation* (1) *in the domain Ω, satisfies the classical initial conditions* (2)*, and the following nonlocal boundary conditions:*

$$u_x(0, y) = u_x(1, y), \quad 0 \leq y \leq T, \tag{5}$$

$$u(0, y) = \beta\, u(1, y), \quad 0 \leq y \leq T, \tag{6}$$

where β is a given real parameter, and $\tau(x)$ and $\nu(x)$ are sufficiently smooth given functions.

Using the spectral analysis method, the main results on the existence and uniqueness of solutions to Problems P_α and P_β have been obtained, and the corresponding theorems have been proved.

The key feature of the problem under study is that the system of eigenfunctions of the spectral problem (which arises when applying the Fourier method) does not form an unconditional basis. Therefore, direct application of the Fourier method is impossible. To overcome this, an auxiliary basis is constructed using eigenfunctions.

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Keywords: degenerate hyperbolic equation, initial condition, nonlocal boundary conditions, regular boundary conditions, root functions, Riesz basis,

2020 Mathematics Subject Classification: 35L20

References:

[1] Bimenov M., Omarbaeva A. Nonlocal Initial-Boundary Value Problems for a Degenerate Hyperbolic Equation, in: *Analysis and Applied Mathematics. AAM 2022. Trends in Mathematics, vol. 6*, Birkhäuser, Cham (2024), 3–45. doi:10.1007/978-3-031-62668-5_14

The mixed initial-boundary value problem for the heat equation with a discontinuous coefficient under general conjugation conditions

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Abstract: An initial-boundary value problem for the heat equation with a discontinuous coefficient is considered:

$$\frac{\partial u}{\partial t} = a_i^2 \frac{\partial^2 u}{\partial x^2}, \tag{7}$$

in the domain $\Omega = \Omega_1 \cup \Omega_2$, $\Omega_1 = \{(x,t) : l_0 < x < l_1, 0 < t < T\}$,
$\Omega_2 = \{(x,t) : l_1 < x < l_2, 0 < t < T\}$, $(i = 1,2)$, with initial conditions

$$u(x,0) = \varphi(x), \quad l_0 \le x \le l_2, \tag{8}$$

boundary conditions of the form

$$u(l_0,t) = u_x(l_2,t) = 0, \quad 0 \le t \le T, \tag{9}$$

and conjugation conditions [1]

$$\begin{cases} a_{11}u_x(l_1-0,t) + a_{12}u_x(l_1+0,t) + a_{13}u(l_1-0,t) + a_{14}u(l_1+0,t) = 0, \\ a_{21}u_x(l_1-0,t) + a_{22}u_x(l_1+0,t) + a_{23}u(l_1-0,t) + a_{24}u(l_1+0,t) = 0. \end{cases} \tag{10}$$

The theorem on the existence and uniqueness of a classical solution to the problem (1)-(4) is proved [2].

Keywords: heat equation, discontinuous coefficients, eigenvalues, eigenfunctions, method of separation of variables

2020 Mathematics Subject Classification: 35K10, 35K20, 35P10

References:

[1] U. Koilyshov, M. Sadybekov, Two-phase heat conduction problems with Sturm-type boundary conditions, Boundary Value Problems, vol. 2024, Article ID 149, 2024.

[2] U. Koilyshov, M. Sadybekov, K. Beisenbayeva, Solution of nonlocal boundary value problems for the heat equation with discontinuous coefficients in the case of two discontinuity points, Bulletin of the Karaganda University. Mathematics Series, vol. 117, no 1, 81–91, 2025.

Absolute stable difference scheme for numerical solving source identification elliptic problem with nonlocal condition

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Abstract Source identification problems arise in many areas of science and engineering, where unknown internal sources must be reconstructed from indirect observations. Such inverse problems are encountered in heat transfer, diffusion processes, geophysics, medical imaging, and environmental modeling ([1]-[3]).

In the paper [1], the well-posedness of the source identification problem for an abstract elliptic equation with nonlocal boundary conditions in a Banach space was established. The problem is formulated as

$$(1) \qquad \begin{cases} -v''(t) - Av(t) = p + f(t), 0 < t < 1 \ , \\ v(0) = v(1), \\ v'(0) = v'(1) \end{cases}$$

where A is a positive operator, the parameter $p \in E$ and the function $v : [0,1] \to E$ are unknown, while $f : [0,1] \to E$ is given.

Although the continuous problem has been shown to be well-posed, practical applications require reliable numerical methods. This motivates the development of stable and convergent difference schemes that can efficiently approximate both the unknown source parameter and the solution.

This work is devoted to construct and analyze a finite difference scheme for the source identification problem, establish stability estimates that guarantee the well-posedness of the discrete problem, and demonstrate the accuracy and efficiency of the proposed method through numerical experiments.

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Keywords: difference scheme, elliptic equation, inverse problem, source identification problem, stability estimate.

2020 Mathematics Subject Classification: 39A14, 65N06, 65J22

References:

[1] A. Ashyralyev, C.Ashyralyyev, V. G. Zvyagin, A note on well-posedness of source identification elliptic problem in a Banach space, Bulletin of the Karaganda University, 99 (3), 2020

[2] C. Ashyralyyev, A.Çay, Well-posedness of Neumann-type elliptic overdetermined problem with integral condition. In: AIP Conference Proceedings,Vol.1997, 020020, 2018

[3] A. Ashyralyev, C. Ashyralyyev, Well-posedness of SI problem for an elliptic equation in a Banach space with mixed boundary conditions, Lobachevskii Journal of Mathematics 44 (8), 3241-3249,2023.

Theoretical analysis of variational formulations and finite element approximations for the Navier–Stokes equations

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Abstract: This report is devoted to the variational formulation and finite element approximation of the nonstationary Navier–Stokes equations:

$$\partial_t u + (u \cdot \nabla)u - \nu\Delta u + \nabla p = f, \qquad \nabla \cdot u = 0. \tag{1}$$

The problem is supplemented with the initial condition

$$u(x, 0) = u_0(x) \tag{2}$$

and homogeneous Dirichlet boundary conditions.

Let $V \subset H^1(\Omega)^d$ and $Q \subset L^2(\Omega)/\mathbb{R}$ be the velocity and pressure spaces, respectively. The bilinear and nonlinear forms are defined as follows: $a(u, v)$ is the diffusion term, $b(v, p)$ is the pressure term, and $c(w; u, v)$ is the nonlinear convective term.

The weak formulation consists in finding $(u, p) \in V \times Q$ such that:

$$(\partial_t u, v) + c(u; u, v) + a(u, v) + b(v, p) = (f, v), \tag{3}$$

$$b(u, q) = 0, \qquad \forall (v, q) \in V \times Q. \tag{4}$$

Finite-dimensional approximations satisfying the discrete Ladyzhenskaya–Babuška–Brezzi (LBB) condition are considered. Under suitable regularity assumptions, the finite element approximation satisfies the error estimate:

$$\|u(t^n) - u_h^n\|_{L^2} \leq C(h^2 + \Delta t). \tag{5}$$

The results confirm the convergence of the finite element method for incompressible flow problems.

Keywords: Navier–Stokes equations, variational formulation, finite element method.

2020 Mathematics Subject Classification: 35Q30, 65M60, 76D05. **References:**

[1] Temam R. *Navier–Stokes Equations: Theory and Numerical Analysis*, North-Holland, Amsterdam (1977).

[2] Brenner S.C., Scott L.R. *The Mathematical Theory of Finite Element Methods*, Springer, New York (2008).

Geometric criteria for controllability in arbitrary Banach spaces

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Abstract: We present geometric criteria ensuring controllability properties for dynamical systems evolving in arbitrary (possibly infinite-dimensional) Banach spaces. Let $(X, \|\cdot\|)$ be a real Banach space and consider the set-valued evolution

(1) $$x'(t) \in F(t, x(t)) \subset X,\ t \in [t_0, T],\ x(t_0) = x_0,$$

where $F(t, x)$ is a nonempty subset of X. The reachable set at time t is

(2) $$\mathcal{R}(t; x_0) = \{x(t) \in X :\ x(\cdot) \text{ solves the dynamics with } x(t_0) = x_0\}.$$

Controllability is reformulated as a geometric property of these reachable sets. For a closed target $S \subset X$, exact controllability requires $\mathcal{R}(T; x_0) \cap S \neq \emptyset$, while approximate controllability requires $\overline{\mathcal{R}(T; x_0)} \cap S \neq \emptyset$; approximate null-controllability corresponds to $0 \in \overline{\mathcal{R}(T; x_0)}$.

The approach is based on set-valued analysis and viability-type arguments on suitable constraint sets, together with separation theorems and tangent (Bouligand/contingent) cones. For a closed set $K \subset X$ and $x \in K$, the contingent cone is

(3) $$T_K(x) = \{v \in X :\ \exists\, h_n \downarrow 0^+,\ \exists\, v_n \to v,\ x + h_n v_n \in K\}.$$

A tangency (viability) condition of the form $F(t, x) \cap T_K(x) \neq \emptyset$ keeps trajectories in K. Encoding the control objective through an epigraph set $\mathcal{K} = \{(x, r) \in X \times \mathbb{R} : r \geq \phi(x)\}$, where ϕ is a lower semicontinuous distance-to-target functional (e.g. $\phi(x) = \|x\|$ or $\phi(x) = \operatorname{dist}(x, S)$), near-viability of $\mathcal{K}$ under an augmented dynamics yields quantitative decrease estimates of $\phi(x(t))$ and hence approximate steering to the target. Finally, in the dual space X^*, approximate controllability is shown to be equivalent to the nonexistence of a separating functional between the target and the reachable sets, providing the infinite-dimensional analogue of finite-dimensional geometric separation arguments.

Throughout this note we mainly use techniques from set-valued analysis and viability theory [1, 2], control theory [3], and semigroup methods [4].

Keywords: Banach spaces, controllability, reachable set, viability, contingent cone, set-valued dynamics, separation

2020 Mathematics Subject Classification: 93B05, 49J53, 34A60, 47D06

References:

[1] J.-P. Aubin and A. Cellina, Differential Inclusions, Springer, 1984.
[2] J.-P. Aubin and H. Frankowska, Set-Valued Analysis, Birkhäuser, 1990.
[3] A. Bressan and B. Piccoli, Introduction to the Mathematical Theory of Control, AIMS, 2007.
[4] A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer, 1983.

Pathwise Fourier continuation for Bermudan and American options: a sinh-accelerated NUFFT–RQMC framework

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Abstract: We develop a pathwise Fourier-continuation framework for Bermudan and American option pricing. The method combines randomized quasi-Monte Carlo path simulation, sinh-accelerated Fourier inversion, and type-3 nonuniform FFT evaluation so that, at each exercise date, continuation values are computed from the one-step characteristic function directly at the simulated states. This replaces the regression fit in least-squares Monte Carlo by a transform-based conditional expectation while retaining a simulation architecture. The framework is formulated for one-factor exponential models with explicit one-step characteristic functions; the continuously monitored barrier treatment considered here is specific to geometric Brownian motion, where a Brownian-bridge survival correction is applied in log space without changing the Fourier continuation step.

Numerical experiments recover standard GBM American-put benchmarks accurately, show competitive bias and RMSE relative to polynomial and Laguerre-basis LSMC on representative cases, and demonstrate the continuation machinery under a pure-jump CGMY model. For continuously monitored up-and-out puts under GBM, the barrier extension gives prices in close agreement with published benchmark values.

Keywords: American options; Bermudan options; Fourier continuation; nonuniform fast Fourier transform; randomized quasi-Monte Carlo; sinh acceleration; barrier options; CGMY model.

JEL classification: C63; G12; G13.

Heat equation with a singular potential in domains with non-smooth boundaries

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Abstract. This paper investigates the behavior of nonnegative generalized solutions to the first initial-boundary value problem for a linear heat equation with a singular potential in a cylindrical domain $Q = \Omega \times (0, T)$, where $\Omega \subset R^n$ $(n \geq 3)$ is a neighborhood of the edge containing the origin and $T > 0$. In a neighborhood of the origin, the domain Ω is locally diffeomorphic to a a dihedral angle of opening β_0 $(0 < \beta_0 \leq 2\pi)$. A sharp condition on the potential is found that ensures the existence and/or non-existence of nonnegative solutions to this problem.

Keywords: heat equation, singular potential, generalized solution, nonnegative solution, behavior of solutions, sharp condition, existence and non-existence.
2020-Mathematics Subject Classification: 35K05, 35K20, 35K67

Let Ω be a bounded domain in R^n $(n \geq 3)$, whose boundary $\partial\Omega$ consists of two smooth $(n-1)-$dimensional surfaces Γ_1 and Γ_2 intersecting along an $(n-2)-$dimensional manifold (edge) Γ_0, such that the origin lies on the edge: $0 \in \Gamma_0$. It is assumed that in a neighborhood of the origin, the domain Ω is locally diffeomorphic to a dihedral angle of opening β_0 $(0 < \beta_0 \leq 2\pi)$.
We consider the first initial-boundary value problem in the cylinder $Q = \Omega \times (0, T)$:

$$u_t - \Delta u - V(x)u = 0, \quad (x, t) \in \Omega \times (0, T), \tag{1}$$

$$u(x, t) = 0, \quad x \in \partial\Omega, \ \ t \in (0, T); \quad u(x, 0) = u_0(x), \quad x \in \Omega. \tag{2}$$

It is assumed that $0 \leq V \leq L^1_{loc}(\Omega)$ and $0 \leq u_0 \leq L^2(\Omega)$.

The initial condition holds in the sense of

$$ess \lim_{t \to +0} \int_{\Omega} u(x, t)\eta(x)dx = \int_{\Omega} u_0(x)\eta(x)dx$$

for any function $\eta \in D(\Omega) = C_0^\infty(\Omega)$.

Definition. Let $Q = \Omega \times (0, T)$. A function $u(x, t)$ is called a *nonnegative weak (generalized) solution* to the problem (1)-(3) in the sense of distributions if $u \in L^1_{loc}(Q)$, $Vu \in L^1_{loc}(Q)$, and for any function $\psi \in C^\infty(\overline{Q})$ such that $\psi(x, T) = 0$ and $\psi|_{\partial\Omega \times (0,T)} = 0$, the following integral identity holds:

$$\iint_{Q} u(-\psi_t - \Delta\psi - V(x)\psi)dxdt = \int_{\Omega} u_0(x)\psi(x, 0)dx.$$

In a neighborhood of the point $0 \in \Gamma_0$, we introduce local spherical coordinates: $r = |x| \in (0, \delta)$, $(\delta > 0)$ and $\omega \in \Omega_\omega \subset S^{n-1}$, where Ω_ω is the spherical cross-section of the dihedral angle on the unit sphere. The Laplace operator takes the form:

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{n-1}{r} \cdot \frac{\partial}{\partial t} + \frac{1}{r^2}\Delta_\omega,$$

where Δ_ω is the Laplace-Beltrami operator on S^{n-1}.

Consider the Dirichlet eigenvalue problem on the spherical segment Ω_ω :

$$-\Delta_\omega Y(\omega) = \lambda Y(\omega), \quad \omega \in \Omega_\omega; \qquad Y(\omega) = 0, \quad \omega \in \partial\Omega_\omega.$$

Let λ_1 be the principal (first) eigenvalue of this problem and let $Y_1(\omega)$ be the eigenfunction corresponding to the principal eigenvalue λ_1.

Let $\varphi(x) = |x|^{-\alpha}Y_1(\omega)$. Let us find the function $V_0(x)$ satisfying the equation $\Delta\varphi + V_0(x)\varphi = 0$ in $D'(\Omega)$. Since $-\Delta\varphi = [\alpha(n-2-\alpha) + \lambda_1]|x|^{-\alpha-2}Y_1(\omega)$, we obtain: $-\Delta\varphi/\varphi = c|x|^{-2}$, where $c = \alpha(n-2-\alpha) + \lambda_1$. From the last equality, it follows that the number α is defined by the formula

$$\alpha = \frac{n-2}{2} - \sqrt{\frac{(n-2)^2}{4} + \lambda_1 - c} = \frac{n-2}{2} - \sqrt{C_H - c}.$$

It is clear that when $0 \leq c \leq C_H = \frac{(n-2)^2}{4} + \lambda_1$, the number α is a real and $\Delta\varphi \in L^1_{loc}(\Omega)$.

We set: $V_0(x) = c|x|^{-2}, \ \ x \in \Omega$.

Note that the constant C_H is the best Hardy constant for non-smooth domains and is not achievable by functions in the Sobolev space.

The main result of this paper is the following theorem:

Theorem. 1. If $0 \leq c \leq C_H$ and $V(x) \leq V_0(x)$ in the domain Ω, then problem (1),(2) has a nonnegative generalized solution for any nonnegative initial function $u_0 \in L^2(\Omega)$.

2. If $c > C_H$ and $V(x) \geq V_0(x)$ in the domain Ω, then for $u_0(x) > 0$ problem (1),(2) has not nonnegative generalized solutions.

Note that in the case of a conical domain with vertex at the origin, problem (1),(2) was studied in [1].

References

[1]. B. A. Khudaykuliyev. The heat equation with a singular potential.// Differensial'nye Uravneniya. 1997, Vol/33, No.4, pp.567-569 (in Russsian).

Method of generalized jacobi elliptic functions for solving some classes of nonlinear elliptic systems of second-order equations on the plane

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Abstract: In this paper, generalized Jacobi elliptic functions $dn\,\omega(z)-$ delta-amplitude are applied to the construction of particular solutions of certain classes of elliptic systems of partial differential equations of second order on the complex plane C with Cauchy–Riemann operators $2\partial_{\overline{z}} = \partial_x + i\partial_y,\ 2\partial_z = \partial_x - i\partial_y$, and the Bitsadze operator $4\partial^2_{\overline{z}\,\overline{z}} = 4\partial_{\overline{z}}(\partial_{\overline{z}}) = \partial^2_{xx} - \partial^2_{yy} + 2i\partial^2_{xy}$, where the variables $z = x + iy,\ \overline{z} = x - iy$ are independent.

The function $\omega(z)$ is a quasi-periodic homeomorphism of the Beltrami equation

$$\omega_{\overline{z}} - q\omega_z = 0, \quad |\,q\,| \neq 1,$$

satisfying the condition

$$\omega(0) = 0, \quad \omega(z + h_j) = \omega(z) + \tau_j, \quad j = 1, 2,$$

where $\mathrm{Im}(h_2/h_1) \neq 0,\ \mathrm{Im}(\tau_2/\tau_1) \neq 0$.

The function $\omega(z)$ quasiconformally maps any parallelogram Ω with vertices $z_0,\ z_0 + h_1,\ z_0 + h_1 + h_2,\ z_0 + h_2$ onto a quadrilateral Ω' with vertices $\omega(z_0),\ \omega(z_0) + \tau_1,\ \omega(z_0) + \tau_1 + \tau_2,\ \omega(z_0) + \tau_2$, and for $|\,q\,| < 1$ the orientation of the boundaries of Ω and Ω' is preserved.

Keywords: solution of equations, modulus of elliptic function, Jacobi elliptic functions, analytic function.

2020 Mathematics Subject Classification: 5A0,9 35B36

Heat transfer model with involution: analytical approaches and FDM analysis

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Abstract: In this study, a heat transfer model on a metal rod of length L, folded in half from its exact center ($t = 0$) into a U-shape, is investigated theoretically and numerically. While differential equations with involutions have been extensively studied by various researchers in the literature [2-5], the symmetric interaction resulting from the geometric structure of the rod and the proximity of its arms is modeled using a first-order differential equation with involution. Two different analytical methods frequently encountered in the literature, namely the separation into even-odd components and the order-elevation method, are evaluated for the solution of the model; both approaches are demonstrated to yield consistent general solutions in integral form [1]. For the numerical analysis of the equation, a Finite Difference Method (FDM) discretization based on the centered difference approximation for derivative expressions is formulated. Finally, under a contraction condition defined on the coefficients of the system, the existence, uniqueness, and uniform convergence of the sequence of successive approximations to the analytical solution are theoretically proven within the framework of the Banach fixed-point theorem.

Keywords: Heat transfer model, finite difference method, Banach fixed point theorem

2020 Mathematics Subject Classification: 34K06, 34A30, 34K10

References:

[1] A. Cabada, F. A. F. Tojo, Differential Equations with Involutions, Atlantis Briefs in Differential Equations, Atlantis Press, 2015.

[2] A. Cabada, F. A. F. Tojo, Existence results for a linear equation with reflection, non-constant coefficient and periodic boundary conditions, Journal of Mathematical Analysis and Applications, vol. 412, no. 1, 529–546, 2014.

[3] L. V. Kritskov, M. A. Sadybekov, A. M. Sarsenbi, Properties in Lp of root functions for a nonlocal problem with involution, Turkish Journal of Mathematics, vol. 43, 393–401, 2019.

[4] D. O'Regan, Existence results for differential equations with reflection of the argument, Journal of the Australian Mathematical Society (Series A), vol. 57, no. 2, 237–260, 1994.

[5] A. Ashyralyev, M. Ashyralyyeva, O. Batyrova, On the boundedness of solution of the second order ordinary differential equation with damping term and involution, Bulletin of the Karaganda University. Mathematics Series, vol. 102, no. 2, 16–24, 2021.

Historic behaviour for a Lotka–Volterra operator on the simplex

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Abstract: We study the dynamics of a four-dimensional Lotka–Volterra type operator $\mathcal{K}_r$ on the 3-simplex S^3, arising from a discrete-time Kolmogorov system. We first give a complete description of the fixed points: the unique fixed point in the interior is the symmetric centre $c = (1/4, 1/4, 1/4, 1/4)$, which is repelling, while every point on the opposite edges Γ_{13} and Γ_{24} is a (non-hyperbolic) fixed point. A Lyapunov-type function is used to show that, for any initial state in the interior of S^3 distinct from c, the ω-limit set is contained in the boundary of the simplex. By analysing the restrictions of $\mathcal{K}_r$ to the two-dimensional faces, we refine this picture and prove that every ω-limit set coming from the interior is in fact contained in the union of one-dimensional edges, with convergence to vertices excluded.

Our main results concern the interior dynamics. We describe the asymptotic ordering of coordinates and construct a set $X \subset \operatorname{int} S^3$ of positive Lebesgue measure such that the orbits of all $x \in X$ follow a cyclic itinerary of order regions near the four non-fixed edges $\Gamma_{12}, \Gamma_{23}, \Gamma_{34}$ and Γ_{14}. This intertwined motion produces long alternating blocks in which different pairs of coordinates dominate and implies that the averages of $\mathcal{K}_r^m$ fail to converge for every $x \in X$. In particular, $\mathcal{K}_r$ is non-ergodic and exhibits historic behaviour on a set of positive Lebesgue measure, providing a higher-dimensional example of rich historic dynamics in the class of Lotka–Volterra operators

Keywords: Lotka–Volterra operator, stochastic operator, Kolmogorov system, historic behaviour, non-ergodicity

2020 Mathematics Subject Classification: 37C75, 47H25, 37N25

Study of Gaussian primes using quantum mechanics

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Abstract: We introduce the concept of the prime state over Gaussian integers, defined as a uniform quantum superposition of all Gaussian primes within a bounded norm. By mapping Gaussian integers to a Hilbert space basis, we define Hamiltonians, norm operators, Hecke-type observables, and entanglement measures acting on the prime state. Time evolution generates interference patters revealing correlations among Gaussian primes. We analyze the spectrum using random-matrix analogies and discuss quantum primality oracles. Connections to the Prime Number Theorem, Gaussian prime number estimation, Chebyshev-type bounds, and prime-counting functions are provided.

Keywords: Gaussian Primes, Quantum superposition, Prime number theorem

2020 Mathematics Subject Classification: 11A05, 11A04, 11R11

References:

[1] G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, 6th ed., Oxford University Press, 2008

[2] M. L. Mehta, Random Matrices, 3rd ed., Academic Press, 2004.

[3] H. Iwaniec, Topics in Classical Automorphic Forms, AMS, 1997

[4] J. Kempe, "Quantum Rrandom walks: An introductory overview", Contemporary Physics, 44(4):307-327, 2003.

[5] C. P. Hughes, "Special intrepretations of Gaussian primes and random matrix theory", J. Phys. A, 43(33), 2010.

[6] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, 10th Anniversary Edition, Cambridge University Press 2010.

Solution of a nonlocal problem for the heat equation using the potential method

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Abstract: One nonlocal problem for the heat equation is considered

$$L(u) \equiv u_t - u_{xx} = 0 \quad \textit{in domain} \quad \Omega = \{(x,t) : 0 < x < l, \quad 0 < t < T\} \tag{1}$$

with initial conditions

$$u(x,0) = f(x), \quad 0 \le x \le l \tag{2}$$

boundary conditions of the form

$$\begin{cases} a_1 u(0,t) + a_2 u(l,t) = \varphi(t), \\ b_1 u_x(0,t) + b_2 u_x(l,t) = \psi(t), \end{cases} \quad 0 \le t \le T \tag{3}$$

the coefficients $a_i, b_i (i = 1,2)-$ real numbers.

Using the potential method, problem (1)-(3) is reduced to Volterra integral equations. The solution of the Volterra integral equation was found and the solvability of the stated problem (1)-(3) was investigated.

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Keywords: heat equation, nonlocal problems, potential method, integral equations, solvability conditions

2020 Mathematics Subject Classification: 35A09, 35L20, 35M13

M. Bakoev, Probabilistic representation and Monte Carlo solution of initial–boundary value problems for the advection–diffusion equation in air pollution modelling

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Abstract: We study the initial–boundary value problem (IBVP) for the advection–diffusion–reaction equation

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = \nabla\cdot(K\nabla c) - \lambda c, \qquad \mathbf{x}\in\Omega,\ 0<t\le T, \tag{1}$$

$$c(\mathbf{x},0) = c_0(\mathbf{x}), \qquad \mathcal{B}[c] = g \ \text{ on } \ \partial\Omega, \tag{2}$$

which governs the spatial distribution of a pollutant concentration c transported by a wind field $\mathbf{u}$, dispersed by a turbulent diffusivity K, and lost at the first-order rate λ. The boundary operator $\mathcal{B}$ ranges over the physically relevant Dirichlet (prescribed background or inflow), Neumann (impermeable, reflecting surface) and Robin (dry deposition, $-K\,\partial_n c = v_d\, c$) conditions.

We present the probabilistic, Feynman–Kac representation of the solution. Equation (1) is the Kolmogorov forward (Fokker–Planck) equation of the Itô diffusion $d\mathbf{X}_t = \mathbf{u}\,dt + \sqrt{2K}\,d\mathbf{W}_t$, so the concentration coincides, up to the released mass, with the sub-probability density of diffusing pollutant parcels killed at rate λ. The pointwise value of the solution admits the stochastic representation

$$c(\mathbf{x},t) = \mathbb{E}\Big[e^{-\lambda(t\wedge\tau)}\big(\mathbf{1}_{\{\tau>t\}}\,c_0(\mathbf{Y}_t) + \mathbf{1}_{\{\tau\le t\}}\,g(\mathbf{Y}_\tau,\,t-\tau)\big)\Big], \tag{3}$$

where $\mathbf{Y}$ is the adjoint diffusion $d\mathbf{Y}_s = -\mathbf{u}\,ds + \sqrt{2K}\,d\mathbf{W}_s$ started at $\mathbf{Y}_0 = \mathbf{x}$, and τ is its first hitting time of the boundary $\partial\Omega$. In this representation every boundary condition becomes a behaviour of the path: Dirichlet corresponds to absorption, Neumann to reflection, and Robin to partial absorption (an elastic boundary). As a consequence, we show that the classical Ogata–Banks solution of the semi-infinite inflow problem is exactly a first-passage probability, $c(x,t)/c_0 = \mathbb{P}(\tau\le t)$.

