International Conference on Analysis and Applied Mathematics (ICAAM) aims to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why we planned the conference series to provide a forum for researchers and scientists to communicate their recent developments and present their original results in various fields of analysis and applied mathematics.

As the knowledge of different branches of mathematics open new perspectives, it is important to learn more about the developments and advancements in the field of applied mathematics and analysis. As organizers we are proud to see that ICAAM provides a forum for researchers and scientists to share their recent developments and to present their original results in various fields of mathematics.

We welcome you to Antalya, Turkey and look forward to seeing you in upcoming conferences.

The main organizers of the conference are:

Bahcesehir University (Turkey),
Institute of Mathematics and Mathematical Modelling (Kazakhstan),
Analysis & PDE Center, Ghent University (Belgium)

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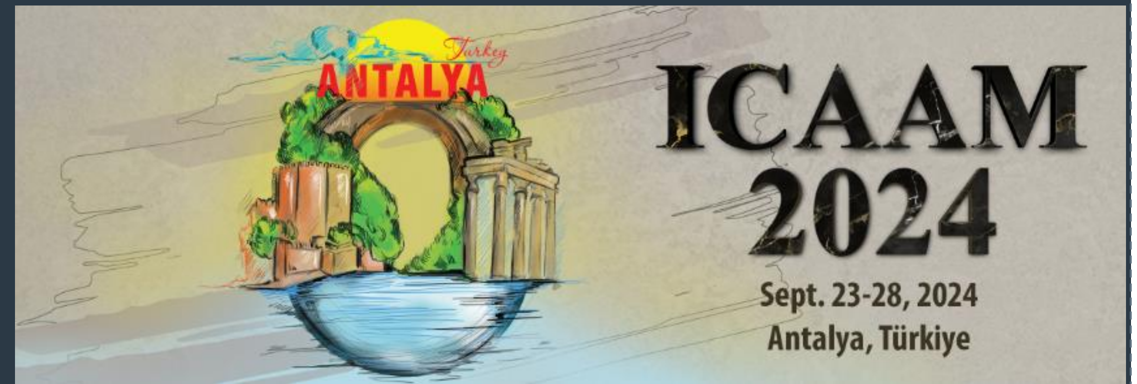

SEVENTH INTERNATIONAL CONFERENCE ON ANALYSIS AND APPLIED MATHEMATICS

ABSTRACT BOOK

Seventh International Conference on Analysis and Applied Mathematics

ABSTRACT BOOK

of the conference ICAAM 2024

Edited by

Allaberen Ashyralyev,

Charyyar Ashyralyyev,

Abdullah S. Erdogan,

Mahmud A. Sadybekov

September 23 – September 28, 2024

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The Organizing Committee of ICAAM, Bahcesehir University, Türkiye, Institute of Mathematics and Mathematical Modelling, Kazakhstan, and Analysis PDE Center, Ghent University, Belgium are pleased to invite you to the Seventh International Conference on Analysis and Applied Mathematics, ICAAM 2024. The meeting will be held from September 23 to 28, 2024, in Antalya, Türkiye.
The conference is organized biannually.
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- Minisymposium MS2: Applied Statistics and Data Analysis
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  Title: A priori estimates of solutions for the Vlasov-Poisson system and plasma confinement
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  Title: Two topologies in the restriction spaces
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FOREWORD AND KEYNOTE TALKS

Opening speech

A. Arif Ergin[1]

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Dear participants!

On behalf of the Rectorate of Bahçeşehir University I welcome you all to the 7-th International Conference on Analysis and Applied Mathematics ICAAM-2024. The ICAAM conference aims to bring together mathematicians working in the area of analysis and applied mathematics to share new trends of applications of mathematics. The developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why this conference provides a forum for researchers and scientists to communicate their recent developments and present their original results in various fields of analysis and applied mathematics. Previous ICAAM conferences were held in Türkiye (2012,2022), Kazakhstan (2014, 2016) and Northern Cyprus (2018, 2020). We are very happy to host you this time again in Türkiye. The proceedings from all previous ICAAM conferences held between 2012 and 2022 have been published in 6 volumes by the American Institute of Physics (AIP) Conference Proceedings and are indexed in the Scopus database. Selected manuscripts were published in the Special issues of various International Journals, such as Filomat, Boundary Value Problems, Numerical Functional Analysis and Optimization, International Journal of Applied Mathematics, Bulletin of the Karaganda University-Mathematics. The main organizers of ICAAM-2024 are:

- Bahçeşehir University, Türkiye
- Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
- Analysis PDE Center, Ghent University, Ghent, Belgium

It is a great pleasure for us to continue collaboration with these important Mathematical Centers of Kazakhstan and Belgium. We are particularly excited about our ongoing collaboration with these institutions through:

- Weekly “Analysis and Applied Mathematics” seminars, both in-person and online.
- The international journal “e-Journal of Analysis and Applied Mathematics”.

The seminars provide a platform for researchers to share their latest findings and encourage international collaboration. Notably, 47 successful seminars have been held since their beginning in 2022. Recordings of almost all seminars are available on YouTube. A book of extended abstracts from these seminars and related talks was recently published by Springer Publishing center. This publication features contributions from researchers from more than 15 different countries. e-Journal of Analysis and Applied Mathematics, established by our teams in 2018, serves as a forum for the latest advancements in the field.

There are 214 mathematicians from 21 different countries participating in the ICAAM-2024 conference this year. It is a great pleasure to welcome our 9 Keynote Speakers from 5 different countries who are the best scientists in the world in the field of Analysis and Applied Mathematics. It is truly a big event, and I would like to thank the organizing committee as well as all the scientists and administrative staff who have contributed to this conference for their tremendous work to make this conference a success.

As Bahçeşehir University, it is a great pleasure for us to be an organizing side of the 7-th ICAAM conference. As a higher education institution dedicated to teaching, research, and service to our society, the mission of Bahçeşehir University is to educate the leading workforce of future who have an inquiring mind and a critical thinking ability; are sensitive to local and global issues; achieve international standards; contribute to scientific, technological, and cultural knowledge; are strong supporters of universal ideas and values. Bahçeşehir University is a leading university representing Turkish higher education in the world with its over 25 thousand students, 20 % of which are international, over 30 thousand alumni all over the world, over 1000 faculty members, over 500 staff, 5 campuses in Istanbul and thousands of national and international collaborations with universities and business as the flagship of BAU Global network. According to UK-based international higher education rating agency Times Higher Education (THE), Bahçeşehir University became the 5-th university in Turkey and rose to the 601-800 band worldwide in the “2023 World University Ranking”.

Dear friends, I sincerely wish you very fruitful discussions over the next few days and I hope you will really enjoy your stay in Antalya. Thank you very much for your attention and as we say in Turkish “Kolay gelsin”.

Foreword

Allaberen Ashyralyev[1], Michael Ruzhansky[2], Makhmud Sadybekov [3]

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Dear Esteemed Guests and Participants,

On behalf of the Organizing Committee for the 7th International Conference on Analysis and Applied Mathematics (ICAAM 2024), we extend a warm welcome to you all. It is a pleasure to have you here in Antalya for this significant event.

We sincerely appreciate the diverse group of scholars and practitioners who have traveled from various countries and cities to join us. Your interest and participation are vital to the success of this conference.

The ICAAM conference serves as a premier platform for mathematicians engaged in analysis and applied mathematics to converge, exchange insights, and explore new trends. The interplay between applied mathematics and analysis fosters innovative research and opens new avenues for exploration. This conference aims to facilitate the exchange of recent advancements and original contributions across these dynamic fields.

Our previous ICAAM conferences have been hosted in Turkiye (2012, 2022), Kazakhstan (2014, 2016), and Northern Cyprus (2018, 2020). We are thrilled to welcome you back to Turkiye for this year's event. This year, we are honored to host 214 mathematicians from 21 different countries. The proceedings from past ICAAM conferences (2012-2022) were published in six volumes by the American Institute of Physics (AIP) Conference Proceedings and are indexed in the Scopus database. Selected papers have also been featured in special issues of renowned journals such as Filomat, Boundary Value Problems, Numerical Functional Analysis and Optimization, International Journal of Applied Mathematics, and the Bulletin of the Karaganda University-Mathematics. ICAAM 2024 is organized by:

- Bahçeşehir University, Turkiye
- Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
- Analysis PDE Center, Ghent University, Belgium

We are delighted to continue our collaboration with these esteemed institutions. Our joint efforts include:

- Weekly "Analysis and Applied Mathematics" seminars, both in-person and online.
- The international journal "e-Journal of Analysis and Applied Mathematics."

These seminars have fostered global collaboration and knowledge sharing, with 47 successful sessions held since their inception in 2022. Most of these sessions are available on YouTube, and a recent Springer publication features extended abstracts from these seminars, highlighting contributions from over 15 countries.

This five-day conference features 10 plenary lectures and 80 presentations, covering a broad spectrum of topics in analysis and applied mathematics. Highlights of ICAAM 2024 include:

- 214 participants from 21 countries.
- An Abstract Book for ICAAM 2024.
- Extended abstracts to be published in AIP Conference Proceedings.
- Selected full papers to be featured in special issues of journals such as Bulletin of the Karaganda University-Mathematics, Journal of Mathematical Sciences (SERIES A), and the e-Journal of Analysis and Applied Mathematics.

We believe your participation will greatly enhance the value of this conference, and we are committed to making your experience enriching and enjoyable. Should you need any assistance, please do not hesitate to contact us. Our heartfelt thanks go to: The Scientific Committee of ICAAM 2024 for their invaluable support. The distinguished speakers for their contributions and keynote addresses. The American Institute of Physics for including our conference materials in their prestigious series. The editors of Bulletin of the Karaganda University-Mathematics and Journal of Mathematical Sciences (SERIES A) for their special issues. Special acknowledgment to Abdullah S. Erdogan, Charyyar Ashyralyyev, Deniz Agirseven, Maksat Ashyraliyev, Yıldırım Özdemir, Özgür Yıldırım, Kheireddine Belakroum, Mesut Negin, Ezgi Soykan Urun, Ramil Salimov, and Akmuhammet Ashyralyyev for their invaluable assistance. We also extend our gratitude to the leadership of Bahçeşehir University, Istanbul; Institute of Mathematics and Mathematical Modeling, Almaty; and Analysis PDE Center, Ghent University for their support and sponsorship. Thank you once again for joining us and contributing to the success of ICAAM 2024. We look forward to a stimulating and productive conference.

Warm regards,

CHAIRS

Generalized q-statistical convergence of order α via ideal

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Abstract: In this paper we present a new and further general concept, namely, generalized $q-$ statistical convergence of order α via ideal. We focus on study some basic properties of this new summability method. Due to the presence of both ideals and the number α, the proofs do not seem to be exactly analogous to those of [7] or [3] and most importantly the study leaves some interesting open problems.

Keywords: Ideal, filter, I-statistical convergence of order α, q-statistical convergence

2020 Mathematics Subject Classification: 40F06, 40G01

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A priori estimates of solutions for the Vlasov–Poisson system and plasma confinement

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Abstract: This lecture is devoted to the second mixed problem for the Vlasov–Poisson system with external magnetic fielf in a half–space. The above problem models the kinetics of the high temperature plasma in a fusion reactor. If plasma reaches a boundary of the domain, it can lead to the destruction of the reactor. Therefore we study solutions of the above mentioned mixed problem such that the supports of density distribution functions for charged particles are located strictly inside domain. For this purpose we use an external magnetic field. We prove existence of global classical solution and obtain a priori estimate for electric field strength. This estimate allows to obtain sufficient conditions for external magnetic field in algebraic form, which garantee that the supports of density distribution functions are lying at a given distance from a boundary [1, 2].

This research was sponsored by the Ministry of Science and Higher Education of the Russian Federation (Megagrant, No. 075-15-2022-1115).

Keywords: a priori estimates, Vlasov–Poisson system, plasma confinement

2020 Mathematics Subject Classification: 35Q83, 35M32

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Hyperbolic equations with differential-difference operators with respect to spatial variables: case of symbols with alternating real parts

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Abstract: This report is devoted to multidimensional hyperbolic differential-difference equations of the kind

$$\frac{\partial^2 u}{\partial t^2} = \Delta u + au(x+h,t),$$

where a is a real parameter and $h := (h_1, \ldots, h_n)$ is a vector parameter.

We explicitly construct a three-parameter family of its infinitely smooth global solutions.

The fundamental novelty (cf. [1] and references therein) is that the real part of the symbol of the differential-difference operator (with respect to the spatial independent variables) of the investigated equation is allowed to change its sign.

This research was carried out at the expense of a grant of the Russian Science Foundation No. 24-11-00073.