On this basis we develop and compare two Monte Carlo solvers: a backward Feynman–Kac estimator for pointwise concentrations, equipped with a Brownian-bridge correction for sub-step boundary crossings, and a forward random-walk particle method for the whole field, in which the dry-deposition (Robin) boundary is realised by absorbing a particle upon contact with probability $v_d\sqrt{\pi\,\Delta t/K}$. Well-posedness is addressed through the parabolic maximum principle and an energy estimate yielding uniqueness and continuous dependence on the data. The estimators are validated against the analytic fundamental solution, the Ogata–Banks formula, and an implicit (Crank–Nicolson) finite-difference solution, in one, two, and three spatial dimensions. For a diagonal diffusivity the multidimensional diffusion factorises into independent one-dimensional processes, which renders the method mesh-free and free of numerical diffusion, the ground reflection, capping inversion, and deposition acting on the vertical coordinate alone. An interactive R-Shiny web application has been developed that lets the user vary the wind, diffusivity, source, and boundary parameters and visualise the resulting concentration fields in real time.

Keywords: advection–diffusion equation, initial–boundary value problem, Feynman–Kac formula, Monte Carlo method, stochastic differential equation, first-passage time, Robin boundary condition, atmospheric dispersion

2020 Mathematics Subject Classification: 35K20, 60H30, 65C05

References:

[1] M.T. Bakoev, Numerical Modeling of Solutions to Diffusion Equations in Financial and Management Problems: A Monograph. Tashkent, Uzbekistan: University of World Economy and Diplomacy, p.324, 2020 (In Russian).

[2] A. Ogata, R.B. Banks, A solution of the differential equation of longitudinal dispersion in porous media, U.S. Geological Survey Professional Paper 411-A, 1961.

[3] B. Øksendal, Stochastic Differential Equations: An Introduction with Applications, 6th ed., Springer, Berlin, 2003.

[4] R. Erban, S.J. Chapman, Reactive boundary conditions for stochastic simulations of reaction–diffusion processes, Physical Biology, vol. 4, no. 1, 16–28, 2007.

[5] D.J. Thomson, Criteria for the selection of stochastic models of particle trajectories in turbulent flows, Journal of Fluid Mechanics, vol. 180, 529–556, 1987.

The ITD approach in quasilinearization of integro-differential equations

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Abstract: Integro-differential equations frequently serve as mathematical frameworks across a variety of disciplines. Investigating Volterra integro-differential equations alongside their real-world uses has emerged as a prominent research domain. Within this context, let us consider the initial value problem (IVP) formulated below:

$$\frac{dx}{dt} = g(t, x(t)) + \int_{t_0}^{t} \varphi(t, s, x(s))ds, \qquad x(t_0) = x_0, \tag{1}$$

with $t \in J_1 = [t_0, t_0 + T], t \geq 0, T > 0$, assuming appropriate conditions on g and φ.

This report focuses on employing the method of quasilinearization incorporating the initial time difference (ITD) technique to analyze a Volterra integro-differential equation (1), thereby yielding monotone sequences of functions that converge uniformly and quadratically to the unique solution of (1).

Keywords: Quasilinearization method, initial value problem, integro-differential equations, lower and upper solutions

2020 Mathematics Subject Classification: 34A12, 45J05

References:

[1] V. Lakshmikantham and A. S. Vatsala, *Generalized Quasilinearization for Nonlinear Problems*, Kluwer Academic Publishers, The Netherlands, 1998.

[2] A. Yakar, *Initial time difference quasilinearization for Caputo Fractional Differential Equations*, Adv. Difference Equ. **2012** (2012), Article ID 92, 12 pp. https://doi.org/10.1186/1687-1847-2012-92

[3] C. Yakar and A. Yakar, *An extension of the quasilinearization method with initial time difference*, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. **14** (2007), no. S2, 275–279.

Weak solvability of Navier–Stokes–Voigt thermal model

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Abstract: We consider the following initial–boundary value problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \text{Div } (\nu(I_2(v))\mathcal{E}(v)) - \varkappa \frac{\partial \Delta v}{\partial t} + \nabla\theta + \nabla p = f;$$

$$\frac{\partial \theta}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial \theta}{\partial x_i} - \chi\Delta\,\theta = \nu(I_2(v))\mathcal{E}(v) : \mathcal{E}(v) + \varkappa \frac{\partial \mathcal{E}(v)}{\partial t} : \mathcal{E}(v) + g;$$

$$\text{div } v = 0; \quad v|_{t=0} = v_0; \quad v|_{[0,T]\times\partial\Omega} = 0; \quad \theta|_{t=0} = \theta_0\Omega; \quad \theta|_{[0,T]\times\partial\Omega} = 0.$$

Here, v is un unknown velocity, p is un unknown pressure, $f = f(t,x)$ is the external force, $\nu > 0$ is the fluid viscosity, $\varkappa > 0$ is the retardation time, $\mathcal{E}(v) = (\mathcal{E}_{ij}(v))$, $\mathcal{E}_{ij}(v) = \frac{1}{2}\big(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\big)$, $i, j = \overline{1, n}$, is the strain-rate tensor, the function $I_2(v)$ is defined by the relation $I_2(v)^2 = \mathcal{E}(v) : \mathcal{E}(v)$.

This initial–boundary value problem describes the flow of a non–linearly elastically retarded Voigt fluid (for example, the motion of a viscous fluid with polymer components) see [1].

Theorem. Let $\Omega \subset \mathbb{R}^n$, $n = 2, 3$, be bounded domain with smooth boundary. Let $f \in L_2(0, T; V^*)$, $v_0 \in V$, $g \in L_1(0, T; H_p^{-2(1-1/p)}(\Omega))$ and $\theta_0 \in W_p^{1-2/p}(\Omega)$. The viscosity function ν have to be continuously differentiable and satisfy the inequalities (ν_1) $0 < C_1 \leq \nu(s) \leq C_2 < \infty$; (ν_2) $-s\nu'(s) \leq \nu(s)$ for $\nu'(s) < 0$; (ν_3) $|s\nu'(s)| \leq C_3 < \infty$. Also let $1 < p < 4/3$ for $n = 2$ and $1 < p < 10/9$ for $n = 3$. Then the initial-boundary value problem has weak solution $v \in W = \{v : v \in C(0, T; V),\ v' \in L_2(0, T; V)\}$.

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Keywords: Navier–Stokes–Voigt system of equations, weak solution, thermodynamic model, existence theorem

2020 Mathematics Subject Classification: 35Q35, 76A05 **References:**

[1] A.V. Zvyagin, On weak solvability for Navier–Stokes–Voigt thermal model with nonlinear viscosity coefficient, Functional Analysis and Its Applications, vol. 60, no 1, 97–101, 2026.

Finite propagation and space localization properties of cross-diffusion systems with time-dependent coefficients and variable density

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Abstract: We investigate the Cauchy problem for a class of degenerate nonlinear cross-diffusion systems with time-dependent diffusion coefficients, variable density, and absorption terms. In the domain

$$Q = \Big\{(t,x) : \ t > 0, \ x \in \mathbb{R}^N\Big\}$$

investigated properties of the solutions of a nonlinear diffusion-reaction system with variable density:

$$\begin{cases} |x|^r \dfrac{\partial u}{\partial t} = \operatorname{div}\Big(\bar{k}_1(t)|x|^{\alpha} v^{m_1-1}|\nabla u|^{p-2}\nabla u\Big) - |x|^r\gamma_1(t)u, \\ \\ |x|^r \dfrac{\partial v}{\partial t} = \operatorname{div}\Big(\bar{k}_2(t)|x|^{\beta} u^{m_2-1}|\nabla v|^{p-2}\nabla v\Big) - |x|^r\gamma_2(t)v. \end{cases} \tag{1}$$

subject to the initial conditions

$$u(0,x) = u_0(x) \geq 0, \qquad v(0,x) = v_0(x) \geq 0, \qquad x \in \mathbb{R}^N, \tag{2}$$

here $k \geq 1$, $n, p, m_i, k_i(t) > 0$, $i = 1,2$, are given positive constants, characterizing the nonlinear media, $\nabla(\cdot) = \operatorname{grad}(\cdot)$, -functions $u_0(x)$, $v_0(x) \geq 0$, $x \in \mathbb{R}^N$, and $0 < k_i(t), \gamma_i(t) \in C(0,\infty)$, $i = 1,2$.
The model describes nonlinear processes arising in porous media, the theory of gas filtration, nonlinear heat conduction, and reaction–diffusion cross-systems with variable density and absorption. Under suitable assumptions on the nonlinear diffusion exponents, absorption coefficients, and initial data, we analyze the qualitative properties of nonnegative weak solutions. By introducing an appropriate nonlinear self-similar transformation, the original evolutionary system is reduced to a family of nonlinear degenerate self-similar differential equations. In the present work, based on the self-similar analysis of solutions to problem (1),(2) and the comparison theorem, we investigate the qualitative properties of solutions to the problem under consideration; these include the finite velocity of disturbance propagation and spatial localization under the influence of absorption and time-dependent coefficients of the cross-system [1–5]. An estimate of the solutions and the free boundary of the problem is obtained for the case of slow diffusion. Furthermore, the asymptotics of the self-similar solution are investigated. It is shown that the coefficient of the leading term of the solutions asymptotic satisfies a certain system of nonlinear algebraic equations for both slow and fast diffusion [2–4]. Critical values of the numerical parameters of the nonlinear medium, at which the behavior of the solution changes and takes on an exponential form, are established. Critical and singular cases are also considered. Finally, the asymptotics of self-similar solutions for weak and quenching solutions are established, and a numerical analysis of the solution is discussed.

Keywords: Degenerate parabolic system; cross-diffusion; weak solutions; finite-speed propagation; free boundary; space localization; self-similar solutions; asymptotic and numerical analysis.

2020 Mathematics Subject Classification. 35K65, 35K67, 35B40, 35B44, 35R35.

References:

[1] A. A. Samarskii, V. A. Galaktionov, S. P. Kurdyumov, A. P. Mikhailov, *Blow-up in Quasilinear Parabolic Equations*, Walter de Gruyter, Berlin, 1995.

[2] M. Aripov, Sh. A. Sadullaeva, Computer modeling of nonlinear diffusion processes, *Tashkent, University Publishing House*, **3** (2020), 687.

[3] N. David, T. Debiec, M. Mandal, M. Schmidtchen, A Degenerate Cross-Diffusion System as the Inviscid Limit of a Nonlocal Tissue Growth Model, *SIAM Journal on Mathematical Analysis*, **56**(2) (2024), 2090–2114.

[4] M. Fellner, A. Jüngel, Existence analysis of a cross-diffusion system with nonlinear Robin boundary conditions for vesicle transport in neurites, *Nonlinear Analysis*, **241**(2) (2024), 113494.

[5] L. Alasio, M. Bruna, S. Fagioli, S. Schulz, Existence and regularity for a system of porous medium equations with small cross-diffusion and nonlocal drifts, *Nonlinear Analysis*, **223** (2022), 113064.

On some inequalities of two-parameter noncommutative martingales

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Abstract: This report is devoted to extending the main results of [1] on two-parameter noncommutative martingales from the finite von Neumann algebra setting to the semi-finite setting. We prove convergence theorems for L_p-bounded martingales ($1 < p < \infty$). Furthermore, we establish the Burkholder–Gundy inequality for general two-parameter martingales and Davis's inequality for strong martingales within this broader semi-finite context. Our approach relies crucially on the powerful interpolation machinery developed in [2], specifically the complex and real interpolation formulae between BMO, $\mathcal{H}_p$, h_p, and their conditioned counterparts. This interpolation framework allows us to transfer the known finite-trace estimates systematically to the semi-finite setting. The results obtained unify and extend the classical L_p-theory for two-parameter martingales, providing essential tools for further developments in noncommutative harmonic analysis and quantum probability. Throughout this note, we mainly use techniques from the works [3–5].

Keywords: von Neumann algebra; two-parameter martingales; Burkholder–Gundy inequality; Davis's inequality; strong martingales

2020 Mathematics Subject Classification: 46L52; 46L53; 60G42

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[1] Cao, F., Hou, Y. L. (2015). Convergence and Inequalities of Two-Parameter Noncommutative Martingales. *Acta Math. Scientia*, 35B(6), 1511-1520.

[2] T.N. Bekjan, Z Chen, M. Raikhan, M. Sun (2022). Interpolation and the John–Nirenberg inequality on symmetric spaces of noncommutative martingales. *Studia Mathematica*, 262, 6241-273.

[3] Pisier, G., Xu, Q. (1997). Non-commutative martingale inequalities. *Comm. Math. Phys.*, 189, 667-698.

[4] Perrin, M. (2009). A noncommutative Davis's decomposition for martingales. *J. London Math. Soc.*, 80(2), 627-648.

[5] Pisier, G., Xu, Q. (2003). Non-commutative L_p-spaces. In *Handbook of the Geometry of Banach Spaces*, Vol. 2, Elsevier.

Optimal control problems with nonlocal conditions for systems of integro-differential equations

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Abstract: This paper considers an optimal control problem in which the system state is determined by a system of integro-differential equations with nonlocal boundary conditions. Admissible controls are selected from the class of bounded and measurable functions with values in an open set. A formula for the second-order increment of the functional is derived. Based on control variations, the necessary optimality conditions for singular controls in the classical sense are obtained.

Similar problems have been considered in [1].

Keywords: nonlocal boundary conditions, optimal control, necessary optimality conditions, second-order functional increment

2020 Mathematics Subject Classification: 34B10, 34A37, 34H05

References:

[1] A. Ashyralyev, Y.A. Sharifov, Optimal control problem for impulsive systems with integral boundary conditions, AIP Conference Proceedings, vol. 1470, no. 1, 12–15, 2012.

Boundedness of convoultion pperators in generalized local Morrey spaces

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Abstract: This paper investigates norm estimates for convolution operators in generalized local Morrey-type spaces. Sufficient conditions for boundedness are established, and the corresponding associated (dual) spaces are characterized.

We prove Young-O'Neil-type inequalities in this setting, extending classical results to a more general framework. The obtained inequalities provide new estimates for convolution operators with kernels from weak Lebesgue spaces.

These results generalize known theorems for Lebesgue, Lorentz, and Morrey spaces, offering refined criteria for the boundedness of integral operators. Applications include potential operators and singular integrals in Morrey-type spaces.

Throughout this note we mainly use techniques from [1].

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Keywords: Morry space, Generalised Local Morrey Space, convolution operator, Young-O'Neil's inequality, Riesz potential

2020 Mathematics Subject Classification: 47A05, 47B06

References:

[1] E. D. Nursultanov, D. Suragan. On the convolution operator in Morrey spaces. *J. Math. Anal. Appl.*, 515 (2022), 126357, 20 pages.

On the class of super-singular integro-differential equation

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Abstract: In this work on $\Gamma = \{x : a \leq x \leq b\}$ we investigate a class of first-order integro-differential equation with super-singular kernel

$$D_x^\beta y + A(x)y - \int_x^b \frac{B(x,t)}{(b-t)^\beta} y(t)\, dt = f(x), \tag{1}$$

where $\beta > 1$, $D_x^\beta = (b-x)^\beta \dfrac{d}{dx}$ is a special differential operator, $A(x)$, $f(x)$ are given continues functions at Γ, $B(x,t)$ is given continues function at rectangular $R = \{(x,t) : a < x < b,\ a < t < b\}$ and $y(x)$ is unknown function.

Since in equation (1) $\beta > 1$, therefore the kernel of equation has singularity of degree greater then 1. On the other hand it is well known that the kernel of type $\frac{B(x,t)}{(b-t)^\beta}$ upon $|B(x,t)| \leq K$ is a divergent kernel, because the improper integral

$$\int_x^b \frac{B(x,t)}{(b-t)^\beta}\, dt$$

diverges.

On the other hand, even though the kernel is divergent, our investigation shows that there exist some classes of functions in which the equation (1) has solutions.

In this work we show that in the class of such functions denoted by $y(x) \in C_{\beta-1}[a,b]$, which at point $x = b$ tends to zero and its behavior determined from the asymptotic formula

$$y(x) = o\left[(b-x)^\delta\right], \qquad \delta > \beta - 1, \qquad \text{when } x \to b.$$

the equation (1) solvable and its solution in general contains two arbitrary constants.

There also exist some cases in which the general solution of investigated equation contains one arbitrary constant or it has only the unique solution.

Keywords: Integro-differential equations, singular coefficients, Hölder class functions, degenerate kernel.

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Analytical solutions for fractional stochastic equations via Martingale processes

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Abstract: The purpose of this paper is to put forward some analytical solutions to solve the fractional stochastic equation (FSDE)

$$\begin{cases} {}^C D_t^\alpha X(t) = a(X(t),t)\,dt^\alpha + \sigma(X(t),t)\,dW_t, \qquad t \in [0,T], \\ \qquad X(0) = X_0, \end{cases}$$

where ${}^C D_t^\alpha$ is a Caputo fractional derivative of order $\alpha,\ 0 < \alpha \leq 1$, $W(t)$ is a Wiener process and triple $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space under some conditions and special relations between drift and volatility.

If functions a and σ are globally Lipschitz in $\mathbb{R}$ and satisfy the linear growth condition, then FSDE has a unique t-continuous solution adapted to the filtration $\mathcal{F}_t,\ t \geq 0$ generated by $W(t)$ and

$$E\left[\int_0^T |X(s)|^2\,ds\right] < \infty.$$

In our paper, we find a couple of analytical solutions of some set of fractional stochastic equations was indicated via the martingale process from a stochastic process. Converting fractional stochastic differential equations to ordinary ones as another suitable method was posed.

Keywords: Fractional order, martingale processes, drift term, diffusion, volatility term.

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On the conditions for the existence of solutions of a difference inclusions

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Abstract: Let X be a complex Banach space and A be a closed linear relation on X, i.e., $A \subset X \times X$ is a closed linear subspace.

In the Banach space $l_p = l_p(\mathbb{Z}, X), 1 \leqslant p \leqslant \infty$, of two-sided sequences $x : \mathbb{Z} \to X$, we consider the difference inclusion

(1) $$x(n) \in Ax(n-1) + f(n), n \in \mathbb{Z},$$

here $f \in l_p, Ax(n-1) = \{y \in X : (x(n-1), y) \in A\}$.

The study of inclusions (1) is based on the closed relation $\mathcal{A} = \overline{A}S(-1)$ on l_p, where $\overline{A} = \{(x, y) \in l_p \times l_p : (x(n), y(n)) \in A\}$ and $(S(-1)x)(n) = x(n-1), n \in \mathbb{Z}, x \in l_p$. The spectrum of $\sigma(\mathcal{A})$ is invariant under rotation around zero in the field $\mathbb{C}$, and if $1 \notin \sigma(\mathcal{A})$, then the representation $\sigma(\mathcal{A}) = \sigma_{int} \cup \sigma_{out}$, is valid, where $\sigma_{int}(\mathcal{A}) = \{\lambda \in \sigma(\mathcal{A}) : |\lambda| < 1\}$ and $\sigma_{out}(\mathcal{A}) = \{\lambda \in \sigma(\mathcal{A}) : |\lambda| > 1\}$.

Theorem 1. The difference inclusion (1) has a unique solution $x \in l_p, 1 \leqslant p \leqslant \infty$ if and only if the spectrum $\sigma(A)$ of A satisfies the condition

(2) $$\sigma(A) \cap \mathbb{T} = \varnothing,$$

where $\mathbb{T}$ is the unit circle in $\mathbb{C}$.

According to condition (2), the Riesz projector decomposes the space $X = X_0 \oplus X_1$, where $X_0 = ImP, KerP = X_1$, and for the restrictions $A|X_0 = A_0$ and $A|X_1 = A_1$ the equalities $\sigma(A_0) = \sigma_{int}(A)$ and $\sigma(A_1) = \sigma_{out}(A)$ hold. Based on this, any solution of difference inclusion (1) can be represented as $x(n) = \sum\limits_{m \in \mathbb{Z}} G(-m)f(m)$, where $G : \mathbb{Z} \to X, G(k) = A_0^k P$ for $k \geqslant 0, G(k) = -A_1^k(I - P)$ for $k \leqslant -1$.

Throughout this note we mainly use techniques from work [1].

Keywords: Banach space, linear relation, difference inclusion, spectrum

2020 Mathematics Subject Classification: 39A70, 39A20

References:

[1] A.G. Baskakov, K.I. Chernyshov. Spectral analysis of linear relations and degenerate operator semigroups. Sbornik: Mathematics, vol. 193, no 1, 3–42, 2002.

Numerical solution of the multidimensional identification elliptic telegraph problem with Dirichlet condition

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Abstract: In the present paper, the theorem on stability estimates for the solution of source identification problem (SIP) of the multidimensional elliptic-telegraph differential equation with Dirichlet's conditions is presented. Furthermore, numerical results for multidimensional of identification elliptic-telegraph problem are given.

Keywords: Source identification problem, elliptic-telegraph differential equation, difference scheme

2020 Mathematics Subject Classification: 65N06, 35M10, 35R30.

References:

[1] A. Al-Hammouri and A. Ashyralyev, A note on the second-order accuracy difference scheme for the elliptic-telegraph identification problem with Dirichlet boundary condition. *International Journal of Applied Mathematics, 38*(1) (2025), 1-13.

[2] S. S. Adavani and G. Biros, Fast algorithms for source identification problems with elliptic PDE constraints. *SIAM Journal on Imaging Sciences, 3*(4) (2010), 791-808.

[3] M. Ashyraliyev and M. A. Ashyralyyeva, A stable difference scheme for the solution of a source identification problem for telegraph-parabolic equations. *Bulletin of the Karaganda University. Mathematics Series, 115*(3) (2024), 46-54.

[4] A. Ashyralyev and A. Al-Hammouri, A note on the elliptic-telegraph identification problem with nonlocal condition. *AIP Conference Proceedings, 2334*(1) (2021), 060006.

Comparative analysis of different classes of adomian polynomials for solving nonlinear volterra integral equation of the second kind

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Abstract: In this research, the Standard Laplace Adomian decomposition Method (SLADM), Modified Laplace Adomian Decomposition Method (MLADM) and new modification in Standard Laplace Adomian Decomposition Method called the Two-Step Laplace Adomian Decomposition Method (TSLADM) for solving nonlinear Volterra integral equation of the second kind was considered. The analytical and numerical solutions of Nonlinear Volterra integral equation of second kind were solved for the classes of the Adomian polynomials. The results obtained are compared with exact solution and it was shown that these three methods are capable to achieve a good agreements with exact solution. This research also demonstrated that new modification in standard Laplace Adomian decomposition method (TSLADM) is reliable and efficient than standard Laplace Adomian decomposition method. However, the comparison between the exact solution and numerical solution obtained using SLADM, MLADM and TSLADM was established. Numerical solutions was carried out and the results shows that the TSLADM is less time consuming when compared to modified Laplace Adomian decomposition method and standard Laplace Adomian decomposition method.

Throughout this note we mainly use techniques from the works [1].

Keywords: Adomian Decomposition Method , nonlinear volterra integral equation , polynomial, numerical solutions.

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

References:

[1] Adomian, G. A review of the decomposition method in applied mathematics. Journal of Mathematical Analysis and Applications, vol. 135, no 2, 501–544, 1988.

Some results for analytic functions related with modified Sigmoid function

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Abstract: In this study, we explore the analytical properties and applications of a Modified Sigmoid Function within the framework of geometric function theory, with a focus on the Schwarz Lemma. The classical sigmoid function, widely used in machine learning, is adjusted to better conform to the unit disk and its boundary behavior, enabling a rigorous examination using tools from complex analysis. We demonstrate that under appropriate normalization, the modified sigmoid satisfies the conditions of the Schwarz Lemma, and we investigate the implications of this conformity on the function's growth, distortion, and fixed-point behavior. Furthermore, we discuss how such modifications influence stability and convergence in iterative schemes, potentially offering refined activation functions for neural networks with provable analytical constraints. These results establish a bridge between classical function theory and modern computational methods, highlighting the role of complex analysis in evaluating and enhancing non-linear mappings in applied contexts.

Keywords: Modified Sigmoid Function, Analytic function, Schwarz lemma

2020 Mathematics Subject Classification: 30C80

References:

[1] T. A. Azeroğlu and B. N. Örnek, A refined Schwarz inequality on the boundary, Complex Variables and Elliptic Equations, 58, 571-577, 2013.

[2] H. P. Boas, Julius and Julia: Mastering the Art of the Schwarz lemma, Amer. Math. Monthly, 117, 770-785, 2010.

[3] V. N. Dubinin, The Schwarz inequality on the boundary for functions regular in the disc, J. Math. Sci., 122, 3623-3629, 2004.

[4] G. M. Golusin, Geometric Theory of Functions of Complex Variable [in Russian], 2nd edn., Moscow 1966.

[5] R. Osserman, A sharp Schwarz inequality on the boundary, Proc. Amer. Math. Soc., 128, 3513-3517, 2000.

[6] B. N. Örnek, S. B. Aydemir, T. Düzenli, and B. Özak, A novel version of slime mould algorithm for global optimization and real world engineering problems: Enhanced slime mould algorithm, Mathematics and Computers in Simulation, 198, 252,288, 2022.

[7] B. N. Örnek, S. B. Aydemir, T. Düzenli, and B. Özak, Some remarks on activation function design in complex extreme learning using Schwarz lemma, Neurocomputing, 492, 23-33, 2022.

[8] Ch. Pommerenke, Boundary Behaviour of Conformal Maps, Springer-Verlag, Berlin. 1992.

Inverse boundary value problem for a second-order hyperbolic integro-differential equation with a periodic condition and an integral condition of the first kind

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Abstract: An inverse boundary value problem for a second-order hyperbolic integro-differential equation with a periodic condition and an integral condition of the first kind is investigated.

In the domain $D_T = \{(x,t) : 0 < x < 1, 0 < t < T\}$ we consider an inverse boundary value problem

$$u_{tt} - u_{xx} = \int_0^t a(\tau)u(x,t-\tau)d\tau + f(x,t) \;\; (x,t) \in D_T, \tag{1}$$

with the initial conditions

$$u(x,0) = \varphi(x), \;\; u_t(x,0) = \psi(x) \;\; (0 \le x \le 1), \tag{2}$$

the periodic boundary condition

$$u(0,t) = u(1,t) \;\; (0 \le t \le T), \tag{3}$$

the integral condition of the first kind

$$\int_0^1 u(x,t)dx = 0 \;\; (0 \le t \le T), \tag{4}$$

and the additional condition

$$\int_0^1 \omega(x)u(x,t)dx = h(t) \;\; (0 \le t \le T), \tag{5}$$

here, $\varphi(x), \psi(x), f(x,t), h(t), \omega(x) \ne const$ are the given functions, and $u(x,t)$ and $a(t)$ are the unknown functions.

First, the original problem has been reduced to an equivalent problem for which the existence and uniqueness theorem has been proved. Then, using these results, the existence and uniqueness of a classical solution to the original problem are established.

Our results develop the results of [1].

Keywords: inverse problem, second-order hyperbolic integro-differential equation, existence, uniqueness, classical solution.

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References:

[1] Ya.T. Mehraliyev, U.S. Alizade, On the Problem of Identifying a Linear Source for the Third-Order Hyperbolic Equation with Integral Condition. Problems of Physics, Mathematics and Technics. 2019. No. 3(40). P. 80–87.

Solvability of a nonlocal problem for a spatially loaded hyperbolic system of differential equations

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Abstract: This paper investigates a nonlocal problem for a second-order hyperbolic system of loaded differential equations with a spatial loading term depending on the value of the solution at a fixed spatial point. To establish the solvability of the problem, we propose a parametrization-based approach. The considered problem is formulated in the domain $\Omega = [0,T] \times [0,X]$ as follows:

$$\frac{\partial^2 u}{\partial x \partial t} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u(t,x) + C_0(t,x)u\left(t,x_0\right) + f(t,x), \tag{1}$$

$$P(x)\frac{\partial u(0,x)}{\partial x} + S(x)\frac{\partial u(T,x)}{\partial x} = \varphi(x), \quad x \in [0,X], \tag{2}$$

$$u(t,0) = \psi(t), \quad t \in [0,T]. \tag{3}$$

To investigate problem (1)-(3), we employ a parametrization-based approach. The proposed approach introduces auxiliary unknown functions and employs the parametrization method. As a result, the original nonlocal problem is transformed into an equivalent family of parameter-dependent Goursat and Cauchy problems.

The proposed approach extends the parametrization method developed in [1, 2] to a class of nonlocal hyperbolic systems with spatial loading.

Keywords: nonlocal problem, hyperbolic loaded differential system, parametrization method, Goursat problem, Cauchy problem, fundamental matrix, iterative method, solvability

2020 Mathematics Subject Classification: 35L15; 34A12; 34B08; 45A05; 39B05

References:

[1] A. T. Assanova, A. Zholamankyzy, A family of two-point boundary value problems for loaded differential equations, Izv. Vyssh. Uchebn. Zaved. Mat., 2021, no. 9, 13-24; Russian Math. (Iz. VUZ), 65:9 (2021), 10-20. DOI: 10.3103/S1066369X21090024.

[2] A.T. Assanova, A. Zholamankyzy. Problem with data on the characteristics for a loaded system of hyperbolic equations, Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'yuternye Nauki, 2021, vol. 31, issue 3, pp. 353-364. DOI: 10.35634/vm210301.

Spectral properties of a second-order differential operator with the spectral parameter in the boundary conditions

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Abstract: We consider the spectral problem for the second-order differential operator

$$L(y) \equiv -y''(x) = \lambda y(x),\ 0 < x < 1, \tag{1}$$

subject to the boundary conditions

$$y(0) = 0, \tag{2}$$

$$\lambda Y_1(y) + Y_0(y) = 0, \tag{3}$$

where

$$Y_i(y) = a_{i1}y'(0) + a_{i2}y'(1) + a_{i3}y(0) + a_{i4}y(1) \tag{4}$$

are linear boundary forms.

The spectral properties of boundary value problems for equation (1) without the spectral parameter in the boundary conditions have been extensively investigated in the literature [1]. However, introducing the spectral parameter into the boundary conditions substantially changes the structure of the characteristic equation and the spectral properties of the problem, which requires a separate investigation.

In the abstract, we study the spectral properties of the boundary value problem (1)–(3), where the spectral parameter enters one of the boundary conditions linearly. A classification of the problem into regular and irregular cases is established in terms of the coefficients of the boundary condition (3). The asymptotic behaviour of the eigenvalues and the properties of the corresponding eigenfunctions are investigated.

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Keywords: Ordinary differential operator, nonlocal boundary conditions, spectral parameter in the boundary conditions, spectral properties, regular boundary conditions, irregular boundary conditions

2020 Mathematics Subject Classification: 34L05,34L10, 34L15

References:

[1] Lang P., Locker J.,Spectral theory of two-point differential operators determined by D^2. Analysis of Cases, J. Math. Anal. And Appl., 146:1 (1990), 148-191.

Deformation of the complex structure in $A(z)$-geometry

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Abstract: This paper introduces $A(z)$-geometry through the space of $A(z)$-analytic functions and its reproducing kernel K_A. The corresponding lemniscates and metric describe the complex structure deformed by the coefficient $A(z)$. Unlike quasiconformal geometry, this framework is based on reproducing-kernel and integral structures.

It is well established that solutions to the Beltrami equation

$$\bar{D}_A = \frac{\partial f}{\partial \bar{z}} - A(z)\frac{\partial f}{\partial z} = 0 \tag{1}$$

are closely connected with the theory of quasiconformal mappings. In the general setting, the coefficient $A(z)$ is assumed to be measurable in the domain $D \subset \mathbb{C}$ and to satisfy the ellipticity condition

$$|A(z)| \leq c$$

almost everywhere in D, where $c < 1$. Functions satisfying equation (1) are referred to as $A(z)$-analytic functions.

Krantz [1] develops the classical foundations of geometric function theory, while Sadullaev and Zhabborov [2] extend them to $A(z)$-analytic functions. These works provide the basis for $A(z)$-geometry and its associated kernel, lemniscates, and metric.

As an illustrative example, consider a multiply connected domain, such as the annulus. In this setting, the Bergman kernel may vanish at points away from the diagonal, a classical phenomenon demonstrating that the finer analytical properties of the kernel are strongly influenced by the topology of the underlying domain:

$$K_A(z,\zeta) = \frac{1}{\pi}\left[\sum_{n=0}^{\infty} \frac{n+1}{1-r^{2n+2}}\left(z\bar{\zeta}\right)^n + \sum_{n=0}^{\infty} \frac{(n+1)r^{2n+2}}{1-r^{2n+2}}\left(\bar{z}\zeta\right)^{n-2}\right].$$

Thus, in contrast to the case of the unit disk, the kernel associated with an annulus may possess zeros outside the diagonal set $z = \zeta$.

Classical Bergman geometry relies on the holomorphic structure of the domain and on invariance under biholomorphic transformations. In the setting of equation (1), however, the underlying complex structure is modified by the prescribed field, with the result that the conventional geometric objects no longer retain their invariance.

Within this framework, a novel geometric structure emerges in a natural manner. We refer to it as $A(z)$-geometry, since it is determined by the space of $A(z)$-analytic functions together with its associated reproducing kernel K_A.

Fix a point $\zeta \in D$ and introduce the function

$$\psi_\zeta(z) = \int_{\gamma(\zeta,z)} A(\tau)d\tau,$$

where $\gamma(\zeta, z)$ denotes an arbitrary curve in D joining ζ to z. If D is convex, the function ψ_ζ is uniquely defined.

Definition 2.1. *For $r > 0$, the set*

$$L_r(\zeta) = \{z \in D : \left|\psi_\zeta(z)\right| < r\}$$

is called the $A(z)$-lemniscate with center at ζ.

Whereas ellipses serve as the fundamental geometric objects in quasiconformal geometry, the natural geometric elements of $A(z)$-geometry are lemniscates. Their form is governed by the integral structure associated with the coefficient field $A(z)$.

The reproducing kernel $K_A(z,\zeta))$ provides a natural basis for defining a metric that serves as an analogue of the classical Bergman metric.

Definition 2.2. *(A(z)-metric). The $A(z)$-metric is defined by*

$$ds_A^2(z) = \partial_z \partial_{\bar{z}} \log K_A(z, z)|dz|^2.$$

This metric has the following properties:

- *it is preserved by those isometries of the space $\mathcal{H}_A^2(D)$ that are generated by $A(z)$-compatible mappings, whenever such mappings exist;*
- *it remains invariant under $A(z)$-automorphisms;*
- *it characterizes the intrinsic geometric structure of $A(z)$-lemniscates;*
- *in the particular case $A(z) \equiv 0$, it reduces to the classical Bergman metric.*

Consequently, the $A(z)$-metric may be regarded as a natural geometric invariant associated with the deformed complex structure.

The essential distinction between $A(z)$-geometry and quasiconformal geometry can be described as follows:

- quasiconformal mappings preserve the moduli of curve families;
- $A(z)$-automorphisms preserve the level sets of the $A(z)$-potential together with the corresponding lemniscatic structure, although they are not necessarily required to preserve extremal length.

Hence, $A(z)$-geometry should not be viewed as a particular subclass of quasiconformal geometry. Rather, it constitutes a distinct geometric framework determined by the kernel of a first-order elliptic differential operator.

The orthogonal decomposition

$$L^2(D) = \ker \bar{D}_A \oplus \operatorname{Ran} \bar{D}_A^*.$$

also admits a natural geometric interpretation. More precisely,

- the subspace $\ker \bar{D}_A$ represents the $A(z)$-analytic, and therefore geometrically intrinsic, component of a function;
- the complementary non-analytic component is described through the fundamental solution and does not contribute directly to the $A(z)$-geometric structure.

Accordingly, $A(z)$-geometry can be interpreted as the geometry induced by the orthogonal projection onto the solution space of the deformed Cauchy–Riemann equation.

In the limiting case $A(z) \equiv 0$, all of the constructions introduced above, namely, $A(z)$-lemniscates, the $A(z)$-metric and the $A(z)$-kernel reduce to their classical counterparts in Bergman theory. This limiting correspondence supports both the consistency and the natural character of the proposed geometric framework.

Remark 2.3. ***(Relation to quasiconformal geometry).*** *In general, the $A(z)$-geometry generated by the reproducing kernel K_A differs from the quasiconformal geometry developed by Ahlfors. Its structure is governed not by conformal moduli or extremal-length invariants, but by the reproducing-kernel Hilbert space of $A(z)$-analytic solutions and by the integral characteristics of the coefficient field $A(z)$. In the particular case $A(z) \equiv 0$, this geometric framework reduces to the classical Bergman geometry.*

Keywords: $A(z)$-analytic functions, the Bergman kernel, reproducing kernel, lemniscates, deformed complex structure, $A(z)$-geometry.

2020 Mathematics Subject Classification: 30C40, 30C62, 30H20

References:

[1] Steven G. Krantz, Geometric Function Theory (Explorations in Complex Analysis), Birkhauser Boston, 2006.

[2] A. Sadullaev, N.M.Zhabborov, On a class of A–analytic functions, Journal Siberian Federal University. Mathematics and Physics, vol. 9, no 3, 374–383, 2016.

An operator-symbol construction of sixth-order difference schemes for a fifth-order differential equation

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Abstract: Taylor's decomposition on four points for a third-order differential equation, introduced by Ashyralyev and Arjmand [1], is reformulated in operator-symbol form. This formulation is extended to the fifth-order differential equation

$$y^{(5)}(t) + a(t)y(t) = f(t), \qquad 0 < t < T.$$

A six-point decomposition is obtained and used to construct formally sixth-order difference schemes for an initial-value problem, a boundary-value problem, and a nonlocal boundary-value problem. The operator-symbol formulation provides a unified derivation of the common interior scheme and the corresponding one-sided closure formulas. Numerical examples exhibit the predicted sixth-order rate.

Keywords: operator-symbol method, six-point decomposition, fifth-order differential equation, sixth-order scheme, nonlocal boundary-value problem

2020 Mathematics Subject Classification: 65L05, 65L10, 65L12

References:

[1] A. Ashyralyev and D. Arjmand, A note on the Taylor's decomposition on four points for a third-order differential equation, *Applied Mathematics and Computation* **188** (2007), 1483–1490.

Existence and uniqueness for a variable-order fractional $p(\cdot)$-Laplacian equation with a causal delay term

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Abstract: This paper investigates the existence and uniqueness of weak solutions to a nonlinear parabolic problem involving the fractional $p(\cdot)$-Laplacian operator of variable order, with a source term containing a causal deviating argument. The analysis is carried out via a semi-discretization approach, where each time step is reformulated as the minimization of a strictly convex and coercive functional on a variable-order, variable-exponent Sobolev space. This formulation guarantees the existence, uniqueness, and stability of the discrete solutions, with estimates independent of the time step, and yields the corresponding weak solution of the continuous problem. The theoretical findings are illustrated through a concrete example, demonstrating the applicability of the proposed framework.

Keywords: Fractional $p(\cdot)$-Laplacian; variable order; deviating argument; semi-discretization method; weak solution.

2020 Mathematics Subject Classification: 35K55; 35R11; 35R10.

References:

[1] J. Giacomoni, S. Tiwari, *Existence and global behavior of solutions to fractional p-Laplacian parabolic problems*, Electron. J. Differential Equations **2018** (2018), no. 44, 1–20.

[2] J. M. Mazón, J. D. Rossi, J. Toledo, *Fractional p-Laplacian evolution equations*, J. Math. Pures Appl. **105** (2016), no. 6, 810–844.

[3] A. Sabri, *Weak solution for nonlinear fractional $p(\cdot)$-Laplacian problem with variable order via Rothe's time-discretization method*, Math. Model. Anal. **27** (2022), no. 4, 533–546.

[4] R. Haloi, D. Bahuguna, D. N. Pandey, *Existence and uniqueness of solutions for quasi-linear differential equations with deviating arguments*, Electron. J. Differential Equations **2012** (2012), no. 13, 1–10.

[5] C. G. Gal, *Semilinear abstract differential equations with deviated argument*, Int. J. Evol. Equ. **2** (2008), no. 4, 381–386.

[6] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Applied Mathematical Sciences **44**, Springer-Verlag, New York, 1983.

[7] X. Fan, D. Zhao, *On the spaces $L^{p(x)}(\Omega)$ and $W^{m,p(x)}(\Omega)$*, J. Math. Anal. Appl. **263** (2001), no. 2, 424–446.

Solution of a nonlocal problem with boundary conditions for a homogeneous ordinary differential equation with continuously varying second-order derivatives

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Abstract: In recent years, fractional mathematical modelling using nonlocal fractional derivatives has become a reliable tool and a new resource for describing the dynamics of complex systems with memory effects and heritable characteristics. Such systems are found in a variety of fields, including physics, fluid dynamics, materials science, signal processing, engineering, chemistry, biology, medicine, finance, social sciences, economics, and ecology.

In this paper, a new method for constructing an analytical solution to a nonlocal boundary value problem for a homogeneous ordinary differential equation with continuously varying second-order derivatives is proposed. To do this, an invariant expression that allows for algebraization of the problem under consideration is found. The resulting characteristic equation, corresponding to the equation under study, is transcendental, and its roots are found using the Lambert operator.

Keywords: Differential equations of variable order, invariant solution, transcendental characteristic equation, W-Lambert transform

2020 Mathematics Subject Classification: 34A08, 34B05, 34B60

References:

[1] N. Aliyev, M. Rasulov, B. Sinsoysal, The boundary value problem for an ordinary linear half-order differential equation, Bulletin of the Karaganda University. Mathematics Series, vol. 115, no 3, 5–12, 2024.

[2] N.A. Aliyev, R.G. Ahmadov, On an initial problem for an ordinary differential equation of order ν with a continuously varying order of derivative, (Russian) Proceedings of International Scientific-practical Internet conference "Trends and prospects for the development of science and education in the context of globalization", Grigory Skovoroda University, Pereyaslav, Ukraine, 31 October, 2023, no 98, 164–167.

Study of a class boundary value problem for ordinary differential equations whose order varies continuously

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Abstract: Fractional and multifractional differential equations are encountered and widely used in certain areas of physics, such as chemistry, astrophysics, biology, medicine, and so on. The study of flow dynamics observed in inhomogeneous and irregular regions also leads to the solution of differential equations of variable order.

Of particular interest is the study of differential equations whose order varies continuously.

The main goal of this paper is to find an invariant expression that allows for the algebraization of differential equations whose order varies continuously, similar to Euler's method for solving ordinary equations with constant coefficients, and to use it to construct an analytical solution to the problem under consideration

Keywords: multifractional differential equation, differential equations of variable order, invariant solution, W Lambert transform

2020 Mathematics Subject Classification: 34A08, 34B05, 34B60

References:

[1] N. Aliyev, M. Rasulov, B. Sinsoysal, The boundary value problem for an ordinary linear half-order differential equation, Bulletin of the Karaganda University. Mathematics Series, vol. 115, no 3, 5–12, 2024.

New examples of regular, but not strongly regular, boundary value problems for the operator of fourfold differentiation on an interval

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Abstract: We consider the problem

(1) $$y^{(IV)}(x) = \lambda y(x)$$

on the interval $(0,1)$ with two-point boundary conditions

(2) $$\begin{aligned} B_\nu(y) &= a_{\nu 1} y'''(0) + a_{\nu 2} y'''(1) + a_{\nu 3} y''(0) + a_{\nu 4} y''(1) \\ &+ a_{\nu 5} y'(0) + a_{\nu 6} y'(1) + a_{\nu 7} y(0) + a_{\nu 8} y(1) = 0, \end{aligned}$$

where the $B_\nu(y)$, $(\nu = 1,2,3,4)$ are linearly independent forms with arbitrary complex-valued coefficients. We assume that boundary conditions are normalized. The function $y(x)$ is an arbitrary complex-valued function of the class $W_2^4(0,1)$.

We can call boundary conditions normalized, if they can be written in the form

(3) $$B_\nu(y) \equiv B_{\nu 0}(y) + B_{\nu 1}(y) = 0,$$

where

(4) $$B_{\nu 0}(y) = \alpha_\nu y^{k_\nu}(0) + \sum_{j=0}^{k_\nu - 1} \alpha_{\nu j} y^j(0),$$

(5) $$B_{\nu 1}(y) = \beta_\nu y^{k_\nu}(1) + \sum_{j=0}^{k_\nu - 1} \beta_{\nu j} y^j(1),$$

$$3 \geq k_1 \geq k_2 \geq k_3 \geq k_4 \geq 0, \qquad k_\nu > k_{\nu+2},$$

and for each index ν at least one of the numbers α_ν, β_ν are nonzero.

As an example, we give the following boundary conditions here:

Orders (k_1,k_2,k_3,k_4)	Leading part (example)	Strengthened irregularity
$(3,3,2,1)$	$y^{(3)}(0)=0,\quad y^{(3)}(1)=0,$ $y''(0)=0,\quad r\,y'(0)+y'(1)=0$	$r=\pm\sqrt{2}$
$(3,3,1,0)$	$y^{(3)}(0)=0,\quad y^{(3)}(1)=0,$ $y'(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm\sqrt{2}$
$(3,3,2,0)$	$y^{(3)}(0)=0,\quad y^{(3)}(1)=0,$ $y''(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm 1$
$(3,2,2,1)$	$y^{(3)}(0)=0,\quad y''(1)=0,$ $y''(0)=0,\quad r\,y'(0)+y'(1)=0$	$r=\pm 1$
$(3,2,2,0)$	$y^{(3)}(0)=0,\quad y''(1)=0,$ $y''(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm\sqrt{2}$
$(3,2,1,0)$	$y^{(3)}(0)=0,\quad y''(1)=0,$ $y'(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm 2$
$(3,1,1,0)$	$y^{(3)}(0)=0,\quad y'(1)=0,$ $y'(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm\sqrt{2}$
$(2,2,1,0)$	$y''(0)=0,\quad y''(1)=0,$ $y'(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm\sqrt{2}$
$(2,1,1,0)$	$y''(0)=0,\quad y'(1)=0,$ $y'(0)=0,\quad r\,y(0)+y(1)=0$	$r=\pm 1$

These studies continue what was started in [1].

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Keywords: operator of fourfold differentiation, nonlocal boundary conditions, regular boundary conditions, not strongly regular boundary conditions, asymptotic distribution of eigenvalues

2020 Mathematics Subject Classification: 34L10, 34L20

References:

[1] Sadybekov M.A., Akhymbek M.E. Spectral properties of some non strongly regular boundary value problems for fourth order differential operators, in: Conference on Analysis and Applied Mathematics (ICAAM 2014) (Shymkent, Kazakhstan, Sep 11-13, 2014), AIP Conference Proceedings, vol. 1611, (2014), 156–160. DOI: 10.1063/1.4893822.

On weak singularity of Prabhakar kernels

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Abstract: We consider the 3-parametric generalization of the Mittag-Leffler type function

$$E^{\gamma}_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{(\gamma)_k z^k}{k!\,\Gamma(\alpha k + \beta)}, \qquad z, \alpha, \beta, \gamma \in \mathbb{C}, \quad \operatorname{Re}\alpha > 0, \tag{1}$$

where $(\gamma)_k$ is the Pochhammer symbol, i.e.

$$(\gamma)_k = \frac{\Gamma(\gamma + k)}{\Gamma(\gamma)},$$

and $\Gamma(\cdot)$ is Euler's gamma function. In this case the Prabhakar kernel [1] has the following form

$$e^{\gamma}_{\alpha,\beta}(\lambda; t) = t^{\beta-1} E^{\gamma}_{\alpha,\beta}(\lambda t^{\alpha}), \tag{2}$$

$$t \in \mathbb{R}^+, \quad \lambda, \alpha, \beta, \gamma \in \mathbb{C}, \quad \operatorname{Re}\alpha > 0, \ \operatorname{Re}\beta > 0.$$

The regularity properties of the function (1) in terms of the kernel (2) have been obtained.

Keywords: Mittag-Leffler function, Prabhakar kernel, Pochhammer symbol, weak singularity, fractional calculus, regularity properties

2020 Mathematics Subject Classification: 33E12, 26A33, 45D05

References:

[1] T.R. Prabhakar, A Singular Integral Equation with a Generalized Mittag-Leffler Function in the Kernel, Yokohama Mathematical Journal, vol. 19, no. 1, 7–15, 1971.

On biparty graphs with concept lattices

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Abstract: This paper studies bipartite graphs as a natural representation of formal contexts and the concept lattices that arise from them within Formal Concept Analysis (FCA). In this paper we recall the basic apparatus of FCA, discuss the structural properties of concept lattices with a particularly simple, flat partial order, and consider practical applications of such lattices, including data analysis, clustering, and the identification of relations between objects and attributes. Using a concrete bipartite graph, we build the corresponding formal context table, extract the formal concepts, and construct the resulting star-shaped, modular concept lattice. The results are illustrated with examples from access-control systems, e-commerce, and security monitoring.

Keywords: bipartite graphs, formal concept analysis, formal context, formal concepts, concept lattice, modular lattice.

2020 Mathematics Subject Classification: 05C76, 06B15, 68R10

References:

[1] C. Carpineto and G. Romano, Concept Data Analysis: Theory and Applications. Wiley, Chichester, 2004.

Numerical analysis of a nonlocal problem for a loaded parabolic equation

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Abstract: This paper considers a nonlocal boundary value problem for a loaded parabolic equation defined on a closed rectangular domain. Such equations arise in mathematical models of diffusion, heat and mass transfer, environmental processes, and other systems involving nonlocal sources or interactions. The method of lines is applied to the spatial variable, reducing the original partial differential equation to a two-point boundary value problem for a system of loaded ordinary differential equations. The resulting problem is solved using the Dzhumabaev parameterization method. This approach introduces auxiliary parameters and transforms the boundary value problem into a family of initial value problems supplemented by a system of algebraic equations. Based on this transformation, a computational algorithm for approximating the solution of the original nonlocal problem is constructed. The accuracy and effectiveness of the proposed algorithm are evaluated through representative computational experiments. A comparison between the numerical and exact solutions confirms the reliability and applicability of the developed approach.

The analysis presented in this paper is primarily based on the techniques developed in our previous studies [1, 2].

Keywords: loaded parabolic equation, nonlocal boundary condition, method of lines, parameterization method, numerical algorithm

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2020 Mathematics Subject Classification: 35K10, 34K10, 35R10, 65M22

References:

[1] S.K. Kuanysh, Zh. M. Kadirbayeva, Analysis of Nonlocal Problem for Loaded Parabolic Equation via Numerical Method, Lobachevskii Journal of Mathematics, vol. 46, no 10, 5417–5427, 2025.

[2] A.T. Assanova, Zh. Liu, S.K. Kuanysh, Zh. M. Kadirbayeva, Convergence, stability, and error analysis of the method of lines for solving loaded parabolic equation, AIMS Mathematics, vol. 10, no 12, 29454–29469, 2025.

Investigating and solving eigenvalue problems including linear and nonlinear difference equations using discrete additive and multiplicative differential equations

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Abstract: In this paper, we design and study eigenvalue problems involving linear and nonlinear difference equations with parameters. For such problems, first; we study simple cases of first-order equations including parameters and find their eigenvalues and eigenfunctions.

Then, for the general case of a second-order discrete multiplicative equations ,we will calculate their eigenvalues and eigenfunctions. eigenvalue problems involving nonlinear difference equations is one of the unsolved problems of this theory, therefore we try to generalize the Storm-Liouville theory to discrete multiplicative calculus and non-Newtonian analysis.

Finally, we apply the above method to boundary value problems including nonlinear difference equations with parameters, and illustarte the distribution eigenvalues and eigenfunctions in some graphics.

Keywords: Discrete multiplicative and additive derivative, Eigenvalue problem, Sturm-Liouvill problem, Non-linear difference equations

2020 Mathematics Subject Classification: 45J05, 34A08

References:

[1] Guaiz, M.A.Al., (2008), Sturm-Liouville theory and its applications, Springer Publisher.

[2] ladistav, V., Kravchenko, V., (2020), Direct and inverse Sturm-Liouville problems, Birkhauser, Publisher.

[3] Grossman. M., Katz. R., (1979), Non-Newtonian Calculus, Int. Journal of mathematical equations 10(4).

[4] Grossman. M., Katz. R., (1972), Non-Newtonian Calculus, Lee -Press. Pigeon Cove

[5] Bashirov. A.E., Rıza. M., (2011), On complex multiplicative differentiation, TWMS J. App. Eng. Math. (1)pp. 75-85.

[6] Bashirov, A.E., Kurpinar, E.M., Ozyapici, A., (2008), Multiplicative calculus and its applications Journal of Mathematical Analysis and Applications, 337(1). pp. 36-48.

[7] Hosseini, R., Jahanshahi, M., (2017), Invariant functions for solving discrete and countinuous ODEs. Computational Methods for Differential Equations, http://cmde.tabrizu.ac.ir, 5(4), pp. 271-279.

[8] Jahanshahi, M., Aliyev, N., Dehghani. H., (2024), Solving Non-Homogenous Nonlinear Difference Equations, COIA. Stanbul. Turkey

On a time nonlocal problem for a polyharmonic equation with involution

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Abstract: Let $\Omega \subset R^n$ be an arbitrary n-dimensional domain with a sufficiently smooth boundary $\partial\Omega$. Let us assume that the domain Ω is invariant with respect to the transformation $Sx = -x$. Denote by $Q_T = \{(t, x) : 0 < t < T, x \in \Omega\}$, Γ the lateral boundary of the cylinder Q_T. Consider the following problem in Q_T:

$$u_t(t, x) + a_0(-\Delta)^m u(t, x) + a_1(-\Delta)^m u(t, -x) = 0, (t, x) \in Q_T, \tag{1}$$

$$u(0, x) + \gamma u(T, x) = \varphi(x), \tag{2}$$

$$\Delta^k u(t, x) = 0, k = 0, 1, ..., m - 1, (t, x) \in \Gamma, \tag{3}$$

where γ is a real number, $\varphi(x)$ is a given function.