Keywords: Hyperbolic equations, differential-difference operators, smooth solutions, global solutions

2020 Mathematics Subject Classification: 35L10, 35R10

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Two topologies in the restriction spaces

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We discuss the linear topological structure of the restriction spaces $C_\infty(K)$, where K is a compact subset of $[0,1]$. As a factor space of $C^\infty[0,1]$ it can be equipped with the corresponding quotient topology τ_1. On the other hand, $C_\infty(K)$ is a factor space of the Whitney space $\mathcal{E}(K)$, defining the quotient topology τ_2. We compare these topologies when K is a null sequence and analyze the linear topological properties of such spaces.

GENERAL SECTION

Hardy inequalities on cones of monotone functions on a measure space

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Abstract: Consider a Hausdorff topological space X with Borel measure ν. Let $\{\Omega(t)\}$ be a totally ordered family of open subsets of an open set Ω in X parameterized by $t \in [0, \infty)$. This family generates a partial order in the set of functions defined on Ω. We obtain Sawer-type bounds for a linear functional on cones of decreasing and increasing functions in weighted spaces. These results are further applied to obtain criteria of boundedness of averaging operators in those weighted spaces. Previously such results were available on R^n.

Keywords: Hardy operator, weighted inequality, Steklov operator, boundedness on cones of decreasing and increasing functions

2020 Mathematics Subject Classification: 26D10, 26D15

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Calibrating stochastic differential equation models for accurate cryptocurrency pricing

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Abstract: Investors and financial institutions are increasingly employing cryptocurrencies as an alternative means of exchange and an investment object worldwide. However, theoretical research on modeling cryptocurrency prices is very scarce unlike empirical research that is abounded. Recently, [1] introduced a stochastic differential model that could be useful for predicting cryptocurrency prices, especially in highly volatile and changing markets combined with low liquidity that characterize the market for these new financial instruments.

This paper aims at tackling the calibration of the model presented in [1] using real market data to make it more practical. It explores different calibration methods such as the maximum likelihood estimation (MLE), which finds the parameter values that make the observed data most probable. Other methods considered include the method of moments and the Bayesian inference. The relative performance of these different approaches are compared.

By providing a procedure that can adjust the model parameters for matching actual market behavior, the model can provide more reliable and precise cryptocurrency pricing. This calibration process ensures the model stays robust and relevant in the ever-changing and unpredictable cryptocurrency markets.

Keywords: Stochastic differential equations, numerical methods, calibration, cryptocurrency

2020 Mathematics Subject Classification: 62P05, 91G80,91-10

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Wavelet multistep methods for initial-value problems

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Abstract: In this paper we construct a family of wavelet-based multistep methods (WMSM) for solving initial value problems. The WMSM can be explicit or implicit and are based on Daubechies' scaling functions of order p. We make use of the approximation property of Daubechies' scaling functions where polynomials of degree less than or equal to $p-1$ are exactly generated by integral translates of the scaling function. We show that the local truncation error is $O(h^p)$. We also study the stability regions of the obtained WMSM. Numerical examples are considered to check the accuracy of the methods.

Keywords: Wavelets, Daubechies wavelets, multi-step methods, initial value problems

2020 Mathematics Subject Classification: 65L05, 65L06, 65L70, 65T60

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Dynamics of a population model in two-patch environment: effect of prey and predator dispersal

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Abstract: This paper presents a comprehensive analysis of a discrete-time prey-predator model in a homogeneous two-patch environment, incorporating prey and predator dispersal. The model is

$$\begin{aligned}
x_{n+1} &= x_n + rx_n\left(1-\frac{x_n}{K}\right) - \frac{\alpha x_n u_n}{h+x_n} + d_1(y_n - x_n),\\
u_{n+1} &= u_n + su_n\left(1-\frac{u_n}{L}\right) + \frac{\beta x_n u_n}{h+x_n} + d_2(v_n - u_n),\\
y_{n+1} &= y_n + ry_n\left(1-\frac{y_n}{K}\right) - \frac{\alpha y_n v_n}{h+y_n} + d_1(x_n - y_n),\\
v_{n+1} &= v_n + sv_n\left(1-\frac{v_n}{L}\right) + \frac{\beta y_n v_n}{h+y_n} + d_2(u_n - v_n),
\end{aligned}$$

with initial population $x_0 > 0,\ u_0 > 0,\ y_0 > 0$ and $v_0 > 0$.
We discuss equilibrium points and establish stability conditions. By independently varying prey and predator dispersal rates, we observe diverse phenomena such as bifurcations, quasi-periodicity, and chaos. Additionally, a two-parameter space analysis reveals intricate dynamics when both dispersal rates are simultaneously varied, showcasing complex phenomena and the emergence of organized periodic structures. This study contributes to the ongoing understanding of predator-prey interactions in spatially heterogeneous environments, shedding light on their intricate and dynamic nature.

Keywords: Dispersal, stability, Neimark-Sacker bifurcation, flip bifurcation, quasiperiodicity, maximum Lyapunov exponent.

2020 Mathematics Subject Classification: 37N25, 92D25, 39A28 , 65Q10

Normal subgroups of the unitary and general linear groups of certain C^*-algebras

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In this paper, we extend the results of Al-Rawashdeh for involutions, projections and unitary groups in [1] to the case of symmetries, idempotents and general linear groups. The type of a symmetry s is the element $[p]$ in $K_0(\mathcal{A})$, where $s = 1 - 2e$, $e \in \mathcal{I}(\mathcal{A})$, and $e \sim_s p$, for $p \in \mathcal{P}(\mathcal{A})$. As, $K_0(\mathcal{O}_n) \simeq \mathbb{Z}_{n-1}$, the type of a symmetry is associated with an integer. A symmetry s is called of even type, if $[p] = 2k$ and odd type, if $[p] = 2k + 1$. We show that a normal subgroup $\mathcal{N}$ of $GL(\mathcal{O}_n)$, $n < \infty$ contains all the symmetries, if,
1) $\mathcal{N}$ contains a symmetry of the type 1 (i.e. of type [1]), or
2) $\mathcal{N}$ contains a non-trivial symmetry, and $(n-1)$ is a prime number, or
3) $\mathcal{N}$ contains a non-trivial symmetry such that its type and $(n-1)$ are relatively prime.

Then, using M. Leen's result [2], we deduce that $GL_0(\mathcal{O}_n) \subseteq \mathcal{N}$. In the case of $\mathcal{O}_\infty$, we prove that if $\mathcal{N}$ contains a symmetry of odd type, then $\mathcal{N}$ contains all the symmetries of $\mathcal{O}_\infty$. Consequently, by Leen's result, we get that $GL_0(\mathcal{O}_\infty) \subseteq \mathcal{N}$.

Also, we show that if $\mathcal{N}$ is a normal subgroup of unitary groups of compact operator $\mathbb{K}$, which contains some certain type of involution, then $\mathcal{N}$ contains all the involutions of $\mathbb{K}$. Further, we prove that if $\mathcal{N}$ is a normal subgroup of general linear groups of compact operator $\mathbb{K}$, which contains some certain type of symmetry, then $\mathcal{N}$ contains all the symmetries of $\mathbb{K}$.

Keywords: Involutions, symmetries, unital C^*-algebras.

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A note on numerical solution of a delay hyperbolic problem with involution and Robin condition

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Abstract: This study is about investigation of a delay hyperbolic problem with involution and Robin conditions. The research determines the stability of the associated differential equation and presents a stable difference scheme along with stability estimates. Numerical results used to validate the theoretical findings are also given.

Keywords: delay hyperbolic equations, involution, Robin boundary condition, stability estimates, numerical solution

2020 Mathematics Subject Classification: 39A27, 65M06, 65M12, 65N06

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Diabetes model, study and control

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Abstract

This study introduces a compartmental-based mathematical model encompassing susceptible individuals, Insulin resistant individuals, diabetics without complications, and diabetics with major complications to examine evolution of the epidemic without and with interventions. The model consists of a system of four first-order nonlinear differential equations. We demonstrate that the solutions to the model exist and are positive. Local asymptotic stability of the diabetes-free and diabetes-endemic equilibrium points is derived as well as global behaviour of the model. Optimal control policy is studied and Numerical simulations are presented under various scenarios. These results reflect real-life situations, thereby validating the realism of the proposed model.

The mathematical model is given by

$$(1)\qquad \begin{cases} \dot{S}(t) = bS(t)\left(1 - \frac{S(t)}{K}\right) - (\alpha + r)\, S(t) \\ \dot{I}(t) = \alpha S(t) - (\beta + r)\, I(t) \\ \dot{D}(t) = \beta I(t) - (\gamma + r)\, D(t) \\ \dot{C}(t) = \gamma D(t) - (r + r_1)\, C(t) \\ S_0 = S(0), I_0 = I(0), D_0 = D(0), C_0 = C(0). \end{cases}$$

Here, S$(\cdot)$ denotes healthy individuals at time $t \in [0,T]$ $T > 0$; $I(\cdot)$ denotes individuals with insulin resistence, $D(\cdot)$ represents people with type2 diabetes without complications, and $C(\cdot)$ represents people with diabetes with sevear complications at the same time.

b is the constant influx of healthy people, K is the cariying capacity, α, β, γ are constant rates at which healthy people become insulin resistent, then type 2 diabetics without complications then diabetics with sever complications respectively. r_1, r is the naturel death considered the same for all classes of individuals and r_1 modelizes the death rate induced by diabetics complications strictely.

Initial conditions are strictly positive and given by : $S_0 = S(0), I_0 = I(0), D_0 = D(0), C_0 = C(0)$.1001[1, 2]

To fit biological constraints, it is reasonable to suppose that all parameters of the system are strictly positive and less than K.

Keywords: Mathematical modeling, Local and Global stability, Optimal Control.

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Two-dimensional inverse problem for a fourth-order hyperbolic equation with a complex-valued coefficient and involution

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Abstract: The work is devoted to the study of inverse problems for a differential equation of the type

$$u_{tt}(x,y,t) + u_{xxxx}(x,y,t) + \alpha \cdot u_{xxxx}(-x,y,t) + u_{yyyy}(x,y,t) + q(y)\,u(x,y,t) = f(x,y),\ (x,y,t) \in E, \tag{1}$$

where the real number $\alpha \neq \pm 1$, in the rectangular area $E = \{-1 < x, y < 1,\ 0 < t < T\}$. Equation (1) is nonlocal, since there is a dependence on the derivatives at the points x and $-x$.
Find a pair of functions $u(x,y,t)$ and $f(x,y)$ in the domain E satisfying equation (1), initial conditions

$$u(x,y,0) = \varphi(x),\ u_t(x,y,0) = 0,\ u(x,y,T) = \psi(x),\ x,y \in [-1,1], \tag{2}$$

and Dirichlet boundary conditions

$$u(-1,y,t) = u(1,y,t) = 0,\ u_{xx}(-1,y,t)\,u_{xx}(1,y,t) = 0,\ t \in [0,T], \tag{3}$$

$$u(x,-1,t) = u(x,1,t) = 0,\ u_{yy}(x,-1,t) = u_{yy}(x,1,t) = 0,\ t \in [0,T], \tag{4}$$

where $\varphi(x)$ and $\psi(x)$ are given sufficiently smooth functions.

Throughout this note we mainly use techniques from our works [1,2].

Theorem 2.1. *Let $|\alpha| < 1$ and the following conditions are satisfied:*
1) $\varphi(x),\ \psi(x) \in C^5[-1,1]$;
2) there exist positive numbers δ_0 such that $\cos\sqrt{\sigma_{nk}}T \le \delta_0 < 1$;
3) the derivatives of the functions $\varphi(x)$ and $\psi(x)$have the properties $\frac{d^j\varphi(\mp 1)}{dx^j} = 0$, $\frac{d^j\psi(\mp 1)}{dx^j} = 0,\ j = 0,2,4$.
Then there is a unique solution to the inverse problem (1)â“(3), which can be represented as a Fourier series

$$u(x,y,t) = \varphi(x) + \sum_{n,k=1}^{\infty} \frac{1-\cos\sqrt{\sigma_{nk}}t}{1-\cos\sqrt{\sigma_{nk}}T}(\psi_{nk} - \varphi_{nk})\,X_k(x)\,Y_n(y),$$

$$f(x) = \varphi_x^{IV}(x) + \alpha\varphi_x^{IV}(-x) + \varphi_y^{IV}(x) + q(y)\varphi(x) - \sum_{k=1}^{\infty}\frac{\varphi_{nk} - \psi_{nk}}{1-\cos\sqrt{\sigma_{nk}}T}\sigma_{nk}X_k(x)\,Y_n(y).$$

Keywords: fourth-order hyperbolic equations; inverse problem; eigenfunction; the Riesz basis; the Fourier method

2020 Mathematics Subject Classification: 35L35; 35N30; 35E99; 34L10

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Viktor Verbovskiy, On approximation of a membrane by a net of strings for Dirichlet's problem for Poisson's equation

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Abstract: In his doctoral thesis [1], O.M. Penkin considered Laplace's equation on a stratified set, namely, in Section 4.3 he showed the closeness of the spectrum of the boundary value problem, which describes the vibrations of a net of strings with sufficiently small cells, and the spectrum of the membrane.