Direct and inverse problems for equation (1) in the case $m = 1$ and $\gamma = 0$ have been studied by numerous authors (see, e.g., [1, 2]). In these papers, the well-posedness of problem (1)–(3) in the case $a_0 > |a_1|$ was proven.

In the present paper, we find all possible values of the parameters a_0, a_1 and γ for which problem (1)–(3) becomes well-posed.

Keywords: involution, nonlocal problem, polyharmonic operator, Navier problem

2020 Mathematics Subject Classification: 35K35, 35K65, 35R30

References:

[1] N. Al-Salti, M. Kirane, B. Torebek, On a class of inverse problems for a heat equation with involution perturbation, Hacettepe Journal of Mathematics and Statistics, vol. 48, no 3, 669–681, 2019.

[2] B. Turmetov, M. Koshanova, M. Muratbekova, I. Orazov, Direct and inverse problems for a fractional parabolic equation with multiple involution, Fract. Differ. Calc, vol. 14, 213–233, 2024.

An inverse problem for time fractional pseudo-parabolic equation

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Abstract: In this work, we will discuss the solvability of an inverse problem for the following time fractional pseudo-parabolic equation :

$$\mathcal{D}_t^{\alpha}\big(u(x,t)+\beta\mathcal{L}u(x,t)\big)+\mathcal{L}u(x,t)=f(x,t), \qquad x\in\Omega, \quad t\in(0,T), \tag{1}$$

supplemented with the initial condition

$$u(x,0)=a(x), \qquad x\in\Omega, \tag{2}$$

the nonhomogeneous boundary condition encoded by

$$\mathcal{B}u(x,t)=p(t)h(x) \qquad x\in\partial\Omega, \quad t\in[0,T], \tag{3}$$

and the overdetermination condition

$$\mathcal{P}u(t)=e(t). \qquad t\in[0,T]. \tag{4}$$

Let $\mathcal{H}$ be a separable Hilbert space. The inverse problem (1)-(4) consists of finding the pair of functions :

- $p:[0,T]\to\mathbb{R}$ and
- $u:Q_T:=\Omega\times(0,T)\mapsto\mathbb{R}$,

which to be determined by the given data: functions $a(x), f(x,t), h(x), e(t)$ and a positive constant β. Here,$\mathcal{L}$ is a second order linear differential operator,$\mathcal{B}$ is a linear boundary (trace) operator $\mathcal{B}:\mathcal{H}^{\gamma_b}\to\mathcal{H}_{\partial\Omega}$, $\mathcal{P}$ is linear bounded functional and $\mathcal{D}_t^{\alpha}$ denotes the Caputo fractional derivative of order $\alpha\in(0,1)$.

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Keywords: Inverse problem,pseudo- parabolic equation, Caputo fractional derivative, boundary condition, existence, uniqueness, stability.

2020 Mathematics Subject Classification: 35R30, 35A01, 35A02

On a generalization of the Robin problem for a nonlocal analogue of the Poisson equation

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Abstract: Let S be an orthogonal transformation satisfying the condition $S^l = E, l \in N$, where E denotes the identity transformation. In the unit ball Ω, we consider the following boundary value problem:

$$\sum_{j=0}^{l-1} a_j(-\Delta u)(S^j x) = f(x), x \in \Omega, \tag{1}$$

$$\sum_{j=0}^{l-1} b_j \frac{\partial u}{\partial \nu}(S^j x) + \sum_{j=0}^{l-1} c_j u(S^j x) = g(x), x \in \partial\Omega \tag{2}$$

where a_j, b_j, c_j are given real constants, while f(x) and g(x) are prescribed functions. We note that, in [1], the main boundary value problems for equation (1) with the classical Dirichlet, Neumann, and Robin boundary conditions were previously investigated. The method used to investigate problem (1)–(2) is based on the decomposition of the function space into a direct sum of invariant subspaces by means of a system of orthogonal projections. This decomposition makes it possible to reduce the original nonlocal problem to a collection of classical local Dirichlet, Neumann, and Robin boundary value problems for the Poisson equation. Conditions ensuring the existence and uniqueness of a solution are obtained, together with necessary and sufficient solvability conditions in the case where a Neumann-type problem arises. An integral representation of the solution is constructed in terms of the Green functions corresponding to the associated classical boundary value problems. Furthermore, the corresponding spectral problem is investigated. It is shown that its spectral characteristics are determined by the spectra of the associated Dirichlet, Neumann, and Robin problems posed in the corresponding invariant subspaces.

Keywords: involution, nonlocal boundary value problem, Poisson equation with involution, projector, Dirichlet problem, Neumann problem, Robin problem, Green function, spectral problem, eigenvalues, eigenfunctions

2020 Mathematics Subject Classification: 35J05, 35J25, 35P05

References:

[1] V. Karachik, A. Sarsenbi, B. Turmetov, On the solvability of the main boundary value problems for a nonlocal Poisson equation, Turkish Journal of Mathematics, vol. 43, no 3, 1604–1625, 2019.

On a method of transferring of non-local boundary value problem for ordinary differential equations of the first order to the Cauchy problem

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Abstract: In this article a new method for transforming a nonlocal boundary value problem for a first-order ordinary differential equation into a Cauchy problem with an initial condition defined at an arbitrary point in the domain is proposed. A fundamental solution of the considered equation is established, with help which yields two necessary conditions. Using those conditions, the value of the solution at point c, i.e., $y(c)$, is found, that allowing the transformation of the boundary value problem into a Cauchy problem. The proposed method provides to easy find as an analytical as well numerical solution of the considered problem.

Keywords: Ordinary differential equation, nonlocal boundary value problem, main relationship, necessary conditions.

2020 Mathematics Subject Classification: 34A08, 34B05

References:

[1] N. Aliyev, M. Rasulov, B. Sinsoysal, The boundary value problem for an ordinary linear half-order differential equation, Bulletin of the Karaganda University. Mathematics Series, vol. 115, no 3, 5–12, 2024.

[2] B. Sinsoysal, N. Aliyev, M. Rasulov, On the method of transforming a nonlocal boundary value problem into a Cauchy problem for a second order ordinary differential equation, Logic Journal of the IGPL, vol. 34, no 3, 2026.

Optimal parameter identification of a discrete gas-lift process

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Abstract: The motion of gas in the annular space and the gas fluid in the lift of a gas-lift well are described by the following system of nonlinear diferential equations, the right side of which includes a small parameter:

$$\begin{cases} \dot{Q}_i = -\dfrac{2a_i \rho_i F_i Q_i^2}{c_i^2 \rho_i^2 F_i^2 \mu - Q_i^2}, & Q(0) = u, \\ \dot{P}_i = \dfrac{2a_i c_i^2 \rho_i^2 F_i^2 Q_i^2}{c_i^2 \rho_i^2 F_i^2 \mu - Q_i^2}, & P(0) = P_0, \qquad i = 1, 2. \end{cases} \tag{1}$$

where the parameter a_i is found from the expression

$$2a_i = \frac{g_i}{\omega_i} + \frac{\lambda_i \omega_i}{2D_i}, \qquad i = 1, 2. \tag{2}$$

Here, g_1 and g_2 denote the gas release rates, ω_1 and ω_2 are the mixture velocities, λ_1 and λ_2 are the hydraulic resistance coefficients, D_1 and D_2 are the effective diameters, F_1 and F_2 are the cross-sectional areas, Q_i represents the mass flow rates of gas and gas-liquid mixture, and μ is a small parameter.

İnvestigates the determination of the hydraulic resistance coefficient (HRC) in a discrete gas-lift process. An asymptotic approach was applied by dividing the well into two sections, with the reciprocal of the well depth used as a small parameter. The HRC parameters were identified using the least squares method with Tikhonov regularization to ensure solution uniqueness. The obtained results are consistent with those from other methods and are close to the exact solution.

Throughout this note we mainly use techniques from our work [1].

Keywords: Gas-lift process, parameter identification, asymptotic method, nonlinear difference equations.

2020 Mathematics Subject Classification: 93B30, 65L12, 49J20

References:

[1] F.A. Aliev, N.S. Hajiyeva, G.H. Mammadova, A.A. Namazov, M.S. Khalilov, Identification of the Parameters of a Discrete Gas-Lift Process, Journal of Computer and Systems Sciences International, vol. 61, no. 5, 805–812, 2022.

The oscillatory of one class of the high-order Sturm-Liouville difference equation

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Abstract: In this work, we consider the problem of oscillatory of the higher-order Sturm-Liouville difference equation

$$(-1)^n \Delta^n \left(\rho_k \Delta^n y_k\right) - v_k y_{k+n} = 0, \tag{3}$$

where $n \in \mathbb{N}$, $n > 1$, $\Delta y_k = y_{k+1} - y_k$, $\rho = \{\rho_k\}_{k=0}^{\infty}$ is a sequence of positive numbers, $v = \{v_k\}_{k=0}^{\infty}$ is a sequence of non-negative numbers, and $\mathbb{N}$ is a set of natural numbers.

The oscillatory properties of the equation (3) are determined by the behavior of its solutions in a neighborhood of infinity. However, since the behavior of these solutions depends on the behavior of the equation coefficients, the determination of the oscillatory properties of equation (3) in terms of the coefficients ρ, v is important. We work directly with the variation principle [1]. We establish conditions equivalent to the fulfillment of the variational principle, on the basis of which the oscillatory conditions of the equation (3) are determined.

Our main result reads as follows:

Let

$$B_n(N) = \max_{0 \le t \le n-1} \left(\frac{1}{t!(n-t-1)!}\right)^2 \sup_{i \ge N} B_{n,i}(t),$$

$$B_{n,i}(t) = \sum_{k=i}^{\infty} \left[(k-i)^{(t)}\right]^2 u_k \sum_{j=N}^{i} \rho_j^{-1} \left[(i-j)^{(n-t-1)}\right]^2.$$

Theorem 1. If $\limsup_{N\to\infty} B_n(N) > 1$, then the equation (3) is oscillatory.

Keywords: Sturm-Liouville difference equation, discrete Sobolev space, oscillatory, variational principle

2020 Mathematics Subject Classification: 34C10, 39A70, 26D15

References:

[1] O. Došlý, R. Hilscher, A Class of Sturm-Liouville Difference Equations: (Non)Oscillation Constants and Property BD, Computers and Mathematics with Applications, vol. 45, 961–981, 2003.

Nonlinear integro-differential model of pest population dynamics with age structure for plant protection tasks

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Abstract: This paper investigates a nonlinear mathematical model of an agroecosystem consisting of three trophic levels (plants, harmful insects, and beneficial insects) that takes into account the age structure of insect populations. The model is described by the system of integro-differential equations with partial derivatives (1). Necessary and sufficient conditions for the existence of solutions to the plant protection problem are established.

Consider the agroecosystem "plant – harmful insects – beneficial insects" with age structure:

$$
(1)\qquad \begin{cases}
\dfrac{dN_0}{dt} = Q - \alpha_0 N_0 N_1, \\
\dfrac{dN_1}{dt} = k_0\alpha_0 N_1 - V_1(N_1)\widetilde{N}_2 - m_1 N_1, \\
\dfrac{\partial N_2}{\partial t} + \dfrac{\partial N_2}{\partial a} = k_1 V_1(N_1) N_2 - V_2(N_2)\widetilde{N}_3 - m_2 N_2, \quad N_2|_{t=0} = N_2^0(a), \\
\dfrac{\partial N_3}{\partial t} + \dfrac{\partial N_3}{\partial a} = k_2 V_2(N_2) N_3 - \varepsilon N_3^2 - m_3 N_3, \quad N_3|_{t=0} = N_3^0(a), \\
N_2(0,t) = \displaystyle\int_0^\infty B_2(\xi, t, N_1) N_2(\xi, t)\, d\xi, \\
N_3(0,t) = \displaystyle\int_0^\infty B_3(\xi, t, \widetilde{N}_2) N_3(\xi, t)\, d\xi,
\end{cases}
$$

where $N_0(t)$ is external resource, $N_1(t)$ is plant biomass, $N_i(a,t)$ $(i = 2,3)$ are age-specific densities of harmful and beneficial insects; $V_i(\cdot)$ are trophic functions; $B_i(\cdot)$ are birth rates; $\widetilde{N}_i(t) = \int_{\alpha_i}^{\beta_i} N_i(a,t)\, da$.

Keywords: mathematical model, agroecosystem, plant protection problem, age structure, insect population, trophic function, integro-differential equations, pest dynamics

2020 Mathematics Subject Classification: 35R09, 92D25

References:

[1] Odinaev R.N., Gaforov A.B. Mathematical and computer modeling of cotton agrocenosis taking into account the age structure and with arbitrary trophic functions // Systems and means of computer science. – 2021. – Vol. 31, No. 2. – pp. 173-183. – DOI 10.14357/08696527210216. – EDN HQKZFJ.

An efficient high-order numerical method for the transverse vibration of a damped beam

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Abstract: In this paper, a high-order and unconditionally stable difference method is proposed for the numerical solution of the one-dimensional damped beam vibration equation (Euler-Bernoulli beam equation with damping).
We apply a fourth-order compact finite difference approximation for discretizing the fourth-order spatial derivative of this equation and a fifth-order Padé approximation for the resulting system of ordinary differential equations. It is shown through analysis that the proposed scheme is unconditionally stable.
This new method is easy to implement, produces very accurate results and requires short CPU time. Several numerical examples are included to demonstrate the validity and applicability of the technique

Keywords: Padé approximation, compact finite difference scheme, unconditionally stable, damped Euler-Bernoulli beam equation, fourth-order PDE, high accuracy.

References:

[1] Mohebbi,A.,Dehghan,M.:High-order compact solution of the one-space dimensional heat and advectiondi¤usion equations. App. Math. Model. 34, 3071 3084(2010)
[2] Richtmyer, R.D., Morton, K.W.: Di¤erence Methods for Initial Value Problems, 2nd edn. Interscience,New York (1967).
[3] Mohebbi, A.: A fourth-order nite di¤erence scheme for the numerical solution of 1D linear hyperbolic equation. Commun. Numer. Anal. 2013, 1 11 (2013)

A generalized averages based variational joint image segmentation and registration model

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Abstract: Image segmentation and registration are tightly coupled tasks, yet many variational models struggle when object and background intensities vary or when multiple structures are present. We propose a generalized averages based variational joint segmentation-registration (**GAV-JSR**) model that replaces Chan-Vese averages with generalized averages parameterized by γ, and embeds them in a non-rigid registration framework regularized by linear curvature. This yields an energy functional that (i) remains backward-compatible with prior **NJSR** formulations when $\gamma = 1$, (ii) better handles high- and low-intensity backgrounds without pre-segmentation, and (iii) produces smooth, realistic deformations suitable for substantial affine changes. We validate the method on clinical (brain MRI, hand X-ray) and synthetic datasets, demonstrating consistent improvements over **GV-JSR** and **NJSR** baselines in both registration error and segmentation quality for example, error drops from 69.1% to 30.85% on one white backgroud multi-object case while maintaining clear object boundaries with across multiple structures along with positive F-measure value shown by suggested model. These results indicate that integrating generalized averages with curvature-based regularization provides a robust and accurate solution for monomodal joint segmentation and registration.

Keywords: segmentation, registration, generalized averages, medical images, synthetic images

Mathematical modeling of cotton agrocenosis with age structure and arbitrary trophic functions

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Abstract: This paper considers mathematical modeling of the process of protecting cotton agrocenosis taking into account age structure and with arbitrary trophic functions.

Assume that the state of the model agrocenosis is described by the following system of differential equations [1]:

$$\begin{cases} l\dfrac{dN_0}{dt} = Q - \alpha_0 N_0 N_1, \\ \dfrac{dN_1}{dt} = k_0\alpha_0 N_0 N_1 - V_1(N_1)\displaystyle\sum_{i=1}^{3}\tilde{N}_{i+1} - m_1 N_1, \\ \dfrac{\partial N_{i-1}}{\partial t} + \dfrac{\partial N_{i-1}}{\partial a} = k_{i-2}V_1(N_1)N_{i-1} - V_{i-1}(N_{i-1})\tilde{N}_{i+2} + \delta N_{i+2} - m_{i-1}N_{i-1}, \quad i = \overline{3,5}, \\ \dfrac{\partial N_{i+2}}{\partial t} + \dfrac{\partial N_{i+2}}{\partial a} = k_{i+1}V_{i-1}(N_{i-1})N_{i+2} - \epsilon\tilde{N}_{i+2}^2 - m_{i+2}N_{i+2}, \quad i = \overline{3,5}, \\ N_i|_{t=0} = N_i^0, \quad i = \overline{0,7}, \\ N_i(0,t) = \displaystyle\int_0^\infty B_i(\xi)N_i(\xi,t)\,d\xi, \quad i = \overline{2,7} \\ \tilde{N}_i = \displaystyle\int_{\alpha_i}^{\beta_i} N_i(a,t)\,da, \quad i = \overline{2,7} \end{cases}$$

where $N_0(t)$ is the mass of the external resource at time t, $N_1(t)$ is the biomass of cotton, Q is the rate of external resource supply, $N_i(a,t)$ is the biomass (or abundance) of harmful and beneficial insects of age a at time t, $B_i(\cdot)$ are the birth rates of harmful and beneficial insects, $V_i(\cdot)$ is the trophic function, $\alpha_0, m_i, k_i, \varepsilon$ are biological parameters of the populations in the agrocenosis; $t \in [0,\tau]$, $\tau = \text{const} < \infty$, and a is age, $a \in [0,\infty)$.

Keywords: mathematical modeling, agrocenosis, age structure, trophic functions, integral equations, biological control

2020 Mathematics Subject Classification: 35R09, 92D25

References:

[1] R.N. Odinaev, Mathematical model of the plant protection problem in a "pest–beneficial insect" biosystem with arbitrary trophic functions, *Systems and Means of Informatics*, vol. 29, no. 1, pp. 96–108, 2019. doi:10.14357/08696527190109. – EDN BDQEJR.

Solving a transport problem with fuzzy tariffs on the example of cargo transportation at the Rogun HPP

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Abstract: This report is devoted to solving the transport problem with fuzzy tariffs on the example of cargo transportation at the Rogun HPP. A homogeneous cargo is available at m supply points A_i with volumes a_i and must be delivered to n demand points B_j with volumes b_j. The cost c_{ij} of transporting a unit from i to j is minimized:

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \to \min, \quad \sum_{j=1}^{n} x_{ij} = a_i, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad x_{ij} \geq 0. \tag{1}$$

For fuzzy tariffs $\tilde{c}_{ij} = (a_{ij}, m_{ij}, b_{ij})$ of triangular type, we use defuzzification via level sets. The deterministic equivalent is:

$$s(\tilde{c}_{ij}) = \sum_{p=0}^{k} w_p L_p, \quad L_p = \frac{1}{2}\{(m_{ij} - a_{ij}(1-\alpha_p)) + (m_{ij} + b_{ij}(1-\alpha_p))\}. \tag{2}$$

The problem is solved by the potential method. For the Rogun HPP case: two suppliers (18,951 and 41,439 tons) deliver cement to four warehouses (17,019, 4,996, 36,444, 1,931 tons). Using level sets $\alpha = 0.05, 0.1, 0.2, 0.3$ with weights $W = (0.05, 0.1, 0.15, 0.3, 0.4)$, the optimal plan yields total cost $Z = 6,208,772$ somoni.

Keywords: transport problem, northwest corner method, potential method, fuzzy tariffs, Fuzzy set, optimal plan, triangular membership function

2020 Mathematics Subject Classification: 90B06, 90C05, 90C70

References:

[1] R.N. Odinaev, S.H. Mavlonzoda, Mathematical model of cargo transportation of the Republic of Tajikistan on the example of the Rogun HPP using the linear programming method, Bulletin of the Tajik National University, Series of Natural Sciences 4 (2020) 5–14.

On the strong solvability of the Laplace equation with a non-classical boundary condition

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Abstract: This paper is devoted to the question of strong solvability of the following nonlocal boundary value problem for the Laplace equation in the domain $D = (0, 1) \times (0, h)$:

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0, \qquad (x,y) \in D, \tag{1}$$

$$u(x,0) = \varphi(x), \qquad u(x,h) = \psi(x), \qquad 0 \leq x \leq 1, \tag{2}$$

$$u(0,y) = u(1,y), \qquad \frac{\partial u}{\partial x}(0,y) = \beta \frac{\partial u}{\partial x}(1,y) + \alpha u(1,y), \qquad 0 \leq y \leq h, \tag{3}$$

where $\alpha, \beta \in \mathbb{R}$, $\beta \neq -1$, $\varphi(x)$ and $\psi(x)$ are given functions. The solution to problem (1)-(3) is sought in a weighted Sobolev space $W^2_{p,\omega}(D)$ generated with a mixed norm

$$\|u\|_{L_{p,\omega}(D)} := \left(\int_0^h \int_0^1 \omega(x) |u(x,y)|^p \, dx \right)^{1/p} dy,$$

where the weight function $\omega(x)$ belongs to the Muckenhoupt class A_p. The boundary conditions of the corresponding spectral problem are regular, but not strongly regular. For $\beta \neq \pm 1$ and $\alpha \neq 0$, the spectral problem has only eigenfunctions that do not form a basis in $L_{p,\omega}[0,1]$, and direct application of the Fourier method is impossible. From the linear combinations of these eigenfunctions, a new system is constructed, which already forms a basis in $L_{p,\omega}[0,1]$, and this system is used to solve the equation under consideration by the Fourier method. In this work we use the approach from [1].

Keywords: Laplace equation, nonlocal boundary value problem, spectral problem, regular boundary conditions, eigenvalues, eigenfunctions, basis

2020 Mathematics Subject Classification: 35J05, 35J08, 46E35, 46M05

References:

[1] T. Gasymov, B. Akhmadli, U̇. Yıldız, On strong solvability of one nonlocal boundary value problem for Laplace equation in rectangle, Turkish Journal of Mathematics, vol. 48, no. 1, 21–33, 2024.

Positive invariance of an admissible region in a discrete predator–prey model

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Abstract: Discrete predator–prey models are widely used to describe population processes [1,2]. The construction of positively invariant regions is an important step in the analysis of discrete predator–prey systems [3]. Consider the following model with rational prey growth:

$$\begin{cases} x_{n+1} = \dfrac{\alpha x_n}{1+\beta x_n+\gamma x_n^2} - a x_n y_n, \\ y_{n+1} = y_n + b x_n y_n - d y_n, \end{cases} \tag{1}$$

where $x_n, y_n \geq 0$ denote the prey and predator populations, respectively, and $\alpha > 0$, $\beta \geq 0$, $\gamma > 0$, $a > 0$, $b > 0$, $d > 0$.

Here α characterizes the reproductive potential of the prey, β, γ describe intraspecific self-regulation, a describes the predator's impact on the prey, b accounts for predator growth due to prey consumption, and d denotes the natural mortality of the predator.

Set

$$F(x) = \frac{\alpha x}{1+\beta x+\gamma x^2}.$$

The entire nonnegative quadrant $\mathbb{R}^2_+$ is not automatically invariant: if

$$y_n > \frac{\alpha}{a(1+\beta x_n+\gamma x_n^2)},$$

then $x_{n+1} < 0$. Therefore, consider the biologically admissible region

$$D_K = \left\{ (x,y) \in \mathbb{R}^2_+ : \ 0 \leq x \leq K, \quad 0 \leq y \leq \frac{\alpha}{a(1+\beta x+\gamma x^2)} \right\}.$$

Theorem 1.

Let $\alpha > 0$, $\beta \geq 0$, $\gamma > 0$, $a > 0$, $b > 0$, $0 < d \leq 1$, and

$$K \geq \frac{\alpha}{\beta + 2\sqrt{\gamma}}. \tag{2}$$

Assume that for all $x \in [0,K]$ and $z \in [0,1]$,

$$z(1-d+bx)\Big[1+\beta F(x)(1-z)+\gamma F^2(x)(1-z)^2\Big] \leq 1+\beta x+\gamma x^2. \tag{H_D}$$

Then D_K is a positively invariant region for system (1):

$$(x_0,y_0) \in D_K \quad \Longrightarrow \quad (x_n,y_n) \in D_K \qquad \text{for all } n \geq 0.$$

Since $\max_{x\geq 0} F(x) = \alpha/(\beta+2\sqrt{\gamma})$, condition (2) controls the prey component, whereas condition (H_D) controls the predator component relative to the upper boundary of D_K. Together, these conditions guarantee positive invariance of the biologically admissible region and provide a basis for further investigation of fixed points, local stability, and bifurcation regimes of the system.

Keywords: discrete predator–prey model, positive invariance, invariant region, population dynamics, rational growth

2020 Mathematics Subject Classification: 39A60, 92D25

References

[1] S. Elaydi, *An Introduction to Difference Equations with Applications in Science and Engineering*, 2nd ed., Chapman & Hall/CRC, 2007.

[2] J. D. Murray, *Mathematical Biology I: An Introduction*, Springer, 2002.

[3] Z. AlSharawi, N. Pal, J. Chattopadhyay, The role of vigilance on a discrete-time predator–prey model, *Discrete Contin. Dyn. Syst. Ser. B*, 27 (2022), 6723–6744.

Basicity of the sine and cosine systems in weighted Sobolev spaces

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Abstract: In this work, we present a method for establishing basicity in weighted Sobolev spaces. Under suitable conditions on the weight function, and assuming that the systems $\{\sin \lambda_n x\}_{n\in\mathbb{N}}$ and $\{\cos \lambda_n x\}_{n\in\mathbb{N}}$ form Schauder bases in $L_{p,\rho}(0,\pi)$, we establish sufficient conditions under which the cosine system $\{\cos \lambda_n x\}_{n\geq 0}$ forms a Schauder basis in the weighted Sobolev space $W^1_{p,\rho}(0,\pi)$.

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Keywords: Schauder basis, weighted Lebesgue space, weighted Sobolev space, biorthogonal system, sine system, cosine system

2020 Mathematics Subject Classification: 46B15, 46E35, 42C15

References:

[1] B.T. Bilalov, Bases from exponents,cosines and sines that are eigen functions differential operators, Diff. Uravn., vol. 39, no 5, pp. 1-5, 2003 (in Russian).

[2] B.T. Bilalov, Basicity of some systems of exponents,cosines and sines, Diff. Uravn., vol. 26, no 1, 10-16, 1990 (in Russian).

On the integral representation of regular solutions to the heat equation and boundary conditions

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Abstract: Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a smooth boundary, $Q = \Omega \times \{t > 0\}$.

Definition 2.4. *The operator L is called a coercively solvable operator of the heat equation*

$$Lu(x,t) = \left(\frac{\partial}{\partial t} - \Delta_x\right) u(x,t) = f(x,t), \quad (x,t) \in Q \tag{1}$$

if for any $f \in L_2(Q)$ there exists a unique solution $u \in W_2^{2,1}(Q)$ of equation (1) satisfying the inequality

$$\|u\|_{W_2^{2,1}(Q)} \leq c\, \|f\|_{L_2(Q)}. \tag{2}$$

We seek a class of regular solutions of equation (1) in the following form

$$\begin{aligned} u(x,t) &= \int_0^t \int_\Omega \varepsilon(x,t,\xi,\tau) f(\xi,\tau) d\xi d\tau \\ &+ \int_\Omega \varepsilon(x,t,\xi,0)\varphi(\xi) d\xi + \int_0^t \int_{\partial\Omega} \frac{\partial \varepsilon(x,t,\xi,\tau)}{\partial n_\xi} \nu(\xi,\tau) dS_\xi d\tau, \end{aligned} \tag{3}$$

where

$$\varepsilon(x,t) := \frac{\Theta(t)}{(2\sqrt{\pi t})^n} e^{-\frac{|x|^2}{4t}} \tag{4}$$

is the fundamental solution of the heat equation.

Next, using the Riesz representation theorem for linear functionals, we derive a unified integral representation of the solutions and find the boundary conditions. For this purpose, we apply the method developed by the authors in [1], [2].

Keywords: heat equation, fundamental solution, boundary condition.

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2020 Mathematics Subject Classification: 35K05, 35K20

References:

[1] Kalmenov T.Sh., Suragan D. A boundary condition and spectral problems for the Newton potential, Modern Aspects of the Theory of Partial Differential Equations, 1:1 (2011), 187–210.

[2] Kal'menov T.S., Tokmagambetov N.E. On a nonlocal boundary value problem for the multidimensional heat equation in a noncylindrical domain, Siberian Mathematical Journal, 54:6 (2013), 1023–1028.

Maximal F-regularity of an abstract Cauchy problem with sectorial operators

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Abstract: We consider the abstract Cauchy problem

$$\frac{du(t)}{dt} + Au(t) = f(t), \quad t > 0,$$
$$u(0) = 0,$$

where A is a closed linear operator in a Banach space X with dense domain $D(A)$, and f belongs to a Bochner–Luxemburg space $F(\mathbb{R}_+; X)$. The space $F(\mathbb{R}_+)$ is assumed to be rearrangement invariant, with Boyd indices α_F and β_F satisfying

$$0 < \alpha_F \leq \beta_F < 1.$$

We define maximal F-regularity by requiring that, for every $f \in F(\mathbb{R}_+; X)$, the corresponding solution satisfies

$$\frac{du}{dt} \in F(\mathbb{R}_+; X), \qquad Au \in F(\mathbb{R}_+; X).$$

The main result is obtained by transferring the classical maximal L_p-regularity result to the rearrangement invariant setting. For this purpose, we use R_F-bounded families of operators and the relation between R_p-boundedness and R_F-boundedness. If X is a UMD space and A is sectorial with spectral angle less than $\pi/2$, then the existence of maximal L_p-regularity for one admissible exponent implies maximal F-regularity. More precisely, it is sufficient that one of the standard equivalent L_p-regularity conditions holds, including R_p-boundedness of the corresponding resolvent family or R_p-sectoriality of A. Thus, the result applies simultaneously to a broad class of rearrangement invariant spaces, including Lebesgue, grand Lebesgue, Orlicz, Lorentz and Marcinkiewicz spaces. The obtained framework also covers Bochner–Luxemburg spaces and provides a convenient setting for further applications to elliptic operators in symmetric function spaces.

Keywords: abstract Cauchy problem, maximal regularity, rearrangement invariant spaces, Boyd indices, sectorial operators, UMD spaces

2020 Mathematics Subject Classification: 35J40, 35K50, 42B15

References:

[1] R. Denk, M. Hieber, J. Prüss, R-Boundedness, Fourier Multipliers and Problems of Elliptic and Parabolic Type, Mem. Amer. Math. Soc., 166(788), 2003.

About an explicit solutions of a second-order differential equation of Riemann-type with many singular points

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Abstract: The paper is devoted to the investigation of the second order differential equation of the Riemann-type with three singular points

$$\frac{d^2w}{dz^2} + \left(\frac{1 - r_1^{(1)} + r_2^{(1)}}{z - a_1} + \frac{1 - r_1^{(2)} + r_2^{(2)}}{z - a_2} + \frac{1 - r_1^{(3)} + r_2^{(3)}}{z - a_3} \right) \frac{dw}{dz} + \left(\frac{r_1^{(1)} r_2^{(1)} (a_1 - a_2)(a_1 - a_3)}{z - a_1} + \frac{r_1^{(2)} r_2^{(2)} (a_2 - a_1)(a_2 - a_3)}{z - a_2} + \frac{r_1^{(3)} r_2^{(3)} (a_3 - a_1)(a_3 - a_2)}{z - a_3} \right) \frac{1}{(z - a_1)(z - a_2)(z - a_3)} w = 0, \tag{1}$$

where $r_i^{(j)} (i = 1, 2; j = 1, 2, 3)$–are the roots of the indicial equations corresponding to the singular points a_1, a_2, a_3 and these roots satisfy the Fuchs condition

$$r_1^{(1)} + r_2^{(1)} + r_1^{(2)} + r_2^{(2)} + r_1^{(3)} + r_2^{(3)} = 1. \tag{2}$$

and many singular points

$$w'' + \left(\sum_{r=1}^{n} \frac{1 - \alpha_r - \beta_r}{z - a_r} \right) w' + \left(\sum_{r=1}^{n} \frac{\alpha_r \beta_r}{(z - a_r)^2} + \sum_{r=1}^{n} \frac{E_r}{z - a_r} \right) w = 0, \tag{3}$$

where α_r, β_r are the roots of the indicial equations corresponding to singular points a_r, and the following conditions are satisfied

$$\sum_{r=1}^{n} (\alpha_r + \beta_r) = n - 2, \sum_{r=1}^{n} E_r = 0, \sum_{r=1}^{n} (a_r E_r + \alpha_r \beta_r) = 0, \quad \sum_{r=1}^{n} (a_r^2 E_r + 2a_r \alpha_r \beta_r) = 0. \tag{4}$$

In the work, a three-pointed differential operator and n-pointed differential operator are introduced, and with the help of these operators, second-order operator-differential equations constructed. Further, it is shown the equivalence of the constructed operator-differential equations to the second-order Riemann equations with three and n singular points, but only with the distinction that the Fuchs condition is replaced by a Fuchs-type condition with zero right-hand side. It is also obtained the explicit solutions of the second order operator-differential equation with three and many singular points.

The classical results about these type equations was obtained in works [1] – [5].

Keywords: differential equation, Riemann-type differential equation, singular point, characteristic equation

2020 Mathematics Subject Classification: 44A15, 45J05, 45E99, 45D99

References:

[1] B. Riemann, Works, Moscow, GITTL, 1948. (in Russian).

[2] J. Plemelj, Problems in the sense of Riemann and Klein, New York; Sydney, Intern, publ., 1964. 175p.

[3] I. A. Lappo-Danilevsky, Application of functions from matrices to the theory of linear systems of ordinary differential equations, Moscow, 1957, 456 p. (in Russian).

[4] N. P. Erugin, The Riemann problem. I., Differential Equations, **11(5)**1975, 771-781. (in Russian).

[5] A. A. Bolibrukh, The Riemann-Hilbert problem , UMN, **45(2)**1990, 3-47. (in Russian).

On the solvability of a problem for a nonlinear pseudoparabolic equation

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Let $\Omega = [0,1] \times [0,T]$ be a rectangle, and let $a_i(x,t) \in C(\Omega)$, $\quad i = \overline{0,3}$. We consider the differential operator L defined by the equality:

$$Lu = u_{xtt} - a_0 u_{xt} - a_1 u_x - a_2 u_t - a_3 u.$$

We study the following nonlinear boundary value problem in the domain Ω:

$$Lu = f(t,u), \;\; u(0,t) = 0, \;\; u(x,0) = u(x,T), \;\; u_t(x,0) = u_t(x,T). \tag{1}$$

where the function $f(t,u) : [0,T] \times \mathbb{R}^1 \to \mathbb{R}^1$. In what follows, we assume that f is continuous on $[0,T] \times \mathbb{R}^1$ and satisfies the Lipschitz condition with respect to u; that is, there exists a constant $L > 0$ such that

$$|f(t,u) - f(t,u_1)| \leq L\,|u - u_1|\,. \tag{2}$$

for any $t \in [0,T]$ and $u, u_1 \in \mathbb{R}^1$.

We further assume that problem (1) has a unique classical solution, provided that the Lipschitz constant L is sufficiently small.

For this purpose, we make essential use of a result derived from work [1], which provides a uniform estimate for the solution of problem (1) and all its derivatives in the linear case.

In this work, the following theorem is proved:

Theorem. Assume that f is continuous on $[0,T] \times \mathbb{R}^1$ and satisfies the Lipschitz condition (2) on u and

$$LK < 1,$$

then problem (1) has a unique classical solution $u(x,t) \in C_3(\Omega)$. Where the constant K is the coefficient bounding u in the linear case in [1]. $C_3(\Omega)$ is the space of functions u such that u, u_t, u_x, u_{xt}, and u_{xtt} are continuous on Ω and equipped with the norm

$$\|u\|_3 = \max_{(x,t)\in\Omega} \big\{ |u(x,t)|,\, |u_t(x,t)|,\, |u_x(x,t)|,\; |u_{xt}(x,t)|,\, |u_{xtt}(x,t)| \big\}.$$

Keywords: pseudoparabolic equation, uniform convergence, small parameter, perturbation.

2020 Mathematics Subject Classification: 35A09, 35A02

References:

[1] Ospanov, M., Merzetkhan, A. Estimates of the solution and its derivatives for the semiperiodic boundary problem for pseudoparabolic equations. *Mathematical Methods in the Applied Sciences*, **48**(13), 12801–12806 (2025). DOI: 10.1002/mma.11063.

[2] A. M. Samoilenko, M. O. Perestyuk, and I. O. Parasyuk, *Differential Equations: Textbook*, Almaty, 2012, 464 pp.

A coupled dynamical system for the social facilitation of eating: stability and transient network effects

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Abstract: The social facilitation of eating, defined as the increased food consumption observed when individuals are in the company of others has been represented in models as a scaling factor. This approach does not capture the dynamic effects of companions and the relationships in their network. We develop a coupled system of k nonlinear differential equations for the food intake $E_i(t)$ of each agent, parameterized by the agent's satiety $E_{\max,i}$, eating rate θ_i, and meal duration r_i:

$$\dot{E}_i = \frac{\theta_i g_i(t)}{E^2_{\max,i}\tau(1+g_i(t))}\left(E_{\max,i}\tau + g_i(t)E_i\right)\left(E_{\max,i}\tau - E_i\right), \tag{1}$$

with $g_i(t) = (F_i(t)r_i)^{0.45}$. Social facilitation acts as a state variable $F_i(t)$, which decays at rate λ toward a level determined by the unconsumed capacity of companions within a familiarity network $S_{ij} \in [0,1]$ and the coupling strength $\alpha \geq 0$:

$$\dot{F}_i = \lambda\Big(f(n)\Big[1 + \frac{\alpha}{k-1}\sum_{j\neq i} S_{ij}\Big(1 - \frac{E_j}{E_{\max,j}}\Big)\Big] - F_i\Big). \tag{2}$$

The equation simplifies to an individual agent model when $k = 1$ and remains bounded across all parameter values, as intake growth diminishes to zero whenever $E_i \to E_{\max,i}\tau$, irrespective of F_i. This term ensures the Jacobian of the coupled system adopts a block lower-triangular form at the satiety equilibrium of the group: at that point, facilitation cannot feed back into intake, so the spectrum remains negative for any α and any network S, thus precluding a Hopf bifurcation. Applying nonlinear simulation revealed a transient rise and decline in social facilitation whose magnitude depended on the network position rather than on group size alone. Consequently, the model represents heterogeneous social effects that a scalar parameter for group size would not be capable of capturing.

Keywords: social facilitation of eating, coupled nonlinear ODE systems, multi-agent dynamical models, local stability analysis, Jacobian, boundedness, mathematical biology

2020 Mathematics Subject Classification: 34D20, 92B05, 37N25

References:

[1] D.M. Thomas, C.K. Martin, S. Lettieri, C. Bredlau, K. Kaiser, T. Church, S.B. Heymsfield, C. Bouchard, A new universal dynamic model to describe eating rate and cumulative intake curves, American Journal of Clinical Nutrition, vol. 105, no. 2, 323–331, 2017.

[2] V.I. Clendenen, C.P. Herman, J. Polivy, Social facilitation of eating among friends and strangers, Appetite, vol. 23, no. 1, 1–13, 1994.

[3] M.E. Young, K.L. Mizzau, N.T. Mai, A. Sirisegaram, M. Wilson, Food for thought. What you eat depends on your sex and eating companions, Appetite, vol. 53, no. 2, 268–271, 2009.

Age-structured Bayesian SIR modelling with reversible-jump MCMC: a comparative study of two social contact matrix projections

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Abstract: We develop a fully Bayesian age-structured discrete-time stochastic SIR model in which the transmission rate $\beta_i(t)$ and recovery probability $\gamma_i(t)$ for age group i are piecewise constant, with the number and location of changepoints treated as unknown. For n_a age groups with populations N_i and a contact matrix $C = (C_{ij})$, the force of infection is

$$\lambda_i(t) = \beta_i(t) \sum_{j=1}^{n_a} C_{ij} \frac{I_j(t-1)}{N_j}, \qquad i = 1, \ldots, n_a. \tag{1}$$

Estimation is carried out through reversible-jump Markov chain Monte Carlo (RJMCMC) coupled with a Product Partition Model (Barry and Hartigan [1]), extending the single-population changepoint framework of Gu and Yin [2] to the age-structured, contact-matrix-coupled setting. We apply the model to Indonesia's 2022 Omicron BA.2 wave using two independently generated contact matrices, the Bayesian hierarchical projection of Prem et al. [3] and the Poisson generalised additive model of Tierney et al. [4], under both a two-group and a three-group age stratification – four configurations in total. All four RJMCMC runs independently converge on a single structural changepoint within a two-to-three day window, without any prior specification of its number or location. We further establish an identifiability result for this class of models: if two contact matrices satisfy $\sum_j C_{ij}^{(2)} = \rho \sum_j C_{ij}^{(1)}$ for a row-wise scalar ρ, then the two parametrisations produce the same likelihood whenever $\beta_i^{(2)} = \beta_i^{(1)}/\rho$, so that only the product $\beta_i(t) \times \sum_j C_{ij}$ is estimable from aggregate incidence data. This is confirmed empirically: the product is conserved across the two independently generated contact matrices to within 1.3–1.5% in every model configuration, despite the matrices themselves differing substantially in total contact rate. The result gives a precise account of what is, and is not, identifiable in contact-matrix-coupled compartmental models.

Keywords: age-structured epidemic model, reversible-jump MCMC, Product Partition Model, changepoint detection, social contact matrix, parameter identifiability

2020 Mathematics Subject Classification: 62F15, 92D30, 65C40

References:

[1] D. Barry, J.A. Hartigan, A Bayesian analysis for change point problems, Journal of the American Statistical Association, vol. 88, 309–319, 1993.

[2] J. Gu, G. Yin, Bayesian SIR model with change points with application to the Omicron wave in Singapore, Scientific Reports, vol. 12, 20864, 2022.

[3] K. Prem, A.R. Cook, M. Jit, Projecting social contact matrices in 152 countries using contact surveys and demographic data, PLOS Computational Biology, vol. 13, e1005697, 2017.

[4] N. Tierney, C. Saraswati, A. Babu, M. Lydeamore, N. Golding, conmat: generate synthetic contact matrices for a given age-stratified population, Journal of Open Source Software, vol. 11, 8326, 2026.

Existence of radially symmetric nonnegative solutions for a class of nonpositone elliptic problems in a ball using shooting method

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Abstract: In this paper, we investigate the existence of nonnegative radial solutions for a semipositone semilinear elliptic boundary value problem of the form -$\Delta u = \lambda f(u)$ in Ω, with $u = 0$ on $\partial\Omega$, where Ω is the unit ball in $\mathbb{R}^n$, $n \geq 2$, and $\lambda > 0$ is a parameter. The nonlinearity satisfies the semipositone condition $f(0) < 0$, which creates analytical difficulties since the maximum principle cannot be directly applied near the origin.
Assuming that $f \in C(\mathbb{R}^+, \mathbb{R})$ has exactly one positive zero, is convex and satisfies suitable superlinearity and growth conditions, we analyze the structure of radial solutions. Using radial symmetry, the problem is reduced to an ordinary differential equation. We apply the shooting method and study the qualitative behavior of the associated initial value problem.

Keywords: Semipositone problem, Radial solutions, Elliptic equation, Shooting method.

2020 Mathematics Subject Classification: 35J66, 35B18.

References:

[1] Ali, I., Castro, A., & Shivaji, R. (1993). Uniqueness and stability of nonnegative solutions for semipositone problems in a ball. Proceedings of the American Mathematical Society, 117(3), 775– 782.

[2] Brown, K. J., Castro, A., & Shivaji, R. (1989). Nonexistence of radially symmetric nonnegative solutions for a class of semipositone problems. Differential and Integral Equations, 2, 541–545.

[3] Hakimi, S., & Zertiti, A. (2009). Radial positive solutions for a nonpositone problem in a ball. Electronic Journal of Differential Equations, 2009(44), 1–6.

Noetherness of oblique derivative problems in weighted spaces of harmonic functions

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Abstract: We study the Noether property of an oblique derivative problem for the Laplace equation in the unit disk within weighted spaces of harmonic functions. Let $D = \{z \in \mathbb{C} : |z| < 1\}$ and $\Gamma = \partial D$. For a Muckenhoupt weight $\rho \in A_p(\Gamma)$, $1 < p < \infty$, we consider the weighted Lebesgue space $L_{p;\rho}(\Gamma)$ and the corresponding harmonic space $h^{(1)}_{p;\rho}$. The boundary value problem is

$$\Delta_{r,\varphi} u = 0, \qquad (r,\varphi) \in D, \tag{1}$$

$$\gamma^{+}\left(\cos\varphi \frac{\partial u}{\partial r} + \sin\varphi \frac{\partial u}{\partial \varphi}\right) = f(\varphi), \qquad \varphi \in [-\pi, \pi), \tag{2}$$

where $f \in L_{p;\rho}(\Gamma)$ and

$$\Delta_{r,\varphi} u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}.$$

Here $h^{(1)}_{p;\rho}$ consists of harmonic functions whose radial and angular derivatives belong to $h_{p;\rho}$. The main result states that, for every $\rho \in A_p(\Gamma)$, the above oblique derivative problem is Noetherian in $h^{(1)}_{p;\rho}$. Moreover, its index is equal to -2. Thus, the result establishes the Fredholm-type solvability structure of the problem in weighted harmonic spaces.

Keywords: harmonic functions, oblique derivative problem, weighted spaces, Muckenhoupt weights, Noether operator, index.

2020 Mathematics Subject Classification: 35J05, 35J08, 46E35, 46M05. **References:**

[1] T.I. Najafov, N.P. Nasibova, On the Noetherness of the Riemann Problem in Generalized Weighted Hardy Classes, Azerbaijan J. Math., vol. 5, no. 2, 109–124, 2015.

Pre-compactness of sets in the discrete Morrey-type spaces

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Abstract: Let $m \in \mathbb{Z}$, $N \in \omega := \mathbb{N} \cup \{0\}$, and write $S_{m,N} := \{m-N, \ldots, m, \ldots, m+N\}$. Then $|S_{m,N}| = 2N+1$ — the cardinality of $S_{m,N}$. (It is independent of m, but we write it in the subscript to keep track of the set.) Let $\mathbb{K}$ be $\mathbb{R}$ or $\mathbb{C}$ and $1 \leq p \leq q < \infty$. We denote by $\ell_q^p = \ell_q^p(\mathbb{Z})$ the set of sequences $x = (x_k)_{k \in \mathbb{Z}}$ taking values in $\mathbb{K}$ such that $\|x\|_{\ell_q^p} := \sup_{m \in \mathbb{Z},\, N \in \omega} |S_{m,N}|^{\frac{1}{q}-\frac{1}{p}} \left(\sum_{k \in S_{m,N}} |x_k|^p\right)^{\frac{1}{p}} < \infty$. Clearly ℓ_q^p is a vector space, which we shall call a *discrete Morrey space.* We remark that when $p = q$, we have $\ell_p^p = \ell^p$, the space of p-summable sequences with integer indices. For a sequence x to be in ℓ_q^p, x has to have some decay, but not as fast as those in ℓ^p. In general, for $p < q$, ℓ_q^p is a larger space than ℓ^p, as we shall see below. In the following theorem we establish conditions for the pre-compactness of sets in discrete Morrey space.

Theorem 2.5. *Let $1 \leq p \leq q < \infty$ and let $\ell_q^p = \ell_q^p(\mathbb{Z})$ be the discrete Morrey space. A subset $\mathcal{F} \subset \ell_q^p$ is relatively compact (precompact) in ℓ_q^p if and only if the following conditions are satisfied:*

(i) *(Boundedness) $\mathcal{F}$ is bounded in ℓ_q^p, i.e., $\sup_{x \in \mathcal{F}} \|x\|_{\ell_q^p} < \infty$.*

(ii) *(Vanishing at infinity uniformly in $\mathcal{F}$) For every $\varepsilon > 0$ there exists $R > 0$ such that*

$$\sup_{x \in \mathcal{F}} \sup_{m \in \mathbb{Z},\, N \in \omega\, S_{m,N} \cap \{k \in \mathbb{Z} : |k| > R\} \neq \varnothing} |S_{m,N}|^{\frac{1}{q}-\frac{1}{p}} \left(\sum_{k \in S_{m,N}} |x_k|^p\right)^{\frac{1}{p}} < \varepsilon.$$

(iii) *(Tightness on finite windows) For every fixed finite interval $I \subset \mathbb{Z}$, the set of restrictions $\{\, (x_k)_{k \in I} : x \in \mathcal{F} \,\}$ is relatively compact in $\mathbb{K}^I$.*

In particular, if $\mathcal{F}$ is bounded in ℓ_q^p and satisfies the uniform vanishing at infinity condition (ii)*, then $\mathcal{F}$ is relatively compact in ℓ_q^p.*

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References:

[1] H. Gunawan, Y. Ramadana and Y. Sawano Olsen's inequality for discrete Morrey spaces, *Acta Math. Hungar*, **176** (2) (2025), 447—456

[2] Hendra Gunawan, Eder Kikianty, Christopher Schwanke Discrete Morrey spaces and their inclusion properties, *Mathematische Nachrichten*, **8–9**:291 (2018), 1283–1296.

On the lattices generated varieties

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Abstract: The main result is the determination of an equational basis for the variety of lattices generated by a finite non-modular and non-semidistributive lattice. It is established that the resulting quasivariety is a variety defined by three identities. Structural constraints are determined for minimal non-trivial covers of join-irreducible elements in lattices satisfying the weak modularity identity, and the possible closures of individual join-irreducible elements are described.

Keywords: variety, finite lattice, quasivariety, non-modular lattice, non-semidistributive lattice.

2020 Mathematics Subject Classification: 06B10, 06B20, 06B05

References:

[1] W. Dziobiak and M. V. Schwidefsky, "Duality for bi-algebraic lattices belonging to the variety of (0, 1)-lattices generated by the pentagon," Algebra Logic 63, 114–140 (2024). https://doi.org/10.1007/s10469-025-09776-3
[2] R. Freese, J. Jez ˇ ek, and J. B. Nation, Free Lattices, Vol. 42 of Mathematical Surveys and Monographs (Am.Math. Soc., Providence, RI, 1995).
[3] G. Gratzer, General Lattice Theory (Birkha ̈ user, Basel, 1978).

A characterization: fixed points and weak compactness in $W^{1,1}((a,b) \cup (c,d), \mu)$

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Abstract: Let $\Omega = (a,b) \cup (c,d)$ be the union of disjoint open intervals in $\mathbb{R}$ and (Ω, Σ, μ) be a finite measure space. We examine the Sobolev space $W^{1,1}(\Omega, \mu)$ equipped with its standard norm and establish a precise characterization of weak compactness for closed, bounded and convex subsets of $W^{1,1}(\Omega, \mu)$. It is formulated in terms of the existence of fixed points for specific classes of affine, uniformly Lipschitz mappings.

Keywords: Weak Compactness, Sobolev Space, Lipschitzian mapping

2020 Mathematics Subject Classification: 47H10, 46B50, 46E35

References:

[1] M. Japón, C. Lennard, and R. Popescu, A fixed-point characterization of weakly compact sets in $L^1(\mu)$ spaces, *Journal of Mathematical Analysis and Applications*, 491(1) (2020) 124228.

[2] T. Domínguez Benavides, M. Japón, and S. Prus, Weak compactness and fixed point property for affine mappings, *Journal of Functional Analysis*, 209 (2004) 1–15.

Boundedness and integral inequalities for Sonin-type fractional integral operators on Morrey spaces

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Abstract: This work develops a functional-analytic framework for Sonin-type fractional integral operators acting on Morrey spaces. Let k be a locally integrable kernel admitting an associated kernel ℓ such that the pair (k, ℓ) satisfies the Sonin convolution relation. The corresponding generalized fractional integral operator is defined by

$$(\mathcal{S}_k f)(x) = \int_0^x k(x-t) f(t)\, dt. \tag{1}$$

We investigate the mapping properties of $\mathcal{S}_k$ in Morrey-type spaces, with particular emphasis on the interaction between the nonlocal structure of the kernel and the local scale-sensitive behavior encoded by the Morrey norm. Under appropriate size and integrability assumptions on the Sonin kernel, sufficient conditions are established for the boundedness of $\mathcal{S}_k$ between suitable Morrey spaces. The resulting estimates are subsequently employed to derive a family of integral inequalities associated with Sonin-type operators. The proposed approach is formulated at the kernel level and therefore does not depend on a particular power-law representation. Consequently, suitable choices of the kernel recover important fractional integral structures as special cases and provide corresponding reductions of the obtained estimates. This establishes a unified connection between Sonin-type fractional calculus, operator-theoretic estimates, and scale-sensitive function spaces. The results provide an analytical basis for studying a broader class of nonlocal integral operators and may be useful in the analysis of fractional integral and differential problems exhibiting memory and multiscale effects.

Keywords: Sonin kernel, Sonin pair, generalized fractional integral operator, Morrey spaces, boundedness, integral inequalities, nonlocal operators

2020 Mathematics Subject Classification: 26A33, 42B35, 46E30

References:

[1] Y. Luchko, General fractional integrals and derivatives with the Sonine kernels, *Mathematics*, **9**(6) (2021), Article 594.

[2] A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland Mathematics Studies, Vol. 204, Elsevier, Amsterdam, 2006.

[3] A. Eridani, V. Kokilashvili, A. Meskhi, Morrey spaces and fractional integral operators, *Expositiones Mathematicae*, **27**(3) (2009), 227–239.

[4] A. Yalcin, E. Gul, A. O. Akdemir, Hermite–Hadamard type inequalities for co-ordinated convex functions with variable-order fractional integrals, *Applied and Computational Mathematics*, **24**(2) (2025), 326–343.