This report is devoted to the closeness of a solution of Dirichlet's problem of Poisson's equation on a net of strings with square cells to a solution of Dirichlet's problem of Poisson's equation on a membrane, provided that the cells are sufficiently small.

In a rectangular domain $D \subseteq \mathbb{R}$ we consider Dirichlet's problem of Poisson's equation $\Delta u = -f$ with zero boundary condition:

Let $\mathcal{S}$ be a rectangular net of strings with square cells, $\mathcal{S} \subset D$ and the boundary points of the strings are on the border of D. We consider Dirichlet's problem for the equation $T \cdot \mathbb{\Delta}\, u = -f$ with zero boundary condition, where $\mathbb{\Delta}$ is the stratified Laplacian, that is

(1) $$T \cdot \sum_{i=1}^{4} u'_{v_i} = -f$$

for the points in which some strings intersect, where u'_{v_i} is the derivative along the string in the direction from the point, and

(2) $$T \cdot u''_{xx} = -f \text{ or } T \cdot u''_{yy} = -f$$

according to the case the point belongs to a horizontal string or a vertical one.

We prove that if the sides of the cells tends to 0, the solution of $T \cdot \mathbb{\Delta}\, u = -f$ on $\mathcal{S}$ converges to the solution of $\Delta u = -f$ on D.

Keywords: Poisson's equation, geometric graph, differential equation on a graph, stratified set

2020 Mathematics Subject Classification: 35J05

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On the stability of the stationary and perturbed degenerate equations

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Abstract: The aim of this research is to study the stability of the stationary degenerate equation $Ax'(t) = Bx(t)$; where A and B are bounded operators in Hilbert space $\mathcal{H}$. The achieved results are the generalization of the Liapounov theorem for the spectrum of the corresponding pencil. On the other hand, the establishment of the stability conditions for the stationary and perturbed degenerate equation:

$$Ax'(t) = (B + B(t))x(t), \quad x \in \mathcal{H}.$$

Keywords: Stability, pencil of operators, stationary degenerate equation, perturbed degenerate equation, spectral theory.

2020 Mathematics Subject Classification: 93D23, 34M46, 34K21

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Well-posed problem for differential operator on a two-dimensional sphere with cuts

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Abstract: This report is devoted to a punctured two-dimensional sphere connected to straight lines at the punctured point, expressed through the equation

$$-y''(x) + q(x)y(x) = \lambda y(x). \tag{1}$$

Here the equation describes the quantum mechanical scattering problem, $y(x)$ is the wave function describing the propagation of a particle along a line, $q(x)$ is the potential energy along a line, and λ is the energy of the particle. In this case, we solve a uniquely solvable problem: a solution to this equation in the space passing through a punctured two-dimensional sphere, with suitable initial and boundary conditions. The boundary conditions will depend on the specific geometry of the sphere and how the line intersects it [1]-[2]. Then we can consider the scattering of a particle along this line when interacting with potential energy $q(x)$, which may depend on the position on the line.

Keywords: Laplace-Beltrami operator, two-dimensional punctured sphere, well-posed problems, Green's functions for elliptic equations.

2020 Mathematics Subject Classification: 53C20, 58D15, 58C25

Funding: This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP22685565).

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Spectrum of the perturbed bi-harmonic operator in the cubic domain

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Abstract: The work found eigenvalues and eigenfunctions for one perturbed bi-harmonic operator

$$(\partial_x^4+\partial_y^4+\partial_z^4)u=\lambda^2(-\Delta)u \quad (1)$$

in cubic domain $\Omega=(0,l)^3\subset\mathbb{R}^3$ with Dirichlet conditions

$$u=\partial_{\vec{n}}u=0 \quad (2)$$

on the boundary of the cube $\partial\Omega$.

Theorem 1. *The spectral problem (1) and (2) has the following solution*

$$u_n(x,y,z)=X_n(x)Y_n(y)Z_n(z), \quad n\in\mathbb{N},$$

$$\lambda_n^2=\left(\frac{n\pi}{l}\right)^2,\ n\in\mathbb{N}\cap\{\text{odd}\},\ \ \lambda_n^2=\left(\frac{2\nu_{n/2}}{l}\right)^2,\ n\in\mathbb{N}\cap\{\text{even}\},$$

where $X_n(x)=\Phi_n(\zeta)_{\zeta=x},\ \ Y_n(y)=\Phi_n(\zeta)_{\zeta=y},\ \ Z_n(z)=\Phi_n(\zeta)_{\zeta=z}$*:*

$$\begin{cases}\Phi_n(\zeta)=\sin^2\frac{\lambda_n\zeta}{2},\ \ n\in\mathbb{N}\cap\{\text{odd}\},\\ \Phi_n(\zeta)=[\lambda_n l-\sin\lambda_n l]\sin^2\frac{\lambda_n\zeta}{2}-\sin^2\frac{\lambda_n l}{2}\left[\lambda_n\zeta-\sin\lambda_n\zeta\right],\ \ n\in\mathbb{N}\cap\{\text{even}\},\end{cases}$$

$\{\nu_k,\ k\in\mathbb{N}\}$ *be the positive roots of the equation* $\tan\nu=\nu$.

Keywords: Spectrum, perturbed bi-harmonic operator, cubic domain

2020 Mathematics Subject Classification: 35P05, 35P15, 47A75

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Estimates for the stiffnesses of co-axial elastic beams

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Abstract: We investigate the stiffnesses of a non-uniform beam of a coaxial structure made of isotropic materials. In our research, the stiffnesses are introduced by the the asymptotic homogenization method as described in [1, 2].

It is demonstrated, that the stiffnesses of a beam of a coaxial structure made of isotropic materials may be calculated using the formulas predicted engineering (Bernoulli-Euler) theory if and only if $\nu(x, y)x^{\mu}$ and $\nu(x, y)y^{\mu}$ ($\mu = 0, 1$) are harmonic functions. Here: x, y are coordinates in the cross-section of the beam, $\nu(x, y)$ is the local Poisson's ratio (the Young modulus does not influence this effect).

If $\nu(x, y)x^{\mu}$ and $\nu(x, y)y^{\mu}$ ($\mu = 0, 1$) are not harmonic function, the stiffnesses of the inhomogeneous beam always exceed the stiffnesses predicted by engineering (Bernoulli-Euler) theory. It means that the engineering stiffnesses are exact low boundaries for the actual stiffnesses of beam.

It is the case of interest if some $\nu(x, y)x^{\mu}$, $\nu(x, y)y^{\mu}$ are harmonic function while other are not harmonic.

Our numerical calculations demonstrate that the differences in the real and engineering stiffnesses can be very large [3]. The large difference is observed if negative Poisson's ratio [4] material(s) presents in the design of the beam.

Keywords: elastic beams, coaxial, homogenization, stiffnesses, estimates, Laplace equation, Poisson's ratio

2020 Mathematics Subject Classification: 74Q20, 35Q74

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Pseudo-differential, amplitude, adjoint, and transpose operators generated by the Dunkl operator

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Abstract: In this work, we consider pseudo-differential, amplitude, adjoint, and transpose operators on Schwartz spaces, generated by the Dunkl operators on the real line. We prove that these operators are continuous linear operators on Schwartz spaces.

Readers can become familiar with this work in detail in [1].

Keywords: The Dunkl operator, pseudo-differential operator, amplitude operator, adjoint operator, and transpose operator.

2020 Mathematics Subject Classification: Primary: 47G30, Secondary: 47A05

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Solutions of a second-order differential equation with a complex-valued coefficient and involution

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Abstract: The work is devoted to the study of the behavior of solutions to the equation

$$l_\alpha u \equiv -u''(x) + \alpha u''(-x) + q(x)\,u(x) = \rho^2 u(x), \; -1 < x < 1. \tag{1}$$

The asymptotic form of linearly independent solutions of the equation under study is established. Analogs of classical asymptotic Birkhoff-type representations are obtained for the fundamental system of solutions of an ordinary differential equation with respect to a spectral parameter at in sectors of the complex plane C.

Throughout this note we mainly use techniques from our works [1,2].

Theorem 2.2. *If the function $q(x)$ is continuous in the interval $[-1,1]$, then in the region $S_\nu,\ \nu = 0,1,\ \alpha_1 \geq \alpha_0$ and $S_\nu,\ \nu = 2,3,\ \alpha_0 \geq \alpha_1$ of the complex plane ρ, the equation*

$$-u''(x) + \alpha u''(-x) + q(x)\,u(x) = \rho^2 u(x)$$

it has two linearly independent solutions $u_0,\ u_1$ satisfying the relations

$$u_k(x) = \left(e^{\alpha_k \rho i x} + (-1)^k e^{-\alpha_k \rho i x}\right)\left[1 + O\left(\frac{1}{\rho}\right)\right],$$

$$\frac{\partial}{\partial x} u_k(x) = \alpha_k \rho i \left(e^{\alpha_k \rho i x} - (-1)^k e^{-\alpha_k \rho i x}\right)\left[1 + O\left(\frac{1}{\rho}\right)\right],\; k = 0,1,$$

where $\alpha_0 = \sqrt{\frac{1}{1+\alpha}},\ \alpha_1 = \sqrt{\frac{1}{1-\alpha}},\ \rho \in S_\nu$ with a sufficiently large $|\rho|$.

Keywords: Asymptotics, differential equation, involution

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Wellposedness of parabolic source identification problem with integral condition

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Abstract: In the papers [1],[2], source identification problems for parabolic equation with multipoint nonlocal condition were investigated. In the paper [3], the well-posedness of the time-nonlocal boundary value problems studied. In the paper [4], theorems on the stability difference schemes for approximate solution of the integral type time-nonlocal parabolic direct problems in a Hilbert space with self-adjoint positive definite operator were established. As well as numerical implemetations were carried out. The aim of present work is to study source identification problems for the parabolic equation with nonlocal integral condition. We give stability estimates for solution of source identification problem. Later, we apply obtained result to study multidimensional parabolic equation with unknown source.

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Keywords: Source identification problem, parabolic equation, well-posedness, stability estimates, nonlocal problem, integral condition

2020 Mathematics Subject Classification: 35M12

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Solvability of mixed problems for the wave equation with reflection of the argument

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Abstract: The paper studies existence and uniqueness of solution to mixed problems for the one-dimensional wave equation with involution

$$u_{tt}(x,t) = u_{xx}(x,t) - \alpha u_{xx}(x,t) - q(x)\,u(x,t)\,, \quad (x,t)\in\Omega, \tag{1}$$

with the initial data

$$u(x,0) = \varphi(x)\,, \quad u_t(x,0) = \psi(x)\,, \quad -1 \le x \le 1, \tag{2}$$

and periodic (anti-periodic) boundary conditions

$$u(-1,t) = \pm u(1,t)\,, \; u_x(-1,t) = \pm u(1,t)\,, \quad t \ge 0, \tag{3}$$

where $\Omega = \{-1 < x < 1, \; t > 0\}\,, \; 0 < |\alpha| < 1, \; q(x)$ is a complex-valued function. Equation (1) contains a linear transformation of the involution $f(f(x)) = x$.

The main result of the work is the following theorem.

Theorem 2.3. *Let the following conditions be satisfied:*
1) all eigenvalues of the spectral problem (1) and (3) are simple;
2) function $q(x) \in C^2[-1,1]$;
3) function $\varphi(x) \in C^4[-1,1]$ and functions $\varphi(x)\,, \; L_\alpha\varphi$ satisfy the conditions (3);
4) function $\psi(x) \in C^2[-1,1]$ and satisfy the conditions (3).
Then the mixed problem (1)-(3) has a unique solution.

Throughout this note we mainly use techniques from our works [1].

Keywords: Riesz basis, eigenfunction, eigenvalue problem, involution perturbation, wave equation

2020 Mathematics Subject Classification: 34B09, 34K06, 35L05, 35L20

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An inverse problem for a nonlinear parabolic equation

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Abstract: In this work, we discuss on unique solvability of the inverse problems for the following parabolic equation perturbed with p-Laplacian diffusion and damping

$$u_t - \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) = \gamma|u|^{m-2}u + h(t)u + f(x,t), \quad (x,t) \in Q_T, \tag{1}$$

which is equipped with initial and boundary conditions

$$u(x,0) = u_0(x), \ x \in \Omega, \tag{2}$$

$$u(x,t) = 0, \quad (x,t) \in \Gamma_T, \tag{3}$$

and the integral overdetermination condition

$$\int_\Omega u(x,t)\omega(x)dx = e(t), \quad t \in [0,T]. \tag{4}$$

Here Ω is a bounded domain in R^d, $d \geq 2$ with smooth boundary $\partial\Omega$, and $Q_T = \Omega \times (0,T)$ is a bounded cylinder with a lateral $\Gamma_T = \partial\Omega \times [0,T]$. The coefficient γ is a finite number such that it might be positive either negative. Exponents p and m are also given numbers such that

$$1 < p, m < infty.$$

The functions f, u_0, ω, and $e(t)$ are given while the functions u and the coefficient $h(t)$ are unknown, which need to find from (1)-(4).