Subword pattern statistics in Carlitz and palindromic compositions

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Abstract: A composition $\pi = \pi_1\pi_2\cdots\pi_m$ of a positive integer n is an ordered collection of one or more positive integers whose sum is n. The number of summands, namely m, is called the number of parts of π. A composition with distinct adjacent parts is called a *Carlitz composition*, while a composition that reads the same forwards and backwards is called a *palindromic composition*. We study pattern statistics in these classes of compositions by enumerating them with respect to the number of occurrences of a consecutive subword pattern τ and the sum of the first entries in these occurrences. Using multivariate generating functions and linear algebraic methods, we obtain formulas that simultaneously encode these statistics. In particular, we derive explicit generating functions for several short patterns in Carlitz compositions, palindromic compositions, Carlitz palindromic compositions, and their k-Carlitz extensions. These results provide a unified framework for studying refined pattern enumeration in restricted classes of compositions.

Keywords: Subwords, generating functions, Carlitz compositions, palindromic compositions, Cramer's method.

2020 Mathematics Subject Classification: 05A15, 15A06, 15A15

References:

[1] M. Archibald and A. Knopfmacher, The largest missing value in a composition of an integer, Discrete Math. 311 (2011) 723–731.

[2] W. Asakly and T. Mansour, Enumeration of compositions according to the sum of the values of the first letters of the occurrences of a 2-letter pattern, Linear Alg. and its Appl. 449 (2014) 43–59.

[3] C. Brennan and A. Knopfmacher, The first ascent of size d or more in compositions, Discrete Math. Theor. Comput. Sci. Proc. AG, Nancy, 2006.

[4] S. Heubach and T. Mansour, Counting rises, levels, and drops in compositions, Integers 5 (2005) Article A11.

[5] S. Heubach and T. Mansour, Combinatorics of compositions and words, CRC Press, Boca Raton, FL, 2010.

[6] T. Mansour and B.O. Sirhan, Counting l-letter subwords in compositions, Discr. Math. Theor. Comput. Sci. 8 (2006) 285–298.

[7] A. Shabani, R. Gjergji, Enumeration of compositions according to the sum of the values of the first letters of the occurrences of a pattern of length three with two distinct letters, Discrete Math. Theor. Comput. Sci. 17 (2016) 147–166.

Existence and uniqueness of an inverse source problem for a degenerate mixed-type equation with a variable coefficient

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Abstract: We investigate an inverse source problem for the degenerate mixed-type partial differential equation

$$u_{yy} + (\operatorname{sgn} y)|y|^m u_{xx} = f(x), \qquad m > 0, \tag{1}$$

in the rectangular domain $\Omega = \{(x, y) : 0 < x < 1, -\alpha < y < \beta\}$, where both the solution $u = u(x, y)$ and the source term $f = f(x)$ are unknown. The problem is supplemented by homogeneous Dirichlet conditions in the x-direction, prescribed boundary data at $y = -\alpha$ and $y = \beta$, and an additional condition for u_y at $y = -\alpha$. Related inverse problems for mixed-type equations were studied in $[1, 2]$.

Applying the method of separation of variables and the Fourier method, we reduce the inverse problem to a sequence of ordinary differential equations whose solutions are expressed in terms of Bessel and modified Bessel functions. Explicit reconstruction formulas for the Fourier coefficients of the unknown source and the corresponding solution components are derived. Sufficient conditions ensuring the non-vanishing of the characteristic denominators are established. Under these conditions, the existence and uniqueness of a classical solution are proved. Moreover, uniform estimates for the Fourier coefficients and the solution components are obtained, which imply the absolute and uniform convergence of the corresponding Fourier series together with their required derivatives.

Keywords: inverse source problem, degenerate mixed-type equation, Fourier method, Bessel functions, characteristic denominator, existence and uniqueness.

2020 Mathematics Subject Classification: 35M10, 35R30, 35A01.

References:

[1] G.Yu. Udalova, Inverse problem for a mixed elliptic–hyperbolic equation, *Vestnik Samara State University*, no. 4(78), 116–122, 2010.

[2] N.V. Martemyanova, Inverse problem for a mixed-type equation with a nonlocal boundary condition, *Vestnik Samara State University*, no. 6(80), 27–38, 2010.

On the Fourier transform of functions from the mixed-metric Lorentz space $L_{\bar{2},\bar{r}}$

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Abstract: Abstract: This extended abstract is devoted to Bochkarev-type inequalities for the Fourier transform and to sharp logarithmic estimates in Lorentz spaces, with particular emphasis on mixed-metric Lorentz spaces on $[0,1]^2$ and on $\mathbb{R}^2$.

1. Mixed-metric Lorentz space Definition 1.

Let $\bar{p} = (p_1, p_2)$ and $\bar{r} = (r_1, r_2)$ satisfy $0 < p_i \le \infty$ and $0 < r_i \le \infty$. The mixed-metric Lorentz space $L_{\bar{p},\bar{r}}\big([0,1]^2\big)$ is defined as the set of all measurable functions f on $[0,1]^2$ for which the quantity

$$\|f\|_{L_{\bar{p},\bar{r}}} = \Big\| \; \|f\|_{L_{p_1,r_1}([0,1])} \; \Big\|_{L_{p_2,r_2}([0,1])}$$

is finite. More explicitly, for $0 < \bar{r} < \infty$,

$$\|f\|_{L_{\bar{p},\bar{r}}} = \left(\int_0^1 \left[t_2^{1/p_2} \left(\int_0^1 \left(t_1^{1/p_1} f^{*1}(t_1, \cdot) \right)^{r_1} \frac{dt_1}{t_1} \right)^{1/r_1} \right]^{r_2} \frac{dt_2}{t_2} \right)^{1/r_2},$$

and for $\bar{r} = \infty$,

$$\|f\|_{L_{\bar{p},\infty}} = \sup_{t_1,t_2} t_1^{1/p_1}\, t_2^{1/p_2}\, f^{*1*2}(t_1, t_2),$$

where f^{*1} and f^{*1*2} denote successive non-increasing rearrangements with respect to the variables.

2. Fourier coefficients in $L_{2,\bar{r}}\big([0,1]^2\big)$

Theorem 1. Let

$$\Phi_{m_1,m_2}(x_1, x_2) = \varphi_{m_1}(x_1)\, \psi_{m_2}(x_2), \qquad m_1, m_2 \in \mathbb{N},$$

be an orthonormal system on $[0,1]^2$ which is uniformly bounded in L_∞, i.e., $\|\varphi_m\|_\infty \le M_1$ and $\|\psi_m\|_\infty \le M_2$. Then for every $f \in L_{2,\bar{r}}\big([0,1]^2\big)$ with $2 < r_1, r_2 < \infty$ the following inequality holds:

$$\sup_{\substack{|A_1|\ge 8\\ A_1\subset\mathbb{N}}} \sup_{\substack{|A_2|\ge 8\\ A_2\subset\mathbb{N}}} \frac{1}{|A_1|^{1/2}\, |A_2|^{1/2} \left(\log_2(|A_1|+1)\right)^{\frac12 - \frac{1}{r_1}} \left(\log_2(|A_2|+1)\right)^{\frac12 - \frac{1}{r_2}}} \sum_{k_1\in A_1} \sum_{k_2\in A_2} \Big|\widehat{f}(k_1, k_2$$

$$\le 9M\, \|f\|_{L_{2,\bar{r}}},$$

where $M = (M_1 + 1)(M_2 + 1)$ and $\widehat{f}(k_1, k_2)$ are the Fourier coefficients of f with respect to the system $\{\Phi_{m_1,m_2}\}$.

3. Bochkarev-type inequality for the Fourier transform in $L_{2,r}(\mathbb{R})$

Theorem 2. Let

$$\mathcal{R}_{\mathbb{N}} = \left\{ A = \bigcup_{i=1}^{N} A_i : \; A_i \text{ are intervals in } \mathbb{R} \right\}.$$

Then for every $f \in L_{2,r}(\mathbb{R})$ with $2 < r < \infty$ one has

$$\sup_{N \geq 8} \sup_{A \subset \langle \mathbb{N} \rangle} \frac{1}{|A|^{1/2} \left(\log_2(1+N)\right)^{\frac{1}{2}-\frac{1}{r}}} \left| \int_A \widehat{f}(\xi)\, d\xi \right| \leq 23 \, \|f\|_{L_{2,r}}.$$

4. Two-dimensional Fourier transform

Definition 2. Let $f \in L_1(\mathbb{R}^2)$. The two-dimensional Fourier transform of f is defined by

$$\widehat{f}(\xi_1, \xi_2) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} f(x_1, x_2)\, e^{\,2\pi i (x_1 \xi_1 + x_2 \xi_2)}\, dx_1\, dx_2.$$

5. Auxiliary lemma

Lemma 1. Let $\frac{4}{3} < q_1, q_2 < 2$ and $f \in L_{\bar{q},\bar{2}}(\mathbb{R}^2)$. For any measurable sets A_1 and A_2 of finite measure in $\mathbb{R}^N$ the following inequality holds:

$$\sup_{A_1 \subset \mathbb{R}^N} \sup_{A_2 \subset \mathbb{R}^N} \frac{1}{|A_1|^{1/q_1} |A_2|^{1/q_2}} \int_{A_1} \int_{A_2} \left| \widehat{f}(\xi_1, \xi_2) \right| d\xi_1 \, d\xi_2$$

$$\leq C \left(\frac{q_1}{2(q_1 - 1)} \right)^{\left(\frac{1}{q_1} - \frac{1}{2}\right)} \left(\frac{q_2}{2(q_2 - 1)} \right)^{\left(\frac{1}{q_2} - \frac{1}{2}\right)} \|f\|_{L_{\bar{q},\bar{2}}}.$$

6. Main theorem for the two-dimensional Fourier transform

Theorem 3 (Main theorem) Let $2 < r_1, r_2 < \infty$. Then for every $f \in L_{2,\bar{r}}(\mathbb{R}^2)$ one has

$$\sup_{\substack{|A_1| \geq 8 \\ A_1 \subset \mathbb{N}}} \sup_{\substack{|A_2| \geq 8 \\ A_2 \subset \mathbb{N}}} \frac{1}{|A_1|^{1/2} |A_2|^{1/2} (\log_2(|A_1| + 1))^{\frac{1}{2} - \frac{1}{r_1}} (\log_2(|A_2| + 1))^{\frac{1}{2} - \frac{1}{r_2}}} \int_{A_1} \int_{A_2} \left| \widehat{f}(\xi_1, \xi_2) \right| d\xi$$

$$\leq \|f\|_{L_{2,\bar{r}}}.$$

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Keywords: Lorentz spaces, mixed metric, Fourier transform, Bochkarev-type inequalities.

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A variable space grid explicit finite difference method for nonlocal boundary value problems with deviated arguments

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Abstract: This work presents a variable space grid (VSG) approach based on the explicit finite difference method for solving a nonlocal boundary value problem with deviated arguments.

The proposed numerical scheme is developed and validated through a series of numerical experiments. The computed results are compared with the corresponding exact solution and exhibit excellent agreement, demonstrating the accuracy and reliability of the proposed method. Furthermore, the convergence of the numerical solution toward the exact solution is verified as the mesh is refined. In addition, the stability of the proposed scheme is analyzed using the von Neumann stability criterion.

Keywords: Numerical solution, Non local partial differential equation, Deviated boundary condition, Variable space grid method

References

(1) A. Ashyralyev, N. Aggez, A note on the difference schemes of the nonlocal boundary value problems for hyperbolic equations, Numer. Funct. Anal. Optim. 25 (2004).
(2) A. Ashyralyev, A. Sirma, Nonlocal boundary value problems for the Schrodinger equation, Comput. Math. Appl. 55 (3) (2008) 392–407.
(3) J. Math. Math. Sci. 20 (1997) 147–164.
(4) S. Mesloub, On a singular two dimensional nonlinear evolution equation with nonlocal conditions, Nonlinear Analysis, Theory, Methods and Applications 68 (2008) 2594–2607.

Spectral properties of a second-order differential operator with the spectral parameter in the boundary conditions

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Abstract: We consider the spectral problem for the second-order differential operator

$$L(y) \equiv -y''(x) = \lambda y(x),\ 0 < x < 1, \tag{1}$$

subject to the boundary conditions

$$y(0) = 0, \tag{2}$$

$$\lambda Y_1(y) + Y_0(y) = 0, \tag{3}$$

where

$$Y_i(y) = a_{i1}y'(0) + a_{i2}y'(1) + a_{i3}y(0) + a_{i4}y(1) \tag{4}$$

are linear boundary forms.

The spectral properties of boundary value problems for equation (1) without the spectral parameter in the boundary conditions have been extensively investigated in the literature [1]. However, introducing the spectral parameter into the boundary conditions substantially changes the structure of the characteristic equation and the spectral properties of the problem, which requires a separate investigation.

In the abstract, we study the spectral properties of the boundary value problem (1)–(3), where the spectral parameter enters one of the boundary conditions linearly. A classification of the problem into regular and irregular cases is established in terms of the coefficients of the boundary condition (3). The asymptotic behaviour of the eigenvalues and the properties of the corresponding eigenfunctions are investigated.

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Keywords: Ordinary differential operator, nonlocal boundary conditions, spectral parameter in the boundary conditions, spectral properties, regular boundary conditions, irregular boundary conditions

2020 Mathematics Subject Classification: 34L05, 34L10, 34L15

References:

[1] Lang P., Locker J.,Spectral theory of two-point differential operators determined by D^2. Analysis of Cases, J. Math. Anal. And Appl., 146:1 (1990), 148-191.

G. Hajiyeva, On spectral properties of one discontinuous second-order differential operator with a spectral parameter in discontinuity conditions

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Abstract: Consider a spectral problem with a point of discontinuity

$$l(y) = -y''(x) = \lambda y(x), \qquad x \in \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, 1\right), \tag{1}$$

$$\begin{cases} \alpha_1 y'(0) + \alpha_0 y(0) = 0, \\ \beta_1 y'(1) + \beta_0 y(1) = 0, \\ y\left(\frac{1}{3} - 0\right) = y\left(\frac{1}{3} + 0\right), \\ y'\left(\frac{1}{3} - 0\right) - y'\left(\frac{1}{3} + 0\right) = \lambda m y\left(\frac{1}{3}\right). \end{cases} \tag{2}$$

where λ is a spectral parameter, $|\alpha_1|+|\alpha_0| \neq 0$, $|\beta_1|+|\beta_0| \neq 0$, $m \neq 0$ are a complex numbers. Such type of spectral problems were investigated in the papers [1], when instead of the boundary conditions $\alpha_1 y'(0) + \alpha_0 y(0) = 0$ and $\beta_1 y'(1) + \beta_0 y(1) = 0$ were taken $y(0) = y(1) = 0$, and in the work [2] were taken $y'(0) = y'(1) = 0$ and theorems on completeness and minimality were proved. We will prove theorems on completeness, minimality, basicity of eigenfunctions of the spectral problem in $L_p[0,1] \bigoplus C$ and $L_p[0,1]$ Lebesgue and $M^{p,\alpha}[0,1] \bigoplus C$ and $M^{p,\alpha}[0,1]$ Morrey type spaces.

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Keywords: discontinuous differential operator, spectral parameter, eigenvalues, eigenfunctions, asymptotic formulas

2020 Mathematics Subject Classification: 34L10, 34L20, 34B24

References:

[1] B.T. Bilalov, T.B. Gasymov, G.V. Maharramova, Basis property of eigenfunctions in Lebesgue spaces for a spectral problem with a point of discontinuity, Differential Equations 55:12, 2019, 1544–1553.

[2] T.B. Gasymov, A.Q. Akhmedov, On basicity of a spectral problem in $L_p \bigoplus C$ and L_p spaces, Baku State University Journal of Mathematics& Computer Sciences, 2024, v.1 (1), 37–51.

A stable second order difference scheme for numerical solution of the source identification problem for telegraph-parabolic equations

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Abstract: We construct a second order of accuracy difference scheme for the approximate solution of a source identification problem for telegraph-parabolic equations with an unknown spacewise dependent source term

$$u''(t) + \alpha u'(t) + Au(t) = p + f(t), \ 0 < t < 1, \tag{1}$$

$$u'(t) + Au(t) = p + g(t), \ -1 < t < 0, \tag{2}$$

$$u(0+) = u(0-), \ u'(0+) = u'(0-), \ u(-1) = \varphi, \ u(\lambda) = \psi, \ -1 < \lambda \leq 1. \tag{3}$$

The problem is considered in a Hilbert space H with a self-adjoint positive definite operator A satisfying $A \geq \delta I$, where $\delta > \frac{\alpha^2}{4} > 0$ (see [1]). A first-order of accuracy difference scheme for this problem was studied in [2].

The main goal of this work is to establish the unique solvability of the constructed second order of accuracy difference scheme and derive stability estimates for its solution. As applications of the abstract results, we obtain stability estimates for the solutions of second order of accuracy difference schemes for the approximate solutions of two source identification problems for telegraph-parabolic equations. We also discuss numerical procedures for implementing these difference schemes and provide an error analysis based on numerical results for simple test problems.

Keywords: Source identification problem, telegraph-parabolic equation, stable difference scheme

2020 Mathematics Subject Classification: 35M10, 35R30, 65M06, 65M32

References:

[1] A. Ashyralyev, M. Ashyraliyev, M.A. Ashyralyyeva, Identification problem for telegraph-parabolic equations, Comput. Math. Math. Phys., Vol. 60, No. 8, 1294–1305, 2020.

[2] M. Ashyraliyev, M.A. Ashyralyyeva, A stable difference scheme for the solution of a source identification problem for telegraph-parabolic equations, Bulletin of the Karaganda University. Mathematics Series, No. 3(115), 46–54, 2024.

Mathematical methods for processing geological data and forecasting prospective ore-bearing areas

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Abstract: In recent years, machine learning methods have been increasingly used in Earth sciences due to their ability to model complex, multidimensional, and nonlinear patterns [1]. Algorithms such as Multilayer Perceptron (MLP) and Extreme Gradient Boosting (XGBoost) demonstrate high efficiency in forecasting and classification tasks. These methods allow for the effective integration of various data types, including geochemical, geological, and remote sensing data, and improve the accuracy of detecting subtle and hidden anomalies.

However, machine learning models often require large datasets and rigorous preprocessing; furthermore, they may be less interpretable from a geological perspective compared to classical geostatistical approaches. Machine learning is successfully applied across various Earth science disciplines, including hydrogeophysics, seismology, and seismic exploration. These studies demonstrate the efficacy of data-driven approaches in identifying significant patterns within complex geological and geophysical datasets. Thus, there is a growing need for a comprehensive comparison between classical interpolation methods and modern machine learning approaches in the context of geochemical anomaly detection. A thorough understanding of the strengths and limitations of each method is essential for selecting the appropriate tools for mineral exploration.

This paper investigates the mathematical aspects of machine learning methods used to detect and predict geochemical anomalies in ore-bearing regions. The research evaluates their effectiveness in terms of accuracy, sensitivity to local changes, and suitability for geological interpretation, while also exploring the potential benefits of combining these approaches.

Keywords:machine learning, geochemical anomalies, mineral exploration, geological data analysis, geospatial data classification, prediction

2020 Mathematics Subject Classification: 68T05, 68T07, 68T10 **References:**

[1] Dramsch J.S. 70 years of machine learning in geoscience in review.In Machine Learning in Geoscience, pages 1-55. Elsevier, 2020. URL: https://doi.org/10.1016/bs.agph.2020.08.002, doi:10.1016/bs.agph.2020.08.002.

Maximal regularity conditions of the solution of a second-order differential equation

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Abstract: The following second-order singular differential equation is considered in this submission:

$$L_0 y = -s(x)\left(\rho(x)y'\right)' + r\left(x\right)y' + q(x)y = f\left(x\right), \quad (1)$$

where $x \in \mathbb{R} = (-\infty, +\infty)$, s, r, ρ are smooth functions, and q is continuous, $f \in L_2\left(\mathbb{R}\right)$. By L we denote the closure in $L_2\left(\mathbb{R}\right)$ of the operator

$$L_0 y = -s(x)\left(\rho(x)y'\right)' + r(x)y' + q(x)y,$$

defined on the set $C_0^{(2)}\left(\mathbb{R}\right)$ of twice continuously differentiable functions with compact support. An element $y \in D(L)$ which satisfies the equality $Ly = f$, we call the solution of equation (1).

The singularity of the second-order differential equation (1) means that it is given on an infinite interval and its coefficients are unbounded functions. For equation (1) we consider the following issues:

a) the existence and uniqueness of the solution y,

b) the fulfillment of the following inequality for the solution y:

$$\left\|s\left(\rho y'\right)'\right\|_2 + \|ry'\|_2 + \|(1+|q|)y\|_2 \leqslant C\,\|f\|_2\,. \quad (2)$$

Also, we give one application of the estimate (2).

Equation (1) in the case of $s = \rho = 1$ and $r = 0$ is a Sturm-Liouville equation and it has been widely studied. The problem of obtaining inequality (2) for the Sturm-Liouville equation was investigated in the works of B. Everitt and W. Geertz, M. Otelbaev, K.Kh. Boymatov and others. A natural extension of the studies above is to study problems a) and b) for equation (1) in the case when the coefficients s and ρ may be unbounded functions. It is also important to describe the simultaneous influence of all the coefficients s, ρ, r, q on the issues considered, while retaining, as far as possible, their independence from each other.

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Keywords: differential equation, generalized solution, closed operator, existence of solution, uniqueness, maximal regularity

2020 Mathematics Subject Classification: 34A30, 34B40, 34C11

On the asymptotic and uniform difference schemes for hyperbolic perturbation problems

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Abstract: The abstract nonlocal boundary value perturbation problem

$$\begin{cases} \varepsilon^2 u^{''}(t) + Au(t) = f(t), 0 < t < T, \\ u(0) = \alpha u(T) + \varphi, \\ u^{'}(0) = \beta u^{'}(T) + \psi \end{cases}$$

for hyperbolic equations with the self-adjoint positive definite operator A in a Hilbert space H and an arbitrary small $\varepsilon \in (0, \infty)$ parameter multiplying the highest derivative term is considered. The asymptotic solution to the problem is presented. A uniform high-order two-step difference scheme corresponding to this problem is obtained. This work covers the general perturbation theory of uniform difference schemes on hyperbolic partial differential equations. The theoretical results are verified by the numerical implementations using Matlab. The error results of the numerical experiments are presented.

Keywords: Hyperbolic perturbation problems, nonlocal conditions, asymptotic formulas, uniform difference schemes

2020 Mathematics Subject Classification: 47A55, 34E10, 39A30, 35G15

References:

[1] A. Ashyralyev, H.O. Fattorini, On uniform difference-schemes for 2nd-order singular perturbation problems in Banach spaces, SIAM J. Math. Anal. 23(1), 29–54, 1992.

[2] A. Ashyralyev, P.E. Sobolevskii, New Difference Schemes for Partial Differential Equations, in: Operator Theory and Appl., Birkhäuser Verlag, Basel, Boston, Berlin, 2004.

[3] H.O. Fattorini, Second Order Linear Differential Equations in Banach Space, Notas de Matematica, North-Holland, 1985.

[4] A.M. Ilin, A difference scheme for a differential equation with a small parameter affecting the highest derivative, Mathematical Notes 6(2):596-602, 1969.

On a family of spectral problems for fourth-order ODE

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Abstract: We introduce

Definition 2.1. *Denote by $V_1(0,l)$ and $V_2(0,l)$ the Hilbert spaces endowed with the inner products and the norms*

$$V_1(0,l): \quad \begin{cases} (u'(x), v'(x))_{L^2(0,l)} \ \forall\, u, v \in \overset{\circ}{W}{}^1_2(0,l), \\ \|u\|_{V_1(0,l)} \equiv \left[(u'(x), u'(x))_{L^2(0,l)}\right]^{1/2}, \end{cases} \tag{1}$$

$$V_2(0,l): \quad \begin{cases} ((u,v)) = (u''(x), v''(x))_{L^2(0,l)} \ \ \forall\, u, v \in \overset{\circ}{W}{}^2_2(0,l) \\ \|u\|_{V_2(0,l)} \equiv \left[((u(x), u(x)))\right]^{1/2}. \end{cases} \tag{2}$$

We consider the following family of spectral problems:

$$\begin{cases} X^{IV}(x) + \mu X(x) = -\lambda X''(x), \quad x \in (0,l), \\ X(0) = X(l) = X'(0) = X'(l) = 0, \ \ \mu \in \mathbb{R}^1, \end{cases} \tag{3}$$

for which the following theorem holds true

Theorem 2.2. *For each fixed value of the parameter $\mu \in \mathbb{R}^1$, the spectral problem* (3) *has a countable spectrum $\{\lambda_n,\ n \in \mathbb{N}\}$ with a single limit point $\lambda = +\infty$. The corresponding system of eigenfunctions $\{X_n(x),\ n \in \mathbb{N}\} \subset V_2(0,l)$* (2) *forms an orthogonal basis in the space $V_1(0,l)$* (1).

Throughout this note we mainly use techniques from our works [1–7]. This theorem is used to construct a basis in the 3-D space of solenoidal functions.

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Keywords: Spectral problem, eigenvalues, eigenfunctions, Stokes operator

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References:

[1] L.V. Kantorovoch, G.P. Akilov, Functional Analysis. Second Edit. [in Russian], M.: Nauka, 1977.

[2] A.N. Kolmogorov, S.V. Fomin, Elements of the theory of functions and functional analysis. 4^{th} Edit. [in Russian], M.: Nauka, 1976.

[3] F. Riesz, B. Sz.-Nagy, Lecons d'Analyse Fonctionnelle, Cinquieme Edition [in French], Budapest: Akademiai Kiado, 1968.

[4] T. Kato, Perturbation Theory for Linear Operators, Berlin: Springer-Verlag, 1995.

[5] O.A. Ladyzhenskaya, Mathematical Theory of Viscous Incompressible Flow, New York: Gordon and Breach, 1987.

[6] R. Temam, Navier-Stokes equations. Theory and numerical analysis, Providence, Rhode Island: AMS CHELSEA PUBLISHING, 1984.

[7] M.T. Jenaliyev, K.S. Sharipov, On a family of spectral problems// Book of Abstracts of the Int. Sci. Conf. "Integral Operators in Function Spaces and Harmonic Analysis", June 11–14, 2026, Astana, Kazakhstan. P.29–30.

On a method for constructing the Green function of the Dirichlet problem

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Abstract: Let $\Omega \subset \mathbb{R}^n$ be a bounded simply connected domain with smooth boundary $\partial\Omega$.

Dirichlet problem. To find the regular solution of the Laplace equation

$$-\Delta u = f(x), \qquad x \in \Omega, \tag{1}$$

satisfying the boundary condition

$$u\big|_{\partial\Omega} = 0. \tag{2}$$

The function $G(x, y)$ is called the Green's function of the Dirichlet problem if

$$u(x) = \int_\Omega G(x, y) f(y)\, dy, \tag{3}$$

where

$$-\Delta_x G(x, y) = \delta(x - y), \quad G(x, y)\big|_{x\in\partial\Omega} = 0. \tag{4}$$

Theorem 2.3. *The Green function of the Dirichlet problem in the domain of Ω is given by the formula*

$$G(x, y) = \chi_- G^-(x, y) - \chi_+ G^+(x, y), \tag{5}$$

where

$$G^-(x, y) = \varepsilon(x, y) + \int_{\partial\Omega} \frac{\partial}{\partial n_\xi^-} \varepsilon(x, \xi)\varepsilon(\xi, y), \quad x \in \Omega^- = \Omega, \tag{6}$$

$$G^+(x, y) = \int_{\partial\Omega} \frac{\partial}{\partial n_\xi^+} \varepsilon(x, \xi)\varepsilon(\xi, y), \quad x \in \Omega^+ = R^n \backslash \Omega, \tag{7}$$

here $\frac{\partial}{\partial n_\xi^-}$ is the derivative on the external normal to $\partial\Omega^-$, and $\frac{\partial}{\partial n_\xi^+}$ is the derivative on the normal to $\partial\Omega^+$,

$$\chi_- = \begin{cases} 1 & x \in \overline{\Omega^-} \\ 0 & x \in \Omega^+ \end{cases}; \qquad \chi_+ = \begin{cases} 1 & x \in \overline{\Omega^+} \\ 0 & x \in \Omega^- \end{cases} \tag{8}$$

Keywords: Green function, Laplace equation, Dirichlet problem.

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

References:

[1] Kal'menov, T. Integral representations of solutions of coercive solvable problems for the Laplace equation. Bound Value Probl 2026, 12 (2026).

[2] Kalmenov, T., D. Suragan, and A. Kadirbek. On a Method for Transforming the Double Layer Potential to a Volume Potential and Its Application to the Description of Coercive Problems for the Laplace Equation. Computational and Applied Mathematics 25, no. 1 (2026): 170–179.

On noncommutative version of the corona theorem

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Abstract: The corona theorem is a result about the spectrum of the bounded holomorphic functions on the open unit disc $\mathbb{D}$, conjectured by Kakutani [1] and proved by L. Carleson [2]. Elementary formulation of corona theorem is that if $f_1, ..., f_n$ is a finite set of functions in $H^(\mathbb{D})$ satisfying $\sum_{j=1}^{n} |f_j(z)| \geq c > 0, \qquad z \in \mathbb{D}$, then there are functions $\{g_j\}_{j=1}^{n}$ in $H^(\mathbb{D})$ with $\sum_{j=1}^{n} f_j(z) g_j(z) = 1, \qquad z \in \mathbb{D}$.

In 1968 Fuhrmann [3] extended Carleson's corona theorem to the finite matrix case. In 1980 Rosenblum [4] and Tolokonnikov [5] proved the corona theorem for infinitely many generators $n =$. This was further generalized to the one-sided infinite matrix setting by Vasyunin in 1981 (see [6]). Finally Treil [7] showed in 1988 that the generalizations stop there by producing a counterexample to the two-sided infinite matrix case. Trent and Zhang [8] extended the matricial corona theorem of Fuhrmann and Vasyunin to more general algebras of functions which satisfy a corona theorem. In [9], Trent and Zhang proved a similar extension of the matricial corona theorem of M. Andersson [11] to the same algebras of functions satisfying a corona theorem.

This report is devoted to a noncommutative version of the corona theorem for finite subdiagonal subalgebras in Arveson's sense and its applications to noncommutative Hardy spaces H^p associated with a finite subdiagonal subalgebra.

Keywords: corona theorem, subdiagonal subalgebra, noncommutative Hardy H^p-spac, finite von Neumann algebra.

2020 Mathematics Subject Classification: Primary 46L51, 47L07; Secondary 47L30.
References:

[1] S. Kakutani, Concrete representation of abstract (M)-spaces(A characterization of the space of continuous functions), Ann. of Math. 42 (4), 994-1024, 1941.

[2] L. Carleson, Interpolations by bounded analytic functions and the corona problem, Ann. of Math. 76 (3), 547–559, 1962.

[3] P. P. Fuhrmann, On the corona theorem and its applications to spectral problems in Hilbert space, Trans. A.M.S. 132, 55-66, 1968.

[4] M. Rosenblum, A corona theorem for countably many functions, Int. Eqs. Op. Theory 3, 125-137, 1980.

[5] V. P. Tolokonnikov, Estimates in Carleson's corona theorem and finitely generated ideals of the algebra H, Funktsional. Anal. i Prilozhen. 14, 85-86, 1980.

[6] V. P. Tolokonnikov, Estimates in the Carleson corona theorem, ideals of the algebra H, a problem of Szökefalvi-Nagy. Investigations on linear operators and the theory of functions, XI. Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) 113, 178-198, 267, 1981.

[7] S. Treil, Angles between coinvariant subspaces and an operator-valued corona problem. A question of Szökefalvi-Nagy., Soviet Math. Dokl. 38, 394-399, 1989.

[8] T. Trent and X. Zhang, A matricial corona theorem, Proc. Amer. Math. Soc. 134, 2549-2558, 2006.

[9] T. Trent and X. Zhang, A matricial corona theorem, Proc. Amer. Math. Soc. 135, 2845–2854, 2007.

[10] S. K. Vodop'yanov, A. D. Ukhlov, Set functions and their applications in the theory of Lebesgue and Sobolev spaces I, Siberian Advances in Mathematics 14 (4), 78-125, 2004.

[11] M. Andersson, The corona theorem for matrices, Math. Z. 201, 121-130, 1989.

Energy method for difference schemes of coupled Klein–Gordon equations with rational terms

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Abstract: In this paper, we study the initial value problem for a coupled system of nonlinear Klein–Gordon equations with rational terms under Neumann boundary conditions. We establish the existence and uniqueness of solution for the difference scheme and prove the associated energy conservation law. A difference scheme of second order accuracy is presented for the approximate solution of this problem. Numerical results for the one-dimensional coupled nonlinear hyperbolic equation are presented. Finally, a numerical example is given to confirm the theoretical analysis.

Keywords: Nonlinear hyperbolic equations, coupled Klein–Gordon system, difference scheme, stability, Hilbert space.

2020 Mathematics Subject Classification: 35L70, 35G50, 35A01, 35A02, 65M06, 65M12.

References:

[1] A. Ashyralyev and D. Agirseven, "Bounded Solutions of Semilinear Time Delay Hyperbolic Differential and Difference Equations," *Mathematics*, vol. 7, Art. no. 1163, 2019. doi: 10.3390/math7121163.

[2] Ashyralyev, A.; Sarsanbi, A. Well-posedness of a parabolic equation with involution. *Numer. Funct. Anal. Optim.* **2017**, *38*, 1295–1305.

[3] Sobolevskii, P. E. *Difference Methods for the Approximate Solution of Differential Equations*; Voronezh State University Press: Voronezh, Russia, 1975. (In Russian)

[4] Ashyralyev, A.; Sobolevskii, P. E. *New Difference Schemes for Partial Differential Equations*; Birkhäuser Verlag: Basel, Switzerland, 2004.

A note on interpolation spaces generated by the second order differential operator with dependent coefficient and Samarskii-Ionkin conditions

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Abstract: In this study, we investigate the operator

$$A^x u = (a(x)u_x)_x + u(x), 1, \ 0 < x < 1,$$

where domain of A^x is

$$D(A^x) = \{u \in C^2[0,1] : u(0) = 0, u'(0) = u'(1) + \mu u(1)\},$$

$\mu > 0$, $a(0) = a(1)$, $a(x) \geqslant \delta_0 > 0$ is continuously differentiable function defined on $[0,1]$.

We construct the Green function of A^x. We also obtain the estimates for the Green function. We prove the positivity ([1]) of differential operator A^x in $C[0,1]$. Furthermore, we investigate the structure of the interpolation spaces $E_\alpha(C[0,1], A^x)$. In addition, we prove that for each $\alpha \in (0, 1/2)$ the norms of the interpolation space $E_\alpha(C[0,1], A^x)$ and the Hölder space $\mathring{C}^{2\alpha}[0,1]$ are equivalent. Hence, we also prove the positivity of A^x in $\mathring{C}^{2\alpha}[0,1]$.

To apply the our theoretical results, we consider the non-local boundary value problems of parabolic type

$$(1) \qquad \begin{cases} v_t(x,t) - (a(x)v_x)_x + \sigma v(t,x) = \varphi(x,t), \sigma \geqslant 1, 0 \leqslant x \leqslant 1, 0 < t < T, \\ v(x,0) = \psi(x), 0 \leqslant x \leqslant 1, \\ v(0,t) = 0, v_x(0,t) = v_x(1,t) + \mu v(1,t), 0 < t < T, \end{cases}$$

and

$$(2) \qquad \begin{cases} v_t(x,t) - (a(x)v_x)_x + \sigma v(t,x) = \varphi(x,t), \sigma \geqslant 1, 0 \leqslant x \leqslant 1, 0 < t < T, \\ v(x,0) = v(x,T), 0 \leqslant x \leqslant 1, \\ v(0,t) = 0, v_x(0,t) = v_x(1,t) + \mu v(1,t), 0 < t < T, \end{cases}$$

where μ is a fixed positive number, φ and ψ are sufficiently smooth functions.

We prove noval coercive inequalities for solution of nonlocal boundary value problems (1) and (2) of parabolic type.

References:

[1] A. Ashyralyev, P.E. Sobolevskii, Well-Posedness of Parabolic Difference Equations, Birkh auser Verlag, Basel, Boston, Berlin, 1994.

On a coupled system of semilinear time-fractional Schrödinger equations

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Abstract: This paper investigates a coupled system of time-fractional semilinear Schrödinger equations with disordered coefficients within a general abstract operator framework. This setting naturally accommodates variable-coefficient and nonstandard spatial operators, thereby extending the applicability of fractional Schrödinger models to heterogeneous media and interacting multi-component systems. Existence and uniqueness of mild solutions are established using operator-theoretic techniques under suitable assumptions. A time discretization scheme is constructed and analyzed within the same abstract framework, applicable to multi-dimensional problems. Numerical experiments in one- and two-dimensional configurations, involving variable-coefficient operators and various nonlinear interactions, confirm the theoretical results and demonstrate the robustness of the proposed method.

Keywords: Time-fractional Schrödinger equations, mild solutions, operator framework, semilinear systems, numerical discretization

2020 Mathematics Subject Classification: 35Q55, 65M12, 35R11

References:

[1] Zu, C., Yu, X.: Time fractional Schrödinger equation with a limit based fractional derivative. Chaos Solitons Fractals **157**, 111941 (2022).

[2] Wang, S., Zhou, X.-F., Liu, J.-B., Pang, D., Liu, B.: On a class of time-space fractional nonlinear Schrödinger equation with the Riesz potential or Bessel potential. Fract. Calc. Appl. Anal. **29**(1), 331–365 (2026).

Stability analysis for advection-diffusion equations and their numerical approximation

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Abstract: The advection-diffusion equation provides a fundamental mathematical framework for describing essential transport processes, including natural gas transport in pipelines and environmental fluid flow [1]. In this study, stability estimates for the advectionâ“diffusion equation are derived [2, 3]. Rothe and Crank-Nicolson finite difference schemes are constructed [4, 5], and their stability properties are established through rigorous theoretical analysis. The results are subsequently extended to one-dimensional advection-diffusion with nonlocal boundary conditions and involution, multidimensional advection-diffusion initial-boundary value problems with Dirichlet and Neumann boundary conditions. The derived stability estimates do not impose restrictions on the ratio of the temporal and spatial step sizes, thereby providing robust performance over a wide range of discretization parameters [6, 7]. Numerical experiments are conducted for two one-dimensional advection-diffusion test problems with Dirichlet and Neumann boundary conditions [8, 9], and the numerical solutions are compared with the exact solutions, with errors evaluated in the L_2 and C norms. The results confirm the theoretical convergence rates, with the Rothe scheme exhibiting first-order accuracy and the Crank-Nicolson scheme exhibiting second-order accuracy.

Keywords: Advection-diffusion equation, Stability estimates, Difference schemes, Rothe scheme, Crank-Nicolson scheme, Initial-boundary value problems

2020 Mathematics Subject Classification: 35K20, 65M06, 65M12 **References:**

[1] Modeling the Processes in Exploration of Gas Deposits and Applied Problems of Theoretical Gas Hydrodynamics, Ilym, Ashgabat, 1998.
[2] A. Ashyralyev, P. E. Sobolevskii, Well-Posedness of Parabolic Difference Equations, Operator Theory: Advances and Applications, vol. 69, Birkhäuser Verlag, Basel, 1994.
[3] A. A. Samarskii, The Theory of Difference Schemes, Marcel Dekker, New York, 2001.
[4] I. Faragó, R. Horváth, Qualitative Analysis of the Crank-Nicolson Method for the Heat Conduction Equation, in: Large-Scale Scientific Computing, Springer, 2009.
[5] A. Ashyralyev, Well-Posedness of the Modified Crank-Nicholson Difference Scheme in Bochner Spaces, Discrete and Continuous Dynamical Systems-Series B, vol. 7, no 1, 29–51, 2007.
[6] A. R. Appadu, H. H. Gidey, Stability analysis and numerical results for some schemes discretising 2D nonconstant coefficient advection-diffusion equations, Open Physics, vol. 22, no 1, 20230195, 2024.
[7] M. Dehghan, Numerical solution of the three-dimensional advection-diffusion equation, Applied Mathematics and Computation, vol. 150, no 1, 5–19, 2004.
[8] E. Soykan, M. Ahlatcioglu, M. Ashyraliyev, On the numerical solution of advection-diffusion problems with singular source terms, AIP Conference Proceedings, 2018.
[9] M. Ashyraliyev, J. G. Blom, J. G. Verwer, On the numerical solution of diffusion-reaction equations with singular source terms, Journal of Computational and Applied Mathematics, 2008.

Stability threshold for a delayed discrete-time epidemic model with periodic transmission

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Abstract: We study a nonlinear delayed discrete-time epidemic model with periodic transmission and a saturation effect. Such models are appropriate when epidemic data are recorded at fixed intervals and transmission varies seasonally [1, 2]. The model is

$$I_{n+1} = \frac{\alpha_n I_n}{1+\beta I_{n-m}}, \qquad \alpha_n = \alpha\left(1+\varepsilon\cos\frac{2\pi n}{T}\right), \tag{1}$$

where I_n is the infectious proportion, $\alpha > 0$ is the mean transmission rate, $\varepsilon \in (0,1)$ is the seasonal amplitude, T is the period, $\beta > 0$ is the saturation parameter, and $m \geq 0$ is the delay. The disease-free solution is $I_n = 0$. Set $\mathcal{R}_T = \prod_{n=0}^{T-1} \alpha_n$.

Theorem. The disease-free solution of (1) is locally asymptotically stable if $\mathcal{R}_T < 1$ and unstable if $\mathcal{R}_T > 1$.

Proof. Let $U_n = (u_n, \ldots, u_{n-m})^\top$. Since $\alpha_n u_n/(1+\beta u_{n-m}) = \alpha_n u_n - \alpha_n \beta u_n u_{n-m} + O(\|U_n\|^3)$, the delay and saturation terms are absent from the linear part, which is $u_{n+1} = \alpha_n u_n$. In the $(m+1)$-dimensional delayed state space, $U_{n+1} = A_n U_n$, where A_n is the lower triangular companion matrix with diagonal $(\alpha_n, 0, \ldots, 0)$. Hence $M = A_{T-1} \cdots A_0$ has diagonal $(\mathcal{R}_T, 0, \ldots, 0)$, so its Floquet multipliers are $\mathcal{R}_T$ and m zeros. Therefore $\mathcal{R}_T < 1$ gives local asymptotic stability, $\mathcal{R}_T > 1$ gives instability, and $\mathcal{R}_T = 1$ is critical. The delay and saturation parameters do not enter this linear threshold, although they affect the nonlinear dynamics away from zero.

Keywords: discrete-time epidemic model, seasonal transmission, time delay, saturation effect, disease-free solution, Floquet multiplier

2020 Mathematics Subject Classification: 39A30, 39A60, 92D30

References:

[1] L. J. S. Allen, Some discrete-time SI, SIR, and SIS epidemic models, *Math. Biosci.* **124** (1994), 83–105.

[2] A.-A. Yakubu and J. E. Franke, Discrete-time SIS epidemic model in a seasonal environment, *SIAM J. Appl. Math.* **66** (2006), 1563–1587.

Spectral properties and global dynamics of interior fixed points in four-dimensional tournament Lotka–Volterra operators

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Abstract: We study the discrete Lotka–Volterra operator

$$(Vx)_i = x_i\left(1 + \sum_{j=1}^{5} a_{ij}x_j\right), \qquad i = 1, \dots, 5,$$

on the simplex S^4, where $A = (a_{ij})$ is a real skew-symmetric matrix. The sign pattern of A determines an associated tournament and the corresponding fixed-point card [1]. We focus on the class of tournaments containing five cyclic triples.

A sufficient condition for the existence of an interior fixed point is obtained in terms of the principal Pfaffians $\theta_1, \dots, \theta_5$. If they have the same nonzero sign, then

$$x^* = \frac{(\theta_1, \dots, \theta_5)}{\theta_1 + \cdots + \theta_5} \in \operatorname{ri} S^4$$

satisfies $Ax^* = 0$ and is an interior fixed point.

The spectrum of the Jacobian at x^* has the form

$$\sigma(DV(x^*)) = \{1,\, 1 \pm i\omega_1,\, 1 \pm i\omega_2\}.$$

For $\omega_1\omega_2 \neq 0$, $|1 \pm i\omega_k| > 1, \qquad k = 1, 2$, so the interior fixed point is non-hyperbolic and is not a local attractor. Lyapunov-type functions are then used to exclude convergence of generic interior trajectories to x^* and to relate their asymptotic behavior to the boundary dynamics.

Keywords: Lotka–Volterra operator, tournament, cyclic triples, Pfaffians, interior fixed point, spectrum, global dynamics

2020 Mathematics Subject Classification: 37B25, 37C25, 37C35 **References:**

[1] D.B. Eshmamatova, M.A. Tadzhieva, N. Temirbekov, Construction of fixed point cards for degenerate Lotka–Volterra mappings, Journal of Mathematical Sciences, 2025.

Directed transition geometry in an eight-signature Lotka–Volterra system

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Abstract: We study the geometric structure and directed transitions of an eight-signature discrete Lotka–Volterra system on the simplex $S^3 = \{(S, E, I, R) \in \mathbb{R}^4 : S, E, I, R \geq 0,\ S + E + I + R = 1\}$. The system is given by

$$
(1) \qquad \begin{cases} S^{(n+1)} = S^{(n)}(1 - bE^{(n)} - cR^{(n)}), \\ E^{(n+1)} = E^{(n)}(1 + bS^{(n)} - dI^{(n)} + fR^{(n)}), \\ I^{(n+1)} = I^{(n)}(1 + dE^{(n)} - eR^{(n)}), \\ R^{(n+1)} = R^{(n)}(1 + cS^{(n)} + eI^{(n)} - fE^{(n)}), \end{cases} \qquad b, c, d, e, f > 0.
$$

Writing (1) as $X_i^{(n+1)} = X_i^{(n)}(1+\Phi_i(X^{(n)}))$, we define the signature $\delta(X) = (\operatorname{sign}\Phi_1, \ldots, \operatorname{sign}\Phi_4)$. The sign conditions generated by $\Phi_1, \ldots, \Phi_4$ define eight nonempty regions $F_i = \{X \in S^3 : \delta(X) = \delta_i\}$, $i = 1, \ldots, 8$. Since $X_i^{(n+1)} - X_i^{(n)} = X_i^{(n)}\Phi_i(X^{(n)})$, their common faces determine adjacency and the signs of Φ_i determine transition directions. Thus, the main result is a directed adjacency graph describing admissible trajectory routes. Following the epidemiological interpretation of discrete Lotka–Volterra systems in [1], system (1) admits an extended SEIR-type interpretation with the principal interaction chain $S \to E \to I \to R$ and additional immunization and reinfection interactions.

Keywords: discrete Lotka–Volterra operator, signature, adjacency, directed transition, SEIR-type model.

2020 Mathematics Subject Classification: 37B25, 37C25, 37C27

References:

[1] Ganikhodzhaev R.N., Eshmamatova D.B., Tadzhieva M.A., Zakirov B.S., Some degenerate cases of discrete Lotka–Volterra dynamical systems and their applications in epidemiology, Journal of Mathematical Sciences, 289 (2024), 755–769.

Solvability of a system of hyperbolic equations with nonlocal conditions

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Abstract: We consider the following nonlocal boundary value problem with two-point and integral boundary conditions for a system of hyperbolic equations in the domain $Q = [0, T] \times [0, l]$:

$$z_{tx} = f(t, x, z(t, x)), \tag{1}$$

$$A_1 z(0, x) + \int_0^T n(t)z(t, x)\, dt + B_1 z(T, x) = \varphi(x), \quad x \in [0, l], \tag{2}$$

$$A_2 z(t, 0) + \int_0^l m(x)z(t, x)\, dx + B_2 z(t, l) = \psi(t), \quad t \in [0, T] \tag{3}$$

where $z(t, x) = z_1(t, x), z_2(t, x), \ldots, z_n(t, x)$ is the unknown n-dimensional vector-function. $f : Q \times \mathbb{R}^n \to \mathbb{R}^n$, $\varphi(x)$, $\psi(t)$ are differentiable n-dimensional vector functions defined on the segments $[0, T]$, $[0, l]$. The functions $n(t)$, $m(x)$ are $n \times n$ matrix-valued functions, $A_1, A_2, B_1, B_2 \in \mathbb{R}^{n\times n}$ are given matrices with $A = A_1 + B_1 + \int_0^T n(t)\, dt$, $\det A \neq 0$ and $B = A_2 + B_2 + \int_0^l m(x)\, dx$, $\det B \neq 0$.

By applying the Banach's contraction mapping principle, sufficient conditions for the existence and uniqueness of a solution to the boundary value problem (1)–(3) are obtained.

Similar problems have been considered in [1].

Keywords: non-local boundary value problems, integral and two-point boundary conditions, Goursat-Darboux problem, existence and uniqueness.

2020 Mathematics Subject Classification: 35G35, 35G46, 35L53, 35L57

References:

[1] Y.A. Sharifov, A.R. Mammadli, F.M. Zeynally. The Goursat-Darboux system with two-point boundary condition, Transactions Issue Mathematics National Academy of Sciences of Azerbaijan, vol. 45, no 1, 142–152, 2025.

On the numerical solution of an inverse source problem by model order reduction

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Abstract: In this study, an inverse time-dependent source problem for a linear parabolic equation arising in numerous practical applications involving diffusion and transport phenomena is considered. Typical examples of such problems aim to reconstruct the unknown time-dependent heat source in heat conduction processes, the temporally varying pollutant emissions in environmental monitoring, the atmospheric emission sources, *etc.* by using some direct observations. The derivation of numerical solutions of such a problem in high-dimensional spaces leads a huge computational cost. Taking into account that it may be necessary to obtain numerical solutions several times in some cases, it is a crucial task to reduce the computational cost, while preserving the accuracy. Motivated by this, for the numerical solution of the considered problem, we propose a model order reduction method by proper orthogonal decomposition [1,2]. In the method, finite difference method is implemented. Particularly, the time discretization is carried out by the implicit Euler's difference scheme. From the theoretical point of view the convergence rate of the method is deduced. The theoretical results and efficiency of the proposed method are illustrated by testing the method on a typical model problem.

Keywords: Inverse source problem, model order reduction, proper orthogonal decomposition

2020 Mathematics Subject Classification: 65J22, 65M32, 65N21, 65K99

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[1] S. Volkwein, Proper orthogonal decomposition: Theory and reduced-order modelling, Lecture Notes, University of Konstanz, 4 (2013).
[2] K. Kunisch, S. Volkwein, Galerkin proper orthogonal decomposition methods for parabolic problems, Numerische Mathematik, vol. 90, 117–148, 2001.

Comparison principle for a nonlocal fractional diffusion problem

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Abstract: In this paper, we study an initial–boundary value problem for a nonlinear time–space fractional diffusion equation in a smooth bounded domain $\Omega \subset \mathbb{R}^n$. The equation involves the Caputo fractional derivative $\mathcal{D}^{\alpha}_{0|t}$ of order $\alpha \in (0,1)$ (see [1]), the fractional p-Laplacian $(-\Delta)^s_p$, and the nonlinear reaction terms $\gamma|u|^{m-1}u + \mu|u|^{q-2}u$, where $s \in (0,1)$, $p \geq 2$, $m > 0$, $q \geq 1$, and $\gamma, \mu \in \mathbb{R}$.

The problem is equipped with the initial condition $u(x,0) = u_0(x)$ and the nonlocal Neumann-type boundary condition $\mathcal{N}^s_p u = 0$ in $\mathbb{R}^n \setminus \overline{\Omega}$, where $\mathcal{N}^s_p$ is defined by a nonlocal integral flux operator.

Using elementary algebraic inequalities, we establish a comparison principle for different ranges of the parameters γ, μ, m, and q. Based on this principle, we classify the finite-time blow-up of solutions and the existence of global weak solutions.

We also note that the corresponding problem with a Dirichlet boundary condition was studied in [2].

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Keywords: parabolic equations, nonlocal operator, polynomial nonlinearity comparison principle.

2010 Mathematics Subject Classification: 35R11, 35A01, 35B51, 35K55 **References:**

[1] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and applications of fractional differential equations, North-Holland Mathematics Studies, (2006).

[2] M.B. Borikhanov, M. Ruzhansky, B.T. Torebek, Qualitative properties of solutions to a nonlinear time-space fractional diffusion equation. Fract. Calc. Appl. Anal., vol. 26, no 1, 111–146, 2023.

On an inverse problem for the Poisson equation with an even source term

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Abstract: An inverse problem for the Poisson equation in the unit disk with an even source term depending only on the angular variable is considered. The problem consists in determining both the solution of the equation and the unknown source term from the Dirichlet boundary condition and an additional Neumann boundary condition. The proposed method is based on the decomposition of the solution into even and odd parts with respect to the angular variable, reducing the original inverse problem to the sequential solution of two auxiliary problems, which are solved by the Fourier method. Theorems on the existence and uniqueness of generalized and classical solutions are established.