Under suitable conditions on the data, we establish existence and uniqueness of weak generalized solutions of the inverse problem (1)-(4).

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Keywords: Inverse problem, parabolic equation, p-Laplacian, weak solution, existence and uniqueness.

2020 Mathematics Subject Classification: 35R30, 35K55, 35D30, 34A12

Fourier series multipliers in the generalized Haar system

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Abstract: This work is devoted to the study of the Fourier series multipliers in the generalized Haar system in Lebesgue and Lorentz spaces.

Let us consider the sequence $\lambda = \{\lambda^k_{l,h}\}^{\infty,\ \ m_k-1,\ \ p_{k+1}-1}_{k=1,\ \ l=0,\ \ \ \ h=1}$. Any sequence λ generates an operator Λ, called a multiplier, which is defined on polynomials in the generalized Haar system by the following way

$$\Lambda\left(\sum_{k=1}^{N}\sum_{l=0}^{m_k-1}\sum_{h=1}^{p_{k+1}-1} a^k_{l,h}(f)\chi^k_{l,h}(x)\right) = \sum_{k=1}^{N}\sum_{l=0}^{m_k-1}\sum_{h=1}^{p_{k+1}-1} \lambda^k_{l,h}a^k_{l,h}\chi^k_{l,h}(x),$$

here $\chi_n(x) := \chi^k_{l,h}(x)$ is the generalized Haar system, see e.g. [1] and [2].

Our aim is to describe the class of multipliers $m(L_{p,r} \to L_{q,s})$, as well as to find characteristics for a sequence to be a multiplier of Fourier series in the generalized Haar system in Lebesgue and Lorentz spaces.

A new theorem on sufficient conditions for the sequence to belong to the class $m(L_{p,r} \to L_{q,s})$ of multipliers of Fourier series with respect to the generalized Haar system is proved.

Keywords: Fourier coefficients, generalized Haar system, Fourier multiplier, Lorentz spaces.

2020 Mathematics Subject Classification: 42B15, 42B35

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Traveling wave speed and profile of a go or grow glioblastoma multiforme model

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Abstract: Glioblastoma multiforme (GBM) is a fast-growing and deadly brain tumor due to its ability to aggressively invade the nearby brain tissue. A host of mathematical models in the form of reactionâ“diffusion equations have been formulated and studied in order to assist clinical assessment of GBM growth and its treatment prediction. To better understand the speed of GBM growth and form, we propose a two population reactionâ“diffusion GBM model based on the â2dcgo or growâ2122 hypothesis. Our model is validated by in vitro data and assumes that tumor cells are more likely to leave and search for better locations when resources are more limited at their current positions. Our findings indicate that the tumor progresses slower than the simpler Fisher model, which is known to overestimate GBM progression. Moreover, we obtain accurate estimations of the traveling wave solution profiles under several plausible GBM cell switching scenarios by applying the approximation method introduced by Canosa.

Keywords: Glioblastoma, Go or grow, Partial differential equations, Traveling wave

2020 Mathematics Subject Classification: 92-10, 92B05, 37N25

Natural frequency of vibration for non-uniform beam

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Abstract: In this paper, the models of Euler–Bernoulli non-uniform beams are considered:

$$\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(EJ(x)\frac{\partial w(x,t)}{\partial x^2}\right)=0,\ 0<x<\ell,\ t>0, \tag{1}$$

The non-uniform of the beam in the model is described by two variable parameters/coefficients: bending stiffness and beam mass per unit length.

The goal of this study is to find conditions for the variable coefficients of bending stiffness and fixing types of a beam under which it is possible to establish the qualitative behavior of the eigenvalues and eigenfunctions by reducing to the consideration of a series of spectral problems with other fixing.

It is shown the influence of agreed symmetry of variable parameters on the spectral properties of problems. Qualitative results have been obtained for the symmetric equivalence (factorization of sets of eigenvalues and eigenfunctions) of eigenvalues of non-uniform beam with hinged-hinged fixation. The results have been verified experimentally and on the Maple computer package.

Throughout this note we mainly use techniques from our works [1]. The obtained results are extended several known results from [1,2].
Funding: The research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan, grant number AP19579114.

Keywords: Euler–Bernoulli beam; non-uniform beam; eigenvalue; symmetry; equivalence, spectral problem

2020 Mathematics Subject Classification: 35B06, 35G15

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O'Neil's inequality in local Morry spaces

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Abstract: Let $\lambda \in \mathbb{R}$, $0 < p, q \leq \infty$. The Local Morry Space

$$\|f\|_{LM^{\lambda}_{p,q}(\mathbb{T})} = \left(\int\limits_0^{\infty} \left(t^{-\lambda} \|f\|_{L_p(B_t(0))} \right)^q \frac{dt}{t} \right)^{1/q} < \infty. \tag{1}$$

where $B_t(0) = \{y \in \mathbb{R}^n : |y| \leq t\}$ is the open ball centered at the point $x = 0$ of radius $t > 0$, were introduced by Guliyev-Burenkov [1]. In this paper, we introduce spaces that generalize local Morrey spaces. We study the Riesz operators, which are dedicated in [2,3]. O'Neil-type inequalities are obtained for convolution operators in these spaces. The methods from [4] are used.

This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan (grant no. AP23488596).

Keywords: Morry space, convolution operator, Young-O'Neil's inequality, Riesz potential.

2020 Mathematics Subject Classification: 47A05, 47B06

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Functional differential equations with dilation and symmetry

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Abstract: This report is devoted to the Dirichlet problem in a plane bounded domain for a strongly elliptic second-order functional differential equation containing arguments' transformations of the form $x \mapsto x/p$ $(p > 0)$ and $x \mapsto -x$ in the highest derivatives. The study of solvability of the problem is based on the Gårding-type inequality for which necessary and sufficient conditions are obtained in algebraic form. Some approaches and results of [1] are used.

In the one-dimensional case, functional differential equations with rescaling model a variety of phenomena. The pantograph equation $\dot{y} = ay(\lambda t) + by(t)$ arises in astrophysics, theory of oscillations, biology, and a number of other areas. On the other hand, the theory of boundary value problems for elliptic differential-difference equations in domains, constructed in the works by A.L. Skubachevskii (1970–1980s), was motivated by applications to elasticity theory (elastic deformations of multi-layer plates and shells with corrugated filler) and elliptic problems with nonlocal conditions of the Bitsadze-Samarskii type. Later, many other applications of elliptic functional differential equations were discovered [2].

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Keywords: elliptic functional differential equation, rescaling, boundary value problem

2020 Mathematics Subject Classification: 35J15, 35J25

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New cubature formulas for Sobolev spaces W_q^α with a dominant mixed derivative

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Abstract: We present new cubature formulae for multivariate periodic functions f w.r.t each variable with period 1 and with some smoothness $\alpha > 0$

$$F_m(f;p) = \frac{1}{p^m} \sum_{\substack{|\mathbf{k}|_1=m \\ \mathbf{k}\in\mathbb{Z}_+^n}} \sum_{r_1=0}^{p^{k_1}-1} \cdots \sum_{r_n=0}^{p^{k_n}-1} \prod_{j=1}^{n-1} \frac{\sum_{l=1}^{p-1} e^{-\pi i\left(\frac{2lr_j}{p}+\varepsilon(k_j)\right)}}{(p-1)^{1-\varepsilon(k_j)}} f\left(\frac{r_1}{p^{k_1}}, \dots, \frac{r_n}{p^{k_n}}\right),$$

where $p > 1$ is prime, $m \in \mathbb{N}$, $\varepsilon(x) = 1$ if $x \geq 1$ and $\varepsilon(x) = 0$ else.

The new cubature formulae are exact for polynomials with frequencies from the corresponding hyperbolic cross, and the upper bound of the worst-case error is $O(N^{-\alpha}(\log N)^{(\alpha+1/\tilde{q})(n-1)})$ for periodic functions from Sobolev spaces W_q^α, where N—number of nodes, $1 \leq q \leq \infty$, $\tilde{q} = \min\{q, 2\}$, n—dimension.

Keywords: Cubature formulae, multivariate numerical integration, Sobolev space, periodic functions, a dominant mixed derivative.

Fundings: This research is supported by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan (grant AP23488613)

2010 Mathematics Subject Classification: 41A55, 41A63

Marcinkiewicz-type interpolation theorem for net spaces

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Abstract: This report is devoted to the interpolation properties of the net spaces $N_{p,q}(M)$. We prove some analogue of Marcinkiewicz's interpolation theorem. This theorem enables us to deduce the strong boundedness of linear operators in net spaces from their weak boundedness in net spaces with local nets.

Let $G = \{G_t\}_{t>0}$ be a local net. We define the net $F_{G,\Omega} = \bigcup_{x\in\Omega} G + x$, where $G + x = \{G_t + x\}_{t>0}$. The net $F_{G,\Omega}$ will be called the net generated by a local net G and a set $\Omega \subset \mathbb{R}^n$.

Example. Let $\Omega = \mathbb{R}^n$, $G = \{Q_t\}_{t>0}$ be the set of cubes centered at 0 with the length of the edge equal to t, then $F_{G,\Omega} = \{Q_t + x\}_{x\in\mathbb{R}^n}$ is the set of all cubes in $\mathbb{R}^n$.

Theorem 1 Let $\Omega \subset \mathbb{R}^n$, $G = \{G_t\}_{t>0}$ be a local net, $F_{G,\Omega} = \bigcup_{x\in\Omega} G + x$. Let $0 < p_0 < p_1 < \infty$ and $0 < q_0, q_1 \le \infty$, $q_0 \ne q_1$, $0 < \theta < 1$, $1 \le \tau \le \infty$,

$$\frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}, \qquad \frac{1}{q} = \frac{1-\theta}{q_0} + \frac{\theta}{q_1}.$$

If the following inequalities hold for a linear operator T and some $M_0, M_1 > 0$

$$\|Tf\|_{N_{q_i,\infty}(G+x)} \le M_i \|f\|_{N_{p_i,1}(G+x)}, \quad x \in \Omega, \quad i = 0, 1, \tag{1}$$

for all functions $f \in N_{p_i,1}(G + x), (i = 0, 1)$, then the following inequality holds

$$\|Tf\|_{N_{q,\tau}(F_{G,\Omega})} \le c M_0^{1-\theta} M_1^{\theta} \|f\|_{N_{p,\tau}(F_{G,\Omega})}, \tag{2}$$

for all functions $f \in N_{p,\tau}(F_{G,\Omega})$, where $c > 0$ depends only on the parameters $p_0, p_1, q_0, q_1, p, q, \tau, \theta$.

Throughout this note we mainly use techniques from work [1].

Keywords: interpolation spaces, net spaces, linear operators.

2020 Mathematics Subject Classification: 46B70

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Necessary and sufficient conditions for the existence of a solution to a discrete inverse problem for a hyperbolic equation

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Abstract: The paper considers a discrete inverse problem for a hyperbolic equation. Based on the Gelfand-Levitan method, using the methodology proposed in works [1,2], necessary and sufficient conditions for the existence of a solution to the inverse problem under study for a hyperbolic equation were obtained.

This research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP 19678469).

Keywords: Gelfand-Levitan method, discrete inverse problem, necessary and sufficient conditions, the existence of a solution, hyperbolic equation

2020 Mathematics Subject Classification: 65M32

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Reduction to block diagonal form for matrices with elements in a multivariate polynomial ring

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Abstract: In the polynomial approach pioneered by [3], matrices over $\mathbb{R}[s]$, $s \equiv d/dt$ are used to represent linear systems of ordinary differential equations. The theory of such systems can be regarded as more or less complete. For more general linear functional systems e.g. partial differential systems or delay-differential systems, the resulting system matrices have elements in a multivariate polynomial rings. The structure of such rings is not as rich as the single variable case making the extension of results from the single variable to the multivariate case often not possible. In this paper, we consider the following problem: let $D = K[x_1, \ldots, x_n]$ denote a commutative multivariate polynomial ring with indeterminates $x_1, \ldots, x_n$ over an arbitrary but fixed field K and $T \in D^{p \times p}$ in an upper-block triangular form:

$$T = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1m} \\ 0 & T_{22} & \cdots & T_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{mm} \end{bmatrix} \tag{1}$$

in which the matrices $T_{ii}, i = 1, 2, \ldots, m$ are square and have pair wise zero-coprime determinants. Then under what conditions the matrix T is unimodular equivalent over D to the block diagonal form

$$T_D = \begin{bmatrix} T_{11} & 0 & \cdots & 0 \\ 0 & T_{22} & \cdots & 0) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{mm} \end{bmatrix} \tag{2}$$

Keywords: Functional systems, Unimodular, Equivalence, Similarity.

2020 Mathematics Subject Classification: 34C41,13B25, 13F20

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On the quadratic proximity of eigenfunctions of "unperturbed" and "perturbed" differentiation operators on an interval

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Abstract: In the function space $W_2^1(-1,1)$, we consider the problem on eigenvalues of the differentiation operator on the interval

$$L_1 y = y'(t) = \lambda y(t), \quad -1 \le t \le 1, \tag{1}$$

$$y(-1) - y(1) = \lambda \int_{-1}^{1} \Phi(t) y(t) dt, \tag{2}$$

where $\Phi(t)$ is a function of bounded variation and $\Phi(-1) = \Phi(1) = 1$, λ is a complex number, spectral parameter.

Conjugate problem.

We consider

$$L_1^* v = v'(t) + \vec{\lambda}\Phi(t)v(-1) = \vec{\lambda}v(t), \quad -1 \le t \le 1, \tag{1*}$$

with boundary value condition

$$v(-1) = v(1), \tag{2*}$$

i.e. a loaded differentiation operator.

In the case, when $\Phi(t) = 0$, we have the "unperturbed" spectral problem:

$$L_0 y = y'(t) = \lambda y(t), \quad -1 \le t \le 1, \quad y(-1) = y(1). \tag{3}$$

The numbers $\lambda_k^0 = ik\pi$, $k = \pm 1, \pm 2, \pm 3, \ldots$ are eigenvaues, moreover, $\forall C > 0$, $y_{k_0}^0 = C \cdot e^{ik\pi t}$ are eigen functions of the "unperturbed" operator L_0, which form a complete orthonormal system and Riesz basis in the space $L_2(-1,1)$. If $\lambda = 0$, $y(t) = C \neq 0$ is an eigenvalue and eigenfunction of the operator L_1.

In [1], the following theorem is proved.

Theorem 1. *Let $\Phi(t)$ be a function of bounded variation and $\Phi(-1) = \Phi(1) = 1$. Then all eigenvalues of the "perturbed" operators L_1 and L_1^* belong to the strip $|Re\lambda| = x < k^1$ for some k^1, form countable set and have asymptotics $\lambda_k^{(1)} = ik\pi + \underline{\underline{O}}(1)$. As $k \to \infty$, the corresponding eigenfunctions are $y_k^{(1)}(t) \approx C \cdot e^{ik\pi t} \cdot e^{\varepsilon t}$, $\varepsilon \approx \underline{\underline{O}}(1)$, $\forall C > 0$.*

In this report, we prove that the systems $\left\{y_{k_0}^0(t)\right\}$ and $\left\{y_k^1(t)\right\}$ are quadratically close in $L_2(-1,1)$.

Lemma. *The system $\left\{y_k^1(t)\right\}$, consequently, and the system $\left\{v_k^1(t)\right\}$ forms a Riesz basis in the space $L_2(-1,1)$, but not orthonormal.*

Remark. *As an example, in a particular case, we can consider $\Phi(t) = \cos 2\pi t$, $\cos 2\pi = \cos(-2\pi) = 1$.*

Funding: Author is supported by the grant

Keywords: differentiation operator, eigenvalues, eigenfunctions, conjugate problem, quadratic proximity.

2020 Mathematics Subject Classification: 34B05, 34B09, 34L10, 34L15

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Numerical algorithms for solving the initial boundary problems for nonlocal time-fractional differential equations

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Abstract: This work is devoted to numerical solution of the initial boundary problem for the subdiffusion equation with nonlocal boundary conditions. Consider the following equation

$$\begin{aligned} &D_t^\alpha u(x,t) - u_{xx}(x,t) = f(x,t),\\ &u(x,0) = 0, \quad 0 \le x \le 1, \qquad (1)\\ &u_x(0,t) - u_x(1,t) - au(1,t) = 0, \quad u(0,t) = 0, \qquad 0 \le t \le T. \end{aligned}$$

To solve problem (1), we apply the method [1] for reducing problem (1) to solving two problems for functions $C(x,t)$ and $S(x,t)$, such as $u(x,t) = C(x,t) + S(x,t)$:

$$D_t^\alpha C(x,t) - C_{xx}(x,t) = f_0(x,t), \quad \begin{cases} C_x(0,t) - aC(0,t) = 0,\\ C_x(1,t) - aC(1,t) = 0, \end{cases} \qquad (2)$$

$$D_t^\alpha C(x,t) - S_{xx}(x,t) = f_1(x,t), \quad \begin{cases} S_x(0,t) = -C(0,1),\\ S_x(1,t) = -C(1,t), \end{cases} \qquad (3)$$

Problem (2) has the boundary condition of the third kind, and problem (3) have the conditions of the first kind. To solve these problems, we apply the difference scheme and Thomas algorithm for solving the SLAEs.

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Keywords: Caputo fractional derivative, nonlocal boundary conditions, initial boundary problem

2020 Mathematics Subject Classification: 35J25, 35R11 **References:**

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The symmetric space Orlicz-Kantorovich

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Abstract: Let B be a complete Boolean algebra, $Q(B)$ the Stone compact of B, and let $C_\infty(Q(B))$ be the commutative unital algebra of all continuous functions $x : Q(B) \to [-\infty, +\infty]$, assuming possibly the values $\pm\infty$ on nowhere-dense subsets of $Q(B)$. We consider the Orlicz-Kantorovich space $(L_\Phi(B, m), \|\cdot\|_\Phi) \subset C_\infty(Q(B))$ with the Luxembourg norm associated with an Orlicz function Φ and a vector-valued measure m, with values in the algebra of real-valued measurable functions. It is shown, that the space $(L_\Phi(B, m), \|\cdot\|_\Phi)$ is a symmetric Banach-Kantorovich space with the Fatou property.

Keywords: Banach-Kantorovich space,vector-valued measure, symmetric space, Orlicz function, Luxembourg norm, Orlicz-Kantorovich space, Fatou property.

2020 Mathematics Subject Classification: 46B04, 46B42, 46E30, 46G10.

Integrability conditions of multiple trigonometric series

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Abstract: This report is devoted to relation between integrability properties of functions of several variables and summability properties of their Fourier coefficients. In particular, we prove multidimensional Hardy-Littlewood type theorem. In [2], F. Moricz proved Hardy-Littlewood's theorem for functions of two variables.

Theorem 1.[2] *Let* $p > 1$ *and* $f(x,y) = \sum\limits_{m=1}^{\infty}\sum\limits_{n=1}^{\infty} a_{mn} \cos \pi m x \cos \pi n y$ *be an integrable on* $[0,\pi]^2$ *function, where the sequence* $\{a_{mn}\}_{m,n=1}^{\infty}$ *is a monotone in the sense of Hardy. Then there exist* $C_1, C_2 > 0$ *such that*

$$C_1 \|f\|_{L_p[0,\pi]^2} \leq \left(\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (mn)^{p-2} a_{mn}^p \right)^{\frac{1}{p}} \leq C_2 \|f\|_{L_p[0,\pi]^2}.$$

Multidimensional analogues of Hardy-Littlewood's theorem were obtained in [1], [3-4] (see also references therein). We prove new multidimensional analogue of Hardy-Littlewood's theorem for functions with general monotone Fourier coefficients and give comparison with some known results.

This work was supported by the Ministry of Science and Higher Education of the Republic of Kazakhstan (grant No AP22688236).

Keywords: Fourier series, multiple series, Lebesgue spaces, general monotonicity

2020 Mathematics Subject Classification: 42B05, 42B35 **References:**

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The approximation of functions of two variables of bounded p-variation by polynomials with respect to Haar system

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Abstract: In this thesis we introduce direct and inverse theorems for approximating a function of two variables by polynomials in the Haar system in the space $C_p[0,1]^2$, $1 \le p < \infty$.

Let $f(x,y)$ be defined on the closed square $[0,1]^2$ and $\tau = \xi \times \eta$, where $\xi = \{x_0 < x_1 < \dots < x_m = 1\}$, $\eta = \{y_0 < y_1 < \dots < y_n = 1\}$- be an arbitrary partition of $[0,1]^2$.Let $(1 \le \ p < \infty)$. The quantity

$$\aleph^p_{\xi,\eta}(f) = \left(\sum_{k=1}^{n} \sum_{l=1}^{m} |f(x_k, y_l) - f(x_{k-1}, y_l) - f(x_k, y_{l-1}) + f(x_{k-1}, y_{l-1})|^p \right)^{1/p}.$$

is called variational sum of order p of the function $f(x,y)$ with respect to the partitions τ.

The quantity

$$\omega_{1-1/p}(f, \delta_1, \delta_2) = \sup_{\substack{|\xi| \le \delta_1 \\ |\eta| \le \delta_2}} \aleph^p_{\xi,\eta}(f),$$

is called variational modulus of continity of order $1 - \frac{1}{p}$ of the function $f(x,y)$.
Here $|\xi| = \max\limits_{1 \le k \le n}(x_k - x_{k-1})$, $|\eta| = \max\limits_{1 \le l \le m}(x_l - x_{l-1})$ [1].

We say that $f \in BV_p[0,1]^2$,$1 \le \ p < \infty$, if $V_p\left(f, [0,1]^2\right) \equiv \omega_{1-1/p}(f,1,1) < \infty$, and if $f \in C_p[0,1]^2$, $1 < \ p < \infty$, if $\lim\limits_{\substack{\delta_1 \to 0 \\ \delta_2 \to 0}} \omega_{1-1/p}(f, \delta_1, \delta_2) = 0$.

Endowed with the norm $\|f\|_{BV_p} = max\left\{ \sup\limits_{(x,y)\in[0,1]^2} |f(x,y)|, V_p\left(f, [0,1]^2\right) \right\}$. This space is a Banach space. Let $h_n(x)$ is functions of the Haar system. For the case of $[0,1)^2$ we put $h_{m,n}(x,y) = h_m(x)h_n(y)$.

Denote by $E^h_{m,n}(f)_X$ the best approximation of $f \in X[0,1]^2$ by Haar polynomials of degree $\le m \times n$ $(m, n \in N)$ in the norm of $X[0,1]^2$, where $X[0,1]^2 = C_p[0,1]^2$, $1 < p < \infty$ or $X[0,1]^2 = BV_p[0,1]^2$, $1 \le p < \infty$

$$E_{m,n}(f)_X = \sup_{c_{ij}} \left\| f(x,y) - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \chi_i(x) \chi_j(y) \right\|_X.$$

Denote by $S^h_{m,n}(f)$ rectangular partial sum of Fourier series of f by Haar system. By $K_{\alpha,\beta,\gamma}$ we denote a constant whose value may be different at each occurrence.

Theorem 1. That $1 < p < \infty$ $f \in C_p[0,1]^2$, $m, n \in P$. Then

$$E^h_{m,n}(f)_{C_p} \le K_p \omega_{1-\frac{1}{p}}\left(f, \frac{1}{m}, \frac{1}{n}\right)$$

Theorem 2. That $1 < p < \infty$ $f \in C_p[0,1]^2$, $m, n \in P$. Then

$$\omega_{1-\frac{1}{p}}\left(f, \frac{1}{m}, \frac{1}{n}\right) \le K_p E^h_{m,n}(f)_{C_p}$$

.

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Keywords: Variational sum, Banach space, Haar system, variational modulus of continity

2020 Mathematics Subject Classification: 12X34, 56Y78

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On systems of fractional nonlinear partial differential equations

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Abstract: Our report is based on joint work [1] with our colleague: A. Mukhiddinova (Tashkent University of Information Technologies)).

The work considers a system of fractional order partial differential equations. The existence and uniqueness theorems for the classical solution of initial-boundary value problems are proved in two cases: 1) the right-hand side of the equation does not depend on the solution of the problem and 2) it depends on the solution, but at the same time satisfies the classical Lipschitz condition with respect to this variable and an additional condition which guarantees a global existence of the solution. Sufficient conditions are found (in some cases they are necessary) on the initial function and on the right-hand side of the equation, which ensure the existence of a classical solution. In previously known works, linear but more general systems of fractional pseudodifferential equations were considered and the existence of a weak solution was proven in the special classes of distributions. Other works consider systems of fractional differential equations in which each differential expression with respect to a spatial variable, unlike the equations considered in this work, has the same order.

Keywords: System of differential equations, fractional order differential equation, matrix symbol, classical solution, Mittag-Leffler function

2020 Mathematics Subject Classification: 35R11, 74S25

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Computational analysis of animal behavior in certain circumstances: integrating mathematical modeling and machine learning methods

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Abstract: This study explores the complexities of animal decision-making in T-maze environments by integrating computational modeling with machine learning techniques. Specifically, we focus on the binary decision-making process in T-mazes, analyzing how animals choose between two paths. Our research utilizes a mathematical model designed to capture the decision-making behavior of fish, providing analytical insights into their intricate behavioral patterns. Additionally, we employ advanced machine learning algorithms to analyze behavioral data from zebrafish and rats. The combination of these techniques yields high predictive accuracies, demonstrating the effectiveness of computational methods in decoding animal behavior in controlled experiments. This study enhances our understanding of animal cognitive processes and highlights the crucial role of computational modeling and machine learning in advancing behavioral science.

Keywords: Animal behavior, decision-making, computational modeling, solution, machine learning methods

2020 Mathematics Subject Classification: 92D50 00A71, 68U07, 91B06, 47H10

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Solution of nonlocal boundary value problems for the heat equation with a discontinuous coefficient in the case of two discontinuity points

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Abstract: The initial-boundary value problem for the heat equation with discontinuous coefficients is considered

$$\frac{\partial u_j}{\partial t} = k_j^2 \frac{\partial^2 u_j}{\partial x^2} \tag{1}$$

in domain $\Omega = \cup\Omega_j,\ \Omega_j = \{(x,t) : l_{j-1} < x < l_j, 0 < t < T\}$, where $(j = 1,\ 2,\ 3)$, with initial conditions

$$u(x,0) = \varphi(x), l_0 \leqslant x \leqslant l_3 \tag{2}$$

boundary conditions of the form

$$u_1(l_0,t) + e^{inm} u_3(l_3,t) = 0, \quad k_1 \frac{\partial u_1(l_0,t)}{\partial x} + e^{inm} k_3 \frac{\partial u_3(l_3,t)}{\partial x} = 0, \tag{3}$$

and pairing conditions

$$u_j(l_j - 0,t) = u_{j+1}(l_j + 0,t), \quad k_j \frac{\partial u_j(l_j - 0,t)}{\partial x} = k_{j+1} \frac{\partial u_{j+1}(l_j + 0,t)}{\partial x},\ j = 1,2. \tag{4}$$

the coefficients $k_j > 0,\ (j = 1,2,3)$. $m = 1,2$.

In the case without a discontinuity, the spectral theory of problems arising in the solution of such problems has been constructed almost completely. In [1], the heat equation with a discontinuous coefficient was considered under Sturm-type boundary conditions (separated boundary conditions), eigenvalues and eigenfunctions were found, and various special cases were investigated. In this paper, the spectral issues of problem (1)-(4) are investigated in the case of two discontinuity points with periodic and antiperiodic boundary conditions. The eigenvalues and eigenfunctions are found, and the existence and uniqueness theorem for the classical solution is proved.

Theorem. Let $\varphi(x)$ a twice continuously differentiable function satisfying the boundary conditions

$$\varphi(l_0) + e^{inm}\varphi(l_3) = 0, \quad k_1\varphi'(l_0) + e^{inm} k_3 \varphi'(l_3) = 0, \tag{5}$$

and pairing conditions

$$\varphi(l_j - 0) = \varphi(l_j + 0), \quad k_j\varphi'(l_j - 0) = k_{j+1}\varphi'(l_j + 0),\ j = 1,2. \tag{6}$$

then there is a unique classical solution to problem (1)-(4).

Keywords: Heat equation, Riesz basis, eigenvalues, eigenfunctions, spectral problem, classical solution

2020 Mathematics Subject Classification: 35J05, 35J08, 35J25 **References:**

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Solvability of nonlinear Goursat-Darboux type problem for hyperbolic equation with integral conditions

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Abstract: Let the function $y = y(t,x)$ be determined as the solution of the equation

$$y_{tx} = f(t,x,y), \ (t,x) \in Q \tag{1}$$

under the constraints

$$\begin{cases} \int\limits_0^T \chi(t) y(t,x)dt = \varphi(x), & x \in [0,l], \\ \int\limits_0^l m(x) y(t,x)dt = \varphi(x), & t \in [0,T], \end{cases} \tag{2}$$

where $Q = \{(t,x) : 0 \leq t \leq T, \ 0 \leq x \leq l\}$, $T, l > 0$ are the given numbers $y = (y_1, y_2, ..., y_n)^T$, is a state vector $f = (f_1, f_2, ..., f_n)$, $\varphi = (\varphi_1, \varphi_2, ..., \varphi_n)^T$ $\psi = (\psi_1, \psi_2, ..., \psi_n)^T$ are vector-function ; $x(t)$ and $m(x)$ are the prescribed matrices of dimension $n \times m$.

Under some conditions on the initial data, the theorem on the existence and uniqueness of the boundary value problem was proved.

The problem (1), (2) was also studied in the paper [1]

Keywords: integral conditions, non-local problem, Goursat-Darboux problem, existence and uniqueness

2020 Mathematics Subject Classification: 34B10, 34B15, 34B37

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On the R_ν-generalized solution for a biharmonic boundary value problem with singularity

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Abstract: Boundary value problems for biharmonic equations with corner singularity are used as mathematical models for problems in fracture mechanics. As is known, the presence of a re-entrant corner at the boundary of the domain leads to a significant decrease in the rate of convergence of the approximate solution of the numerical method. Previously, various numerical methods were developed to find an approximate solution without loss of accuracy [1]. We have created weight and weight vector finite element methods to find solutions with high accuracy for mathematical models with a singularity in electrodynamics, hydrodynamics and elasticity theory [2, 3, 3]. The solution to these problems was defined as an R_ν-generalized solution in the weighted Sobolev space.

This paper introduces the concept of an R_ν-generalized solution with mixed derivatives for a biharmonic boundary value problem in a domain with a re-entrant corner on the boundary. The existence and uniqueness of an R_ν-generalized solution in a special weight set is proven. The uniqueness of the solution is established for all parameters ν from a certain scale. The differential properties of the solution in the weighted space W^3_{2,ν^*} necessary to prove estimates of the rate of convergence of the approximate solution to the exact one is studied.

Keywords: boundary value problems with a singularity, weighted finite element method

2020 Mathematics Subject Classification: 35J40, 65N30

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Finite element method and computational experiments for biharmonic equation with singularity

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Abstract: Boundary value problems for a biharmonic equation in domains with re-entrant corners at the boundary are effectively used as mathematical models of fracture mechanics. A weak solution of such a problem belongs to the fractional Sobolev space. According to the principle of consistent estimates, the rate of convergence of an approximate solution by the finite element method has a fractional exponent relative to the mesh step [1]. Nonconforming Morley finite element method for finding an approximate solution of a boundary value problem with a biharmonic equation in an L-shaped domain is constructed. Computational experiments were carried out for two model problems with one and two singular components in solving the problem. A confirmation of theoretical estimates of the rate of convergence of an approximate solution to an exact one in the norms of Sobolev spaces is established.

To increase the rate of finding an approximate solution, an R_ν-generalized solution for the biharmonic problem with singularity will be introduced and a weighted finite element method will be constructed (see, for example, $[2, 3, 3]$).

Keywords: fracture mechanics, biharmonic equation with singularity, Morley finite element method

2020 Mathematics Subject Classification: 65N30, 35J40

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Continuity of linear dynamical systems

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Abstract: A general important question in both pure and applied mathematics is the continuity question. It is about what may happen with an object under interest when certain parameters change slightly. Will the object also change slightly, or will it change substantially? The question is especially relevant, for instance, in mathematical modelling as all measurements one takes in a modelling process are uncertain.

This report is devoted to the study of the continuity problem for linear dynamical systems, which was initiated by Nieuwenhuis and Willems in [3,4].

Linear dynamical systems, as it is known, are parameterized by means of polynomial matrices. Introducing appropriate topologies, it is shown that this parametrization becomes a quotient map.

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Keywords: Linear dynamical system, AR-model, Chern number, Grassmanian, polynomial matrix, polynomial module, exact sequence

2020 Mathematics Subject Classification: 93B25, 93C05, 93C20

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The criterion of invertibility of the minimal degenerate elliptic Gellerstedt operator

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Abstract: Let $\Omega \subset R^2$ is a bounded domain, lying at $y > 0$ with smooth boundary $\partial\Omega$. By L_0^+ we denote the closure of the Gellerstedt differential operator

$$Lu \equiv y^m u_{xx} + u_{yy} = f(x,y) \tag{1}$$

on the subset of functions $u \in C^\infty(\Omega)$, satisfying next conditions

$$u|_{\partial\Omega} = 0, \quad \left.\frac{\partial u}{\partial n}\right|_{\partial\Omega} = 0, \tag{2}$$

where $\frac{\partial u}{\partial n}$ is a conormal derivative. The following theorem is proved.

Theorem 1. For any $f \in L_2(\Omega)$ the problem (1)-(2) is solvable (the minimal Gellerstedt operator is invertible) if and only if next condition is met

$$u(x,y)|_{(x,y)\in\partial\Omega} = \left.\left(\int_\Omega \varepsilon(x,y,\xi,\eta) f(\xi,\eta)d\xi d\eta\right)\right|_{(x,y)\in\partial\Omega} = 0, \tag{3}$$

where

$$\varepsilon(x_0,y_0,x,y) = k \cdot \left(r_1^2\right)^{-\beta}(1-\sigma)^{1-2\beta}F(1-\beta,1-\beta;2-2\beta;1-\sigma), \tag{4}$$

$$\beta = \frac{1}{2(m+2)}, \quad k = \left(\frac{4}{m+2}\right)^{4\beta-2} \cdot \frac{\Gamma(\beta)}{\Gamma(1-\beta)\cdot\Gamma(2\beta)}, \quad \sigma = \frac{r^2}{r_1^2},$$

$$r^2 = (x-x_0)^2 + \frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}} - y_0^{\frac{m+2}{2}}\right)^2,$$

$$r_1^2 = (x-x_0)^2 + \frac{4}{(m+2)^2}\left(y^{\frac{m+2}{2}} + y_0^{\frac{m+2}{2}}\right)^2.$$

Keywords: overdetermined boundary value problem, Gellerstedt equation, criterion, minimal differential operator, hypergeometric function

2020 Mathematics Subject Classification: 35L80, 35M10, 35N30, 33C05

Solvability of fractional Kelvin-Voigt model

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Abstract: Let Ω be bounded domain in the space $\mathbb{R}^n$, $n = 2, 3$, with a smooth boundary $\partial\Omega$. We consider the following initial–boundary value problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_0 \Delta v - \mu_1 \frac{\partial \Delta v}{\partial t} - 2\mu_1 \mathrm{Div} \left(\sum_{i=1}^{n} v_i \frac{\partial \mathcal{E}(v)}{\partial x_i} \right) -$$

$$(1) \qquad -\frac{\mu_1}{\Gamma(1-\alpha)} \mathrm{Div} \int_0^t (t-s)^{-\alpha} \mathcal{E}(v)\big(s, z(s;t,x)\big)\, ds + \nabla p = f,$$

$$(2) \qquad z(\tau; t, x) = x + \int_t^\tau v\big(s, z(s;t,x)\big)\, ds, \quad t, \tau \in [0,T], \quad x \in \Omega,$$

$$(3) \qquad \mathrm{div}\, v(t,x) = 0, \quad t \in [0,T], \quad x \in \Omega,$$

$$(4) \qquad v|_{t=0} = v_0, \qquad v|_{[0,T]\times\partial\Omega} = 0.$$

Here, v is un unknown vector–valued velocity function, p is un unknown pressure, f is the external force, μ_0 is the fluid viscosity, $\mu_1 > 0$ is the retardation time, $\mathcal{E}(v) = (\mathcal{E}_{ij}(v))$, $\mathcal{E}_{ij}(v) = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$, $i, j = \overline{1, n}$, is the strain–rate tensor, $\Gamma(1-\alpha)$ is the Euler gamma function, $0 < \alpha < 1$.

This report is devoted to the weak solvability of initial–boundary value problem (1) – (4) with describes the motion of weakly concentrated aqueous polymer solutions (see [1] – [2]).

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Keywords: Existence theorem, weak solution, viscoelastic model, Kelvin–Voigt model, regular Lagrangian flow

2020 Mathematics Subject Classification: 35Q35, 76A05

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On the asymptotics of the attractors of the 2D Navier–Stokes systems in a medium with obstacles

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Abstract: Let $\Omega \subset \mathbb{R}^2$ be a smooth bounded domain. Let 's introduce the notation

$$\Upsilon_\varepsilon = \left\{ r \in \mathbb{Z}^2 : \operatorname{dist}(\varepsilon r, \partial\Omega) \geq \sqrt{2}\varepsilon \right\}, \quad \square \equiv \left\{ \xi : -\frac{1}{2} < \xi_i < \frac{1}{2}, i = 1, 2 \right\}.$$

By specifying a 1-periodic ξ smooth function $F(x,\xi)$ such that $F(x,\xi)|_{\xi\in\partial\square} \geq \text{const} > 0$, $F(x,0) = -1$, $\nabla_\xi F \neq 0$ by $\xi \in \square \setminus \{0\}$, we define

$$G_\varepsilon^r = \left\{ x \in \varepsilon(\square + r) \mid F\left(x, \frac{x}{\varepsilon}\right) \leq 0 \right\}, \qquad G_\varepsilon = \bigcup_{r\in\Upsilon_\varepsilon} G_\varepsilon^r$$

and the perforated domain: $\Omega_\varepsilon = \Omega \setminus G_\varepsilon$. We will study the asymptotic behavior of the trajectory attractors of the following initial boundary value problem for an autonomous 2D system of Navier–Stokes equations (see [1])

$$(1) \qquad \begin{cases} \dfrac{\partial u_\varepsilon}{\partial t} - 2\nu \operatorname{div} \overline{e}(u_\varepsilon) + (u_\varepsilon, \nabla)u_\varepsilon + \nabla p_\varepsilon = g\left(x, \dfrac{x}{\varepsilon}\right), & x \in \Omega_\varepsilon, \\ (\nabla, u_\varepsilon) = 0, & x \in \Omega_\varepsilon, \\ \sigma_n^\varepsilon + \alpha_n \varepsilon (u_\varepsilon, n) = 0, & x \in \partial G_\varepsilon, t > 0, \\ \sigma_\tau^\varepsilon + \alpha_\tau \varepsilon (u_\varepsilon, \tau) = 0, & x \in \partial G_\varepsilon, \\ u_\varepsilon = 0, & x \in \partial\Omega \\ u_\varepsilon = U(x), & x \in \Omega_\varepsilon, t = 0. \end{cases}$$

Here $p_\varepsilon(x,t)$ is a pressure, $u_\varepsilon = u_\varepsilon(x,t) = (u_\varepsilon^1, u_\varepsilon^2)$, $2\overline{e}(u_\varepsilon) = \nabla u_\varepsilon + (\nabla u_\varepsilon)^T$, $g_\varepsilon(x) = g\left(x, \frac{x}{\varepsilon}\right) = (g^1, g^2) \in \left(L_2(\Omega;\mathbb{R})\right)^2$, $\sigma_n^\varepsilon(u_\varepsilon, p_\varepsilon) = -p_\varepsilon + \nu((\overline{e}(u_\varepsilon)n), n)$, $\sigma_\tau^\varepsilon(u_\varepsilon) = \nu((\overline{e}(u_\varepsilon)n), \tau)$, $n = (n_1, n_2)$ is the vector of the unit external normal to the boundary. It is shown that the trajectory attractors of problem (1) converge in a weak topology to the trajectory attractors of the homogenized problem defined in the domain without obstacles, as $\varepsilon \to 0$.

Keywords: attractors, 2D Navier-Stokes system, perforated domain.

2020 Mathematics Subject Classification: 35B20, 35B27.

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The Cartesian products of algebras with genetic realization

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Abstract: One of the important notions of the mathematical theory of genetics is that of an algebra with genetic realization [3], [2]. It is defined as a finite-dimensional (not necessarily associative or commutative) algebra over the field of real or complex numbers that has a basis $e_1, ..., e_n$ such that the structural constants γ_{ijk} $(i, j, k = 1, ..., n)$ of the algebra relative to this basis, i.e., the coefficients in the decompositions

$$e_i e_j = \sum_{k=1}^{n} \gamma_{ijk} e_k, \tag{1}$$

are real and satisfy the following conditions:

$$0 \leq \gamma_{ijk} \leq 1 \quad (i, j, k = 1, ..., n); \tag{2}$$

$$\sum_{k=1}^{n} \gamma_{ijk} = 1 \quad (i, j, = 1, ..., n). \tag{3}$$

In complex genetic situations the problem of constructing new algebras from a number of particular algebras arises. In the present work, we find sufficient conditions for the Cartesian product of two algebras with genetic realization to be such an algebra. Further, we consider the problem of the existence of finite limits in the category of algebras with genetic realization.

The results of this work are presented in our paper [3].

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Keywords: algebra with genetic realization, natural basis, baric algebra, genetic algebra, Cartesian product of algebras.

2020 Mathematics Subject Classification: 17D92, 18A30, 92D10

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An efficient implementation of the conjugate gradient method with a non-polynomial spline and applications

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Abstract: In this work non-polynomial spline function is proposed to solve a linear fractional differential equation where the derivatives are in the Caputo sense. This method transforms the fractional differential equation to a system of linear equations.

$$C_1p_{i+1} + C_2p_i + C_3y_{i+1} + C_4y_i + C_5p_{i-1} + C_6y_{i-1} = 0 \tag{1}$$

using equation(2),and applying difference formula for $y''_{i+1}, y''_i, y''_{i-1}$ we obtain

$$y^{\alpha} + \phi(x)y'' + \psi(x)y = \tau(x), \qquad x \in [a, b] \tag{2}$$

$$\begin{aligned} p_{i+1} &= -\phi_{i+1}(x)y''_{i+1} - \psi_{i+1}(x)y_{i+1} + \tau_{i+1}(x) \\ p_i &= -\phi_i(x)y''_i - \psi_i(x)y_i + \tau_i(x) \\ p_{i-1} &= -\phi_{i-1}(x)y''_{i-1} - \psi_{i-1}(x)y_{i-1} + \tau_{i-1}(x) \end{aligned} \tag{3}$$

The Gauss-Seidel and conjugate gradient methods were used to iteratively solve the linear system. Finally to validate the method's accuracy, some numerical examples with known analytical solutions have been tested. according to the numerical experiments, the findings are in satisfactory agreement with the exact solution.

Keywords: Fractional calculus, spline approximation, conjugates function, gradient method.

2020 Mathematics Subject Classification: 26A33; 41A15, 42A50, 90C52

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Investigation of the dynamics of the orbits of the composition of quadratic mappings

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Abstract: In the work, we consider the composition of two quadratic Lotka – Volterra mappings acting in S^3 and having the form:

$$V_1 : \begin{cases} x_1' = x_1(1 - \alpha x_2 + \eta x_3 + \delta x_4), \\ x_2' = x_2(1 + \alpha x_1 - \beta x_3 + \gamma x_4), \\ x_3' = x_3(1 - \eta x_1 + \beta x_2 - \varsigma x_4), \\ x_4' = x_4(1 - \delta x_1 - \gamma x_2 + \varsigma x_3). \end{cases} \tag{1}$$

$$V_2 : \begin{cases} x_1' = x_1(1 + \alpha x_2 + \eta x_3 + \delta x_4), \\ x_2' = x_2(1 - \alpha x_1 - \beta x_3 + \gamma x_4), \\ x_3' = x_3(1 - \eta x_1 + \beta x_2 - \varsigma x_4), \\ x_4' = x_4(1 - \delta x_1 - \gamma x_2 + \varsigma x_3). \end{cases} \tag{2}$$

The composition of these mappings can be represented as:

$$V : \begin{cases} x_1' = x_1(1 + \alpha x_2 + \eta x_3 + \delta x_4)(1 - \alpha x_2(1 - \alpha x_1 - \beta x_3 + \gamma x_4)+ \\ +\eta x_3(1 - \eta x_1 + \beta x_2 - \varsigma x_4) + \delta x_4(1 - \delta x_1 - \gamma x_2 + \varsigma x_3)), \\ x_2' = x_2(1 - \alpha x_1 - \beta x_3 + \gamma x_4)(1 + \alpha x_1(1 + \alpha x_2 + \eta x_3 + \delta x_4)- \\ -\beta x_3(1 - \eta x_1 + \beta x_2 - \varsigma x_4) + \gamma x_4(1 - \delta x_1 - \gamma x_2 + \varsigma x_3)), \\ x_3' = x_3(1 - \eta x_1 + \beta x_2 - \varsigma x_4)(1 - \eta x_1(1 + \alpha x_2 + \eta x_3 + \delta x_4)+ \\ +\beta x_2(1 - \alpha x_1 - \beta x_3 + \gamma x_4) - \varsigma x_4(1 - \delta x_1 - \gamma x_2 + \varsigma x_3)), \\ x_4' = x_4(1 - \delta x_1 - \gamma x_2 + \varsigma x_3)(1 - \delta x_1(1 + \alpha x_2 + \eta x_3 + \delta x_4)- \\ -\gamma x_2(1 - \alpha x_1 - \beta x_3 + \gamma x_4) + \varsigma x_3(1 - \eta x_1 + \beta x_2 - \varsigma x_4)), \end{cases} \tag{3}$$

where $0 \leq \alpha, \beta, \gamma, \eta, \delta, \varsigma \leq 1$, and $V = V_1 \circ V_2$.

The relevance of studying the dynamics of the considered compositions lies in the fact that it is fully capable of describing the dynamics of virus programs that can work in different operating systems simultaneously for replicating viruses and for the case when viruses are of a replicative nature [1].

For the considered compositional mapping, rest points are found, their cards are constructed, the characters of fixed points are determined by analyzing the Jacobian spectrum, and the basins of attractors are determined. In the paper, we applied the methods from the works [2], [3].

Keywords: Lotka – Volterra mapping, composition, fixed point, eigenvalues, fixed point, basins of attractor

2020 Mathematics Subject Classification: 37B25, 37C25, 37C27 **References:**

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A discrete balance model in ecology

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Abstract: The balance equations in ecology describing the movement of biomass and energy flows between the main components of terrestrial systems are based on the laws of conservation of mass and energy. The paper considers a discrete balance model in the form of a discrete Lotka – Volterra dynamical system with a degenerate skew-symmetric matrix acting in a four-dimensional simplex having the form:

$$
(1) \qquad \begin{cases} L^{(m+1)} = L^{(n)}(1 - aP^{(n)} + bS^{(n)}), \\ P^{(n+1)} = P^{(n)}(1 + aL^{(n)} - eD^{(n)} - kC^{(n)}), \\ S^{(n+1)} = S^{(n)}(1 - bL^{(n)} + lD^{(n)}), \\ D^{(n+1)} = D^{(n)}(1 + eP^{(n)} - lS^{(n)} + fC^{(n)}), \\ C^{(n+1)} = C^{(n)}(1 + kP^{(n)} - fD^{(n)}). \end{cases}
$$

Here, the components of the community are defined as follows:

Producers with biomasses P are mainly green plants capable of converting the light energy L into their own biomass and using simple substances such as substrates S for food. In turn, the substrates are products of the vital activity of the enzymes C. Consumers are animals that feed on producers and other organisms and microorganisms. Also here $0 \leqslant a, b, e, l, k, f \leqslant 1$ and

$$S^4 = \{x = \{L, P, S, D, C\} \in R^5 : L + P + S + D + C = 1, L, P, S, D, C \geqslant 0\}.$$

For the considered model, using the construction of the Lyapunov function, as well as by examining the Jacobian spectrum, the characters of the rest points are determined and the basins of attractors are found. In the paper, we apply the methods from our next works [1], [2].

Keywords: A discrete Lotka – Volterra dynamical system, Lyapunov function, fixed point, the Jacobian spectrum, basins of attractor, producer, substrate, consumers

2020 Mathematics Subject Classification: 37B25, 37C25, 37C27 **References:**

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One mathematical model of the glucose-insulin interaction and feedback loop

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Abstract: One mathematical model of the interaction of glucose and insulin is presented in the form of a nonlinear system of ordinary differential equations. This model takes into account the phenomenon that rapid changes in glucose levels concentration in the blood lead to an increase in secretory take into account the dose of insulin. This provides the base to analyze the glucose-insulin feedback loop mathematically.

Keywords: mathematical model, critical points, nonlinear systems of ordinary differential equations, glucose, insulin, blood lead, feedback loop

2020 Mathematics Subject Classification: 37C75

Robin boundary value problem for higher order equations in the upper half plane

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Abstract: The Dirichlet and the Neumann problems are basic boundary value problems in complex analysis. The Dirichlet problem is related to the Riemann jump problem. The Neumann problem is connected to the Dirichlet problem. Conditions of Robin problem on the boundary of the given domain is a linear combination of Dirichlet and Neumann conditions on the same domain.

The aim of this study is to solve Robin boundary value problem for higher order equations in the upper half plane. Following the idea given by Begehr [1], we reduce the problem into the boundary value problems for first order complex differential equations. Here, solution of inhomogeneous Cauchy- Riemann eqaution $w_{\bar{z}} = f(z)$ in the upper half plane plays a crucial role in solving the main problem.

Keywords: Robin boundary value problem, homogeneous higher order equation, inhomogeneous higher order equation

2020 Mathematics Subject Classification: 30E25, 35C15,32A30

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The model of contact erosion at a nonstationary arc spot

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Abstract: This article presents a mathematical model of arc development and instability in low-current inductive electric circuits. Arc instability refers to the displacement of the arc spot along the contact surface, as well as the possible transition of an arc discharge into a smoldering discharge. The instability of the arc helps to reduce the erosion of the electrode.

The mathematical model analyzing the instability of the arc, which is proposed in the report, includes the characteristics of an open electric circuit (inductance, capacitance, resistivity) and the Stefan thermal problem for an electrode with three adjacent regions of different phase states – vaporous, liquid and solid.

The task is to find criteria and the optimal choice of interdependent parameters (material properties, current, voltage, resistance, inductance, pressure, opening rate, etc.) that ensure the instability of the arc.

The analysis of the model solutions was carried out and the following conclusions were obtained:

A) The immobility of the arc spot is due to the dynamics of contact evaporation and the specific resistance of the arc, depending on the vapor density in the arc.

B) The model allows you to find the time and critical distance between the contacts at the point of immobility of the arc spot.

C) The thermal model based on the Stefan problem allows us to find the dynamics of the power, temperature, radius and resistance of the arc necessary to calculate the parameters of the immobility of the arc.

Keywords: mathematical model, Stefan problem, electrical contacts, arc erosion

2020 Mathematics Subject Classification: 34A34, 35A24, 45D05, 80A22

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Combination of Hadamard derivative and boundary random fractional differential equations

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Abstract: We study the existence of solutions for random system of fractional differential equations with boundary nonlocal initial conditions. Our approach based on random fixed point principles of Schaefer and Perov, combined with a vector approach that uses matrices that converge to zero. We prove existence and uniqueness results for these systems. Some examples are presented to illustrate the theory. In this paper, we consider the systems of random fractional differential equations with boundary conditions in the following form:

$$(1)\qquad \left\{\begin{array}{ccc} D^{\alpha}(D^{\beta}+\lambda_1)\,x\,(t,\omega) & = & f(t,\,x(t,\omega),\,y(t,\omega),\omega) \\ D^{\gamma}(D^{\sigma}+\lambda_2)\,y\,(t,\omega) & = & g(t,\,x(t,\omega),\,y(t,\omega),\omega) \\ i=1]m\sum\theta_i I^{\mu_i}x(\eta_i,\omega) & = & j=1]n\sum\phi_j I^{\gamma_j}x(\xi_j,\omega) \\ k=1]p\sum\varepsilon_k I^{\varsigma_k}x(\psi_k,\omega) & = & l=1]q\sum\nu_l I^{\tau_l}x(\varphi_l,\omega) \\ i=1]m\sum\overline{\theta_i} I^{\overline{\mu_i}}y(\overline{\eta_i},\omega) & = & j=1]n\sum\overline{\phi_j} I^{\overline{\gamma_j}}y(\overline{\xi_j},\omega) \\ k=1]p\sum\overline{\varepsilon_k} I^{\overline{\varsigma_k}}y(\overline{\psi_k},\omega) & = & l=1]q\sum\overline{\nu_l} I^{\overline{\tau_l}}y(\overline{\varphi_l},\omega) \end{array}\right.$$

where D^{ρ} denotes the Hadamard Caputo-type fractional derivative of order ρ, $\rho \in \{\alpha,\beta,\gamma,\sigma\}$ with $0<\alpha,\beta,\gamma,\sigma<1, 1<\alpha+\beta<2, 1<\gamma+\sigma<2$, λ_1,λ_2 are given constants, I^{r} is the Hadamard fractional integral of order $r>0$, $r\in\{\mu_i,\,\gamma_j,\,\varsigma_k,\,\tau_l,\,\overline{\mu_i},\,\overline{\gamma_j},\,\overline{\varsigma_k},\,\overline{\tau_l}\}$ the constants $\eta_i,\,\xi_j,\,\psi_k,\,\varphi_l,\,\overline{\eta_i},\,\overline{\xi_j},\,\overline{\psi_k},\,\overline{\varphi_l}\in(1,\,e)$ and $\theta_i,\,\phi_j,\,\varepsilon_k,\,\nu_l,\,\overline{\theta_i},\,\overline{\phi_j},\,\overline{\varepsilon_k},\,\overline{\nu_l}\in\mathbb{R}$, for all $i=1,2,\ldots,m$, $j=1,2,\ldots,n$, $k=1,2,\ldots,p$, $l=1,2,\ldots,q$ and $f,\,g:[1,e]\times\mathbb{R}^m\times\mathbb{R}^m\times\Omega\to\mathbb{R}^m$, $(\Omega,\mathcal{A})$ is a measurable space.

Keywords: Random fractional differential equation, Hadamard fractional differential equation, existence, fixed point, vector metric space

Mathematics Subject Classification: 34A08, 47B80, 34F05

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Equivalence of several generalized singular number inequalities of τ-measurable operators

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Abstract: In [1], the authors provided some generalized singular number inequalities for products and sums of τ-measurable operators. They also proved some related arithmetic-geometric mean and Heinz mean inequalities for a generalized singular number of τ-measurable operators and demonstrated their equivalence. In this report, we prove some results from [1] for the case where τ-measurable operators are self-adjoint. In [2], the authors established some singular value inequalities for matrices of 2×2 compact operators on a complex separable Hilbert space. As an application of our results, we extend the findings in [2] to the generalized singular number case. We also prove the equivalence of the corresponding inequalities and some symmetric norm inequalities.

Keywords: Generalized singular number, τ-measurable operator, semi-finite von Neumann algebra, noncommutative symmetric space.

2020 Mathematics Subject Classification: 46L52, 47L05

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An interpolation theorem for Lebesgue and Lorentz spaces with mixed metrics

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Abstract: Let $1 \le \bar{q} = (q_1, q_2) \le \infty$, $\bar{\varphi}(t) = (\varphi_1(t), \varphi_2(t)) \ge 0$. Let

$$\Lambda_{\bar{q}}(\bar{\varphi}) := \left\{ f : \left(\int_0^{+\infty} \left(\int_0^{+\infty} \left(f^{*_1 *_2}(t_1, t_2) \varphi_1(t_1) \varphi_2(t_2) \right)^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}} < \infty \right\},$$

where $f^{*_1 *_2} = f^{*_1 *_2}(t_1, t_2)$ is the nonincreasing permutation of a function f [1]. In the paper [2] were studied one-dimensional generalized Lorentz spaces.

In this work the interpolation theorem for Lebesgue and Lorentz spaces with mixed metrics are obtained.

Let $\delta > 0$ and $\varphi(t)$ be nonnegative function on $[0, +\infty)$. Let $C_\delta = \{\varphi(t) : \varphi(t) t^{-\delta}$ is an increasing function and $\varphi(t) t^{-1+\delta}$ is a decreasing function$\}$. The class C is defined as follows: $C = \bigcup_{\delta > 0} C_\delta$.

Theorem 2.4. *Let* $0 < \bar{p}_0 = (p_1^0, p_2^0) < \bar{p}_1 = (p_1^1, p_2^1) < \infty$, $1 \le \bar{q} = (q_1, q_2) \le \infty$, $\gamma_i = \frac{1}{p_i^0} - \frac{1}{p_i^1}$, $i = 1, 2$, $\varphi_1, \varphi_2 \in C$. *Then the following inequality is true*

$$\left(L_{\bar{p}_0}, L_{\bar{p}_1} \right)_{\bar{\varphi}, \bar{q}} = \Lambda^{\bar{q}} \left(\bar{\psi} \right),$$

where $\bar{\psi}(t_1, t_2) = \left(\frac{t_1^{\frac{1}{p_1^0}}}{\varphi_1(t_1^{\gamma_1})}, \frac{t_2^{\frac{1}{p_2^0}}}{\varphi_2(t_2^{\gamma_2})} \right)$.

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Keywords: Lebesgue and Lorentz spaces, interpolation methods, interpolation theorem

2010 Mathematics Subject Classification: 46B70, 46E30

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Inverse problem for parabolic equation with p-Laplacian and damping term

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Let Ω be a bounded domain in $\mathbb{R}^d$ with smooth boundary $\partial\Omega$, and $Q_T = \{(x,t): \ x \in \Omega, \ 0 < t \leq T\}$ is a cylinder with lateral Γ_T. In this work, we study the following inverse problem of determinig the pair of the functions $(u(x,t), f(t))$, which satisfy parabolic equation with $p-$Laplacian and damping term

$$u_t - div\ \left(|\nabla u|^{p-2}\nabla u\right) = \gamma|u|^{m-2}u + f(t)g(x,t) \ \text{ in } \ Q_T, \tag{1}$$

the initial condition

$$u(x,0) = u_0(x) \text{ in } \Omega, \tag{2}$$

the Dirihlet boundary condition

$$u(x,t) = 0 \ \text{ on } \ \Gamma_T \tag{3}$$

and overdetermination condition

$$\int\limits_{\Omega} u(x,t)\omega(x)dx = h(t), \ t \in [0,T], \tag{4}$$

Here the coefficient γ might be positive $\gamma > 0$ either negative $\gamma < 0$. The functions $u_0(x), h(t), \omega(x)$ and $g(x,t)$ are given. The exponents p and m are given positive numbers, such that

$$1 < \ p, \ m < \infty.$$

In this talk, we prove the existence and uniqueness of weak solution above possed problem (1)-(4).

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On the solvability of a nonlocal boundary value problem for integro - differential equation with involution

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Abstract: In this paper, on the interval $[-1, 1]$, we study a boundary value problem of the Samarsky-Yonkin type for an integro-differential equation with involution

$$y''(x) + \varepsilon y''(-x) = -\lambda y(x) + \nu \int_{-1}^{1} K(x,s) y(s)\, ds + F(x), \tag{1}$$

$$-1 \le x \le 1, \ -1 < \varepsilon < 1,$$

$$y(-1) = y(1), \quad y'(1) = 0. \tag{2}$$

Here $K(x, s)$is continuous on $[-1, 1] \times [-1, 1]$, the function $F(x)$ is continuous on the interval under consideration. We will study the solvability of the boundary value problem (1), (2) using the parameterization method proposed by Professor D. Dzhumabaev [1]. The parameterization method was initially used to study the unique solvability of a boundary value problem for systems of differential equations. Later, the solvability of various boundary value problems was studied using the parameterization method.

For the study, the parameters $\mu_1 = y(0)$, $\mu_2 = y'(0)$ are introduced and the replacement $y(x) = u(x) + \mu_1 + \mu_2 x$ is used, i.e. the problem is formally divided into two parts: the Cauchy problem for the original equation and a system of two equations with two unknowns. The condition for the solvability of the problem under study is established.

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Keywords: integro-differential equations, parameterization method, parameter, Cauchy problem, unique solvability.

2020 Mathematics Subject Classification: 34ÃŠ06, 34ÃŠ37, 34ÃŠ10

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Rates of decay for Lord-Shulman elastic system with microtemperatures

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Abstract: In this work, we consider the Lord-Shulman porous-elastic system with dissipation due to microtemperature effects. First, we show that the system is exponentially stable provided that the new stability number $\chi = 0$. Otherwise, we prove the lack of exponential stability under the assumption $\chi \neq 0$. Furthermore, in the last case, we show that the solution decays polynomially.

Keywords: Exponential decay, porous system, microtemperature effects, lack of exponential stability, polynomial stability.

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On convergence of approximations to the complete solution of the tracking problem when nonlinear optimization of oscillation processes

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Abstract: In the paper [1] an algorithm is developed for constructing the complete solution to the tracking problem, where it is required to minimize the following quadratic integral functional

$$J[\bar{u}(t,x),\bar{v}(t,x)] = \int_0^T \int_Q [V(t,x)-\xi(t,x)]^2 \,dx\,dt +$$

$$+\int_0^T \left\{\alpha \int_Q h[t,x,\bar{u}(t,x)]\,dx + \beta \int_\gamma b[t,x,\bar{\vartheta}(t,x)]\,dx\right\} dt, \quad \alpha,\beta>0,$$

on the set of solutions of the following boundary value problem

$$V_{tt} - AV = \lambda \int_0^T K(t,\tau)V(\tau,x)d\tau + f[t,x,\bar{u}(t,x)], \quad x\in Q\subset R^n, \quad 0<t<T,$$

$$V(0,x)=\psi_1(x), \quad V_t(0,x)=\psi_2(x), \quad x\in Q,$$

$$\Gamma V(t,x) \equiv \sum_{i,k=1}^n a_{ik}(x)V_{x_k}(t,x)\cos(\delta,x_i) + a(x)V(t,x) = p[t,x,\bar{\vartheta}(t,x)], \