Problem formulation. Consider the unit disk

$$\Omega = \{(r,\theta) : 0 < r < 1,\ 0 \le \theta \le 2\pi\}, \Gamma = \{(r,\theta) : r = 1,\ 0 \le \theta \le 2\pi\}.$$

Determine a pair of functions $u(r,\theta)$ and $f(\theta)$ satisfying the Poisson equation

$$-\Delta u(r,\theta) = f(\theta), \qquad (r,\theta) \in \Omega. \tag{1}$$

The Dirichlet boundary condition is specified on the boundary of the disk:

$$u(1,\theta) = \tau(\theta), \qquad 0 \le \theta \le 2\pi. \tag{2}$$

As an additional condition, the Neumann boundary condition is imposed on the upper semicircle:

$$\frac{\partial u}{\partial r}(1,\theta) = \nu(\theta), \qquad 0 \le \theta \le \pi. \tag{3}$$

The function $\tau(\theta)$ is assumed to be 2π–periodic. Moreover,

$$f(\theta) = f(2\pi - \theta), \qquad 0 \le \theta \le 2\pi.$$

Theorem 2.1. *Let*

$$\tau \in C^{3+\varepsilon}(\Gamma), \qquad \nu \in C^{2+\varepsilon}[0,\pi], \qquad 0 < \varepsilon \le 1$$

and let the conditions

$$\nu'(0) = \nu'(\pi) = 0$$

hold. Then problem (1)-(3) has a unique classical solution satisfying

$$u \in C^{2+\varepsilon}(\overline{\Omega}), \qquad f \in C^{\varepsilon}(\Gamma).$$

Theorem 2.2. *Let*

$$\tau \in W_2^2(\Gamma), \qquad \nu \in W_2^1[0,\pi].$$

Then problem (1)-(3) has a unique generalized solution satisfying

$$u \in W_2^2(\Omega), \qquad f \in L_2(\Gamma).$$

Keywords: Poisson equation, inverse problem, Sobolev spaces, Hölder spaces

2020 Mathematics Subject Classification: 35J05, 31A25

References:

[1] M.A. Sadybekov, A.A. Dukenbayeva, Direct and inverse problems for the Poisson equation with equality of ows on a part of the boundary , Complex Variables and Elliptic Equations, vol. 64, no 5, 777–791, 2019.

Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

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Abstract: This report is devoted to a spectral problem for a multiple differentiation operator with an integral perturbation of boundary conditions of one type which are regular, but not strongly regular:

$$l(u) \equiv -u''(x) = \lambda u(x),\ 0 < x < 1, \tag{1}$$

$$U_1(u) \equiv u'(0) - \int_0^1 \overline{p(x)} u(x)dx = 0,\ p(x) \in L_2(0,1), \tag{2}$$

$$U_2(u) \equiv u(0) - \alpha u(1) = 0. \tag{3}$$

Here $\alpha \neq 0$ is an arbitrary complex number.

The unperturbed problem ($p(x) \equiv 0$) has an asymptotically simple spectrum, and its system of normalized eigenfunctions creates the Riesz basis. We construct the characteristic determinant of the spectral problem with an integral perturbation of the boundary conditions. The perturbed problem can have any finite number of multiple eigenvalues. Therefore, its root subspaces consist of its eigen and (maybe) adjoint functions. It is shown that the Riesz basis property of a system of eigen and adjoint functions is stable with respect to integral perturbations of the boundary condition.

Throughout this note we mainly use techniques from our works [1].

Keywords: Riesz basis, regular boundary conditions, eigenvalues, root functions, spectral problem, integral perturbation of boundary condition, characteristic determinant

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

References:

[1] M.A. Sadybekov, N.S. Imanbaev, On the basis property of root functions of a periodic problem with an integral perturbation of the boundary condition, Differential Equations, vol. 48, no 6, 896–900, 2012.

Anisotropic Morrey spaces and the boundedness of the Riesz operator in them

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Abstract: In this paper, we define anisotropic Morrey-type spaces and consider the Riesz-type potential operator acting in them.

Let $k \in \mathbb{Z}$ and denote by G_k the set of all cubes in $\mathbb{R}^d$ of the form $[0, 2^k)^d + 2^k \bar{m}$, $\bar{m} \in \mathbb{Z}^d$.

Assume $\bar{k} = (k_1, ..., k_d)$, $\bar{n} = (n_1, ..., n_d)$, G_{k_i} be the set of cubes in R^{n_i}, $\mathbb{G}_{\bar{k}} = \{Q = Q_1 \times \cdots \times Q_d : \ Q_i \in G_{k_i}\}$. $\mathbb{R}^{\bar{n}} = \mathbb{R}^{n_1} \times \cdots \times \mathbb{R}^{n_d}$, $\bar{x} = (x_1, ..., x_d)$, $x_i \in R^{n_i}$, $i = \overline{1, d}$

Let $\bar{p} = (p_1, ..., p_d)$, $\bar{q} = (q_1, ..., q_d)$, $\bar{\lambda} = (\lambda_1, ..., \lambda_d)$ such that $0 < p_i \leq \infty$, $0 < q_i \leq \infty$, $0 < \lambda_i < \infty$. The anisotropic Morrey-type spaces $M_{\bar{p}}^{\bar{\lambda}}$ are defined as the set of all Lebesgue measurable functions $f \in L_{\bar{p}}^{loc}(\mathbb{R}^{\bar{n}})$ for which $\|f\|_{M_{\bar{p}}^{\bar{\lambda}}} = \sup_{\bar{k} \in \mathbb{Z}^d} \left(2^{-\langle \bar{k}, \bar{\lambda} \rangle} \sup_{Q \in G_{\bar{k}}} \|f\|_{L_{\bar{p}}(Q)} \right) < \infty$.

Let $\bar{\delta} = (\delta_1, \ldots, \delta_d)$, $\bar{n} = (n_1, ..., n_d)$, $\delta_i < n_i$, $i = \overline{1, d}$. Consider the Riesz-type potential operator $I_{\bar{\delta}} f(\bar{x}) = \int_{\mathbb{R}^{|\bar{n}|}} \frac{f(\bar{y})}{|\bar{x} - \bar{y}|^{\bar{n} - \delta}} d\bar{y}$. The following theorems holds. Let $\bar{p} = (p_1, \ldots, p_d)$, $\bar{q} = (q_1, \ldots, q_d)$, $\bar{\lambda} = (\lambda_1, \ldots, \lambda_d)$, $\bar{\delta} = (\delta_1, \ldots, \delta_d)$, $\bar{\gamma} = (\gamma_1, \ldots, \gamma_d)$, $\bar{n} = (n_1, \ldots, n_d)$.

Theorem 1. Let $1 < p_i < q_i < \infty$, $0 \leq \lambda_i < \frac{n_i}{q_i}$, $\delta_i = \frac{n_i}{p_i} - \frac{n_i}{q_i}$, $i = \overline{1, d}$. Then $I_{\bar{\delta}}$ is bounded from $M_{\bar{p}}^{\bar{\lambda}}$ to $M_{\bar{q}}^{\bar{\lambda}}$.

Theorem 2. Let $1 < p_i < q_i < \infty$, $1 \leq \lambda_i < \frac{n_i}{p}$, $0 \leq \lambda_i < \frac{n_i}{q_i}$, $0 \leq \gamma_i < \frac{n_i}{p_i}$, $\lambda_i q_i = \gamma_i p_i$, $\delta_i = \lambda_i + \frac{n_i}{p_i} - \frac{n_i}{q_i}$, $i = \overline{1, d}$. Then $I_{\bar{\delta}}$ is bounded from $M_{\bar{p}}^{\bar{\gamma}}$ to $M_{\bar{q}}^{\bar{\gamma}}$.

Keywords: anisotropic Morrey space, Riesz-type potential operator

2020 Mathematics Subject Classification: 35Q79, 35K05, 35K20

References:

[1] J. Jumabayeva, E.D. Nursultanov, Anisotropic Morrey-type spaces and their interpolation properties, Eurasian Math. J., vol. 17, no 1, 47–57, 2026.

A new non-polynomial fractional hyperbolic spline approach with convergence study for fractional Fredholm integro-differential equations

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Abstract: Fractional Fredholm integro-differential equations arise in many mathematical models involving memory, hereditary effects, and nonlocal interactions. Their numerical treatment is challenging because fractional differential operators and Fredholm integral terms act simultaneously on the unknown solution. In this work, a new non-polynomial fractional hyperbolic spline method is proposed for the numerical solution of Caputo fractional Fredholm integro-differential equations. The spline combines fractional-power terms with modified hyperbolic basis functions and is constructed locally on a uniform partition. Six interpolation conditions are imposed on each subinterval to determine the spline coefficients, while a sequential Caputo fractional continuity condition is used to connect adjacent spline segments. The resulting relations lead to a compact matrix formulation for the numerical scheme. The local coefficient matrix is shown to be nonsingular for sufficiently small mesh sizes, which guarantees unique determination of the local spline coefficients. In addition, the reduced discrete formulation is shown to be well posed, and a convergence analysis establishes uniform convergence of the spline approximation as the mesh size tends to zero. Numerical examples with known exact solutions are used to assess the method. The computed maximum absolute and root mean square errors decrease consistently under mesh refinement, confirming the accuracy and reliability of the proposed approach. The results indicate that the fractional hyperbolic spline provides an effective framework for the numerical approximation of fractional Fredholm integro-differential equations.

Keywords: Non-polynomial fractional spline, hyperbolic spline, fractional Fredholm integro-differential equations, Caputo fractional derivative, numerical approximation, convergence analysis

2020 Mathematics Subject Classification: 45J05, 65R20, 41A15, 65D07, 26A33

References:

[1] J.I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.

[2] A.A. Kilbas, H.M. Srivastava and J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, 2006.

A numerical algorithm based on first and second order of accuracy difference schemes for third order delay partial differential equation with involution

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Abstract: In this paper, first and second order of accuracy difference schemes are developed for solving a third order delay partial differential equation with involution. Illustrative numerical results are presented to demonstrate the accuracy and effectiveness of the proposed schemes..

Throughout this note we mainly use techniques from our works [1].

Keywords: Keywords: Time delay, third order partial differential equations, first- and second-order difference schemes

2020 Mathematics Subject Classification: 35G10, 35L90, 58D25

References:

[1] A. Ashyralyev, S. Ibrahim, E. Hincal, Absolutely stable difference scheme for the delay partial differential equation with involution and Robin boundary condition. Bulletin of the Karaganda University. Mathematics Series, vol.3 no 115, 55–65, 2024. https://doi.org/10.31489/2024m3/55-65

Pathwise Fourier continuation for Bermudan and American options: a sinh-accelerated NUFFT–RQMC framework

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Abstract: We develop a pathwise Fourier-continuation framework for Bermudan and American option pricing. The method combines randomized quasi-Monte Carlo path simulation, sinh-accelerated Fourier inversion, and type-3 nonuniform FFT evaluation so that, at each exercise date, continuation values are computed from the one-step characteristic function directly at the simulated states. This replaces the regression fit in least-squares Monte Carlo by a transform-based conditional expectation while retaining a simulation architecture. The framework is formulated for one-factor exponential models with explicit one-step characteristic functions; the continuously monitored barrier treatment considered here is specific to geometric Brownian motion, where a Brownian-bridge survival correction is applied in log space without changing the Fourier continuation step.

Numerical experiments recover standard GBM American-put benchmarks accurately, show competitive bias and RMSE relative to polynomial and Laguerre-basis LSMC on representative cases, and demonstrate the continuation machinery under a pure-jump CGMY model. For continuously monitored up-and-out puts under GBM, the barrier extension gives prices in close agreement with published benchmark values.

Keywords: American options; Bermudan options; Fourier continuation; nonuniform fast Fourier transform; randomized quasi-Monte Carlo; sinh acceleration; barrier options; CGMY model.

JEL classification: C63; G12; G13.

On the positivity of a second-order linear differential operator with constant coefficients in Banach spaces

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Abstract: Positivity of linear differential operators in Hilbert and Banach spaces is a fundamental concept in the analysis of partial differential equations (PDEs). In Hilbert spaces, positivity is defined using the notion of an inner product. This concept can be generalized to Banach spaces, where an inner product is not available. However, establishing positivity in Banach spaces is technically more challenging than in Hilbert spaces [1].

The main goal of this work is to establish sufficient conditions for the positivity of the second-order linear differential operator

$$Av = -\alpha v''(x) + \beta v'(x) + \delta v(x), \quad 0 < x < 1, \tag{1}$$

where α, β, and δ are positive constants, with domain

$$\mathcal{D}(A) = \left\{ v \ : \ v \in C^{(2)}(0,1);\ v(0) = v(1) = 0 \right\}. \tag{2}$$

The positivity is considered in abstract Banach spaces. The obtained result can be applied to the theory and analysis of various initial-boundary value problems for parabolic PDEs.

Keywords: Linear differential operator, positive operator, Banach spaces

2020 Mathematics Subject Classification: 47B01, 47B65

References:

[1] M.A. Krasnoselskii, P.P. Zabreiko, E.I. Pustylnik, P.E. Sobolevskii, Integral Operators in Spaces of Summable Functions, Nauka, Moscow, 1966.

On the new eighthrder-order of approximation difference schemes for fourth-order differential equations

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Abstract: Two types compact four-step difference schemes of eighth-order of approximation for the numerical solutions of local and nonlocal boundary value problems for fourth-order differential equations with dependent coefficients are studied. These these difference schemes are generated by a new Taylor decomposition on five points [1]. The theoretical statements for the solution of these difference schemes are supported by the results of numerical experiments. The main goal of this work is to establish sufficient conditions for the positivity of the second-order linear differential operator.

Keywords: Taylor's decomposition on five points, local and nonlocal problems, difference schemes, approximation, numerical experiments.

2020 Mathematics Subject Classification: 34B10, 35K10, 49K40

References:

[1] Ashyralyev, A., Ibrahım, I. M. (2024). High-Order, Accurate Finite Difference Schemes for Fourth-Order Differential Equations. Axioms, 13(2), 90.

Method of generalized Jacobi elliptic functions for solving some classes of nonlinear elliptic systems of second-order equations on the plane

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Abstract: In this paper, generalized Jacobi elliptic functions $dn\,\omega(z)-$ delta-amplitude are applied to the construction of particular solutions of certain classes of elliptic systems of partial differential equations of second order on the complex plane C with Cauchy–Riemann operators $2\partial_{\overline{z}} = \partial_x + i\partial_y,\ 2\partial_z = \partial_x - i\partial_y$, and the Bitsadze operator $4\partial^2_{\overline{z}\,\overline{z}} = 4\partial_{\overline{z}}(\partial_{\overline{z}}) = \partial^2_{xx} - \partial^2_{yy} + 2i\partial^2_{xy}$, where the variables $z = x + iy,\ \overline{z} = x - iy$ are independent.

The function $\omega(z)$ is a quasi-periodic homeomorphism of the Beltrami equation

$$\omega_{\overline{z}} - q\omega_z = 0, \quad |\,q\,| \neq 1,$$

satisfying the condition

$$\omega(0) = 0, \quad \omega(z + h_j) = \omega(z) + \tau_j, \quad j = 1, 2,$$

where $\mathrm{Im}(h_2/h_1) \neq 0,\ \mathrm{Im}(\tau_2/\tau_1) \neq 0$.

The function $\omega(z)$ quasiconformally maps any parallelogram Ω with vertices $z_0,\ z_0 + h_1,\ z_0 + h_1 + h_2,\ z_0 + h_2$ onto a quadrilateral Ω' with vertices $\omega(z_0),\ \omega(z_0) + \tau_1,\ \omega(z_0) + \tau_1 + \tau_2,\ \omega(z_0) + \tau_2$, and for $|\,q\,| < 1$ the orientation of the boundaries of Ω and Ω' is preserved.

Keywords: solution of equations, modulus of elliptic function, Jacobi elliptic functions, analytic function.

2020 Mathematics Subject Classification: Primary 35A09; Secondary 35B36

Numerical solution of the inverse coefficient problem for an elliptic equation

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Abstract: This report is devoted to the inverse recovery problem of the coefficient $c(y)$ and solving $u(x,y)$ from the equation

$$-\Delta u(x,y) + c(y)u(x,y) = f(x,y), \quad (x,y) \in \Omega \subset R^2, \tag{1}$$

with Robin's condition

$$\frac{\partial u}{\partial n}(x,y) = -q(x,y)u(x,y) + g(x,y) \tag{2}$$

and additional data

$$u\rfloor_{\partial\Omega} = U(x,y), \quad (x,y) \in \partial\Omega \tag{3}$$

The uniqueness theorem of the solution of the inverse problem is proved. A finite-difference scheme and an iterative gradient algorithm based on the sequential solution of direct and adjoint problems are constructed. The results of numerical experiments at different noise levels in the input data are presented. The numerical algorithm is based on the approach described in [1].

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Keywords: coefficient inverse problem, Poisson equation, Robin boundary condition, finite-difference method, adjoint problem, gradient method.

2020 Mathematics Subject Classification: 35R30, 65N21

References:

[1] Vabishchevich P. N. Numerical Solution of the Identification Problem for the Lower Coefficient of an Elliptic Equation. Differential Equations, 38(7), 999–1005. 2002.

Unique determination of the time-dependent source factor and the order of a fractional derivative with an unknown initial value

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Abstract: This paper considers the inverse problem of simultaneously recovering the fractional order $\alpha \in (0, 1)$ and the time-dependent source factor $p(t)$ in a boundary-value problem for the subdiffusion equation with a Dirichlet Laplacian. The overdetermination condition is defined by the scalar product $(u(t), \psi)_X$ for $0 < t < T$, where $\psi \in D(A)$ is an arbitrary fixed element.

Uniqueness is established independently of the specific form of the initial function.

On a class of a loaded overdetermined system of second-order differential equations with regular coefficients

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Abstract: We consider a system of two loaded differential equations: one linear second-order hyperbolic equation and one first-order differential equation with regular coefficients. Under certain conditions on the coefficients of these equations, we obtain a representation of the variety of solutions via two arbitrary constants and prove the unique solvability of the initial value problem.

Key words: overdetermined system, solution manifold, parallelepiped, weak singular coefficient.

An approximation operator for the Haar wavelet spaces

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Abstract: This report is devoted to constructing a concrete approximation operator for multivariate functions. The functions can be written as the Fourier–Haar series and their Fourier–Haar coefficients decay rapidly. We call a set of such functions a Haar wavelet space. We show that the classical Besov and Sobolev spaces with a dominant mixed derivative is continuously embedded into the Haar wavelet space. Thus, the approximation operator can be applied to Besov and Sobolev spaces.

We also investigate the convergence rate of the approximation operator for the Haar wavelet spaces. Due to the embeddings of the Besov and Sobolev spaces with a dominant mixed derivative we also receive the convergence rate for these spaces.

Keywords: multi-dimension, function approximation, Besov space, Sobolev space.

2020 Mathematics Subject Classification: 41A25, 41A63

The BD property of a class of high-order singular difference operators

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Abstract: This report is devoted to the discreteness of the spectrum and its boundedness from below for a high-order one term difference operator with arbitrary positive coefficients.

Let $n \in \mathbb{N}$. Let $\rho = \{\rho_k\}_{k=0}^{\infty}$, $v = \{v_k\}_{k=0}^{\infty}$ be positive real number sequences.

Let us consider the symmetric difference expression

$$ly_{k+n} = (-1)^n \frac{\bar{\Delta}^n(\rho_k \Delta^n y_k)}{v_k},$$

where $\Delta y_k = y_{k+1} - y_k$, $\bar{\Delta} y_k = y_k - y_{k-1}$, $\Delta^n y_k = \Delta(\Delta^{n-1} y_k)$, $\Delta^0 y_k \equiv y_k$, $k \in \mathbb{N}$ and similarly for $\bar{\Delta}^n y_k$.

Let $N \in \mathbb{N}$, and E_N be the set of sequences of real numbers $y = \{y_k\}_{k=N}^{\infty}$ such that $y_N = y_{N+1} = \cdots = y_{N+n-1} = 0$. Let $\mathring{E}_N \subset E_N$ and for any $y \in \mathring{E}_N$ there exists a number $M(y) \in \mathbb{N}$ such that $y_k = 0$ for all $k \geq M(y)$.

Let $\mathbb{N}(N) = \{k \in \mathbb{N} : k \geq N\}$ and $l_{2,v}$ be the Hilbert space of sequences $f = \{f_k\}_{k=0}^{\infty}$ with an inner product $(f,g)_{2,v} = \sum_{i=0}^{\infty} f_i g_i v_i$, $f, g \in l_{2,v}$, and with norm $\|f\|_{2,v} = \left(\sum_{i=N}^{\infty} v_i |f_i|^2\right)^{\frac{1}{2}}$.

We determine the following operator L_{min} with domain $\mathcal{D}(L_{min}) = \mathring{E}_N$

$$L_{min} y_{k+n} = l y_{k+n} = (-1)^n \frac{\Delta^n(\rho_k \Delta^n y_k)}{v_k}, \quad \forall y \in \mathcal{D}(L_{min}),$$

where outer $\Delta^n \equiv \bar{\Delta}^n$.

The operator L_{min} is symmetric in the space $l_{2,v}$. Denote its closure in $l_{2,v}$ by $\bar{L}_{min}$. It is known that all self-adjoint extensions L of the operator $\bar{L}_{min}$ have similar spectral properties.

Here, we investigate the discreteness of the spectrum of the operator L and its boundedness from below. According to the terminology introduced in [1], this property is called the BD property.

Our main result reads as follows.

Theorem 1. The operator L has the BD property if and only if

$$\lim_{N\to\infty} \sup_{s \geq N} \sum_{j=N}^{s} \rho_j^{-1} \left[(s-j)^{(n-t)}\right]^2 \sum_{k=s}^{\infty} v_{k-(n-1)} \left[(k-s)^{(t-1)}\right]^2 = 0 \tag{1}$$

for all $t = 1, 2, \ldots, n$, where $k^{(i)} = k(k-1)\cdots(k-i+1)$.

Keywords: difference operator, spectral properties, BD property, roundabout theorem, variational method.

2020 Mathematics Subject Classification: 39A70, 34C10, 26D15

References:

[1] D.B. Hinton, R.T. Lewis, Discrete criteria for singular differential operators with middle terms, Math. Proc. Cambridge Philos. Soc., no. 77, 337-347, 1975.

Stability analysis of multipoint nonlocal problems for third-order partial differential equations

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Abstract: This paper investigates a finite multipoint nonlocal boundary value problem for a third-order differential equation in a Hilbert space. Assuming that the governing operator is self-adjoint and positive definite, we derive an explicit condition on the multipoint coefficients that guarantees the bounded invertibility of the operator associated with the nonlocal conditions. This result allows the solution to be represented by means of cosine and sine operator functions. Using this representation, we prove the existence and uniqueness of the solution and derive stability estimates with respect to the given data. The proposed formulation extends earlier single-point nonlocal problems to a finite multipoint setting, allowing several temporal evaluation points and their corresponding coefficients. The abstract results are then applied to one-dimensional and multidimensional third-order partial differential equations subject to periodic, Dirichlet, and conormal boundary conditions. These applications demonstrate the direct applicability of the operator-theoretic results to several classes of evolution problems.

Keywords: Third-order partial differential equations, multipoint nonlocal boundary conditions, stability estimates, cosine and sine operator functions, operator methods, Hilbert spaces.

2020 Mathematics Subject Classification: 34B10, 335G15, 47D09.

References:

[1] Y. P. Apakov, *On the solution of a boundary-value problem for a third-order equation with multiple characteristics*, Ukrainian Mathematical Journal **64** (2012), no. 1, 1–12.

[2] A. Ashyralyev and D. Arjmand, *A note on Taylor's decomposition on four points for a third-order differential equation*, Applied Mathematics and Computation **188** (2007), no. 2, 1483–1490.

[3] A. Ashyralyev and K. Belakroum, *On the stability of nonlocal boundary value problem for a third-order PDE*, AIP Conference Proceedings **2183** (2019), Article 070012.

[4] K. Belakroum, A. Ashyralyev, and A. Guezane-Lakoud, *A note on the nonlocal boundary value problem for a third-order partial differential equation*, Filomat **32** (2018), no. 3, 801–808.

Two-sided estimates of the norm of a elliptic-hyperbolic type operator with Parabolic Degeneration

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Abstract: The aim of the work is to construct a solvability theory (invertibility, separability, resolvent compactness) and to obtain sharp two-sided estimates for the s-numbers and eigenvalues of the operator $Lu = k(y)u_x x - u_y y + b(y)u_x + q(y)u$ on the cylinder $\omega = \|x\| \leq \pi, y \in R$ with periodic conditions in x, where $k(y)$ is an arbitrary piecewise continuous (including oscillating) function and $b(y)$, $q(y)$ are unbounded as $|y| \to \infty$.

The continuous invertibility of $L + \mu I$ is proved, and its resolvent is represented as a series over the resolvents of l_n. Under slow-variation conditions on b, q and the relation $q \leq C_0 b^2$, separability of the operator is established. A compactness criterion for the resolvent is obtained. A two-sided estimate for the counting function $N(\lambda)$ of singular numbers is derived in terms of the level-set measures of $Q_n(y)$, $K_n(y)$. For a model example ($k(y) = sin(e^{10|y|}), b(y) = q(y) = |y| + 1$), explicit power-type two-sided s-number estimates are obtained, and, via Weyl's inequality, eigenvalue estimates.

Throughout this note we mainly use techniques from our works [1].

Keywords: Invertibility, separability, compactness of the resolvent, eigenvalues, s-numbers, two-sided estimates

2020 Mathematics Subject Classification: 34B24, 34L05, 35M10, 47A10

References:

[1] Muratbekov M.B., Abylayeva A., Muratbekov M.M. Estimates of singular numbers (s-numbers) and eigenvalues of a mixed elliptic-hyperbolic type operator with parabolic degeneration // Math. Methods in the Appl. Sci. 2022. P. 1–13. DOI: 10.1002/mma.8908.

Two-weight estimates for integral operators with power-logarithmic singularities, $p > q$

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Abstract: Let $I = (0, \infty)$ and the functions v, u are positive almost everywhere, locally integrable on the interval I. Let $1 < p, q < \infty$ and $p' = \frac{p}{p-1}$. Let us denote by $L_{q,v} \equiv L_q(v, I)$ the set of functions f measurable on I for which $\|f\|_{q,v} = \left(\int\limits_0^\infty |f(x)|^q v(x)dx\right)^{\frac{1}{q}} < \infty$. Let W is a positive, strictly increasing and locally absolutely continuous function on the interval I. Assume $\frac{dW(x)}{dx} = w(x)$, for almost all $x \in I$. We consider the operator $T_{\alpha,\beta}$, where $\alpha > 0$, $\beta \geq 0$:

$$T_{\alpha,\beta} f(x) = \int\limits_0^x \frac{\left(ln \frac{W(x)}{W(x)-W(s)}\right)^\beta}{(W(x) - W(s))^{1-\alpha}} u(s) f(s) dW(s), \quad x \in I.$$

Theorem. Let $0 < \alpha < 1$, $0 < q < p < \infty$, $p > \frac{1}{\alpha}$ and $\beta \geq 0$. Let the function u be nonincreasing on I. Then the operator $T_{\alpha,\beta}$ is bounded from $L_{p,w}$ to $L_{q,v}$ if and only if

$$B_{\alpha,\beta} = \left(\int\limits_0^\infty \left(\int\limits_0^z W^{p'\beta}(s) u^{p'}(s) w(s) ds \right)^{\frac{q(p-1)}{p-q}} \times \right.$$

$$\left. \times \left(\int\limits_z^\infty v(x) W^{q(\alpha-\beta-1)}(x) dx \right)^{\frac{q}{p-q}} W^{q(\alpha-\beta-1)}(z) v(z) dz \right)^{\frac{p-q}{pq}} < \infty,$$

and $\|T_{\alpha,\beta}\| \approx B_{\alpha,\beta}$, where $\|T_{\alpha,\beta}\|$ is the norm of the operator $T_{\alpha,\beta}$ from $L_{p,w}$ to $L_{q,v}$.

Keywords: Boundedness; Weight function; Logarithmic singularity

2020 Mathematics Subject Classification: 26A33; 26D10; 47G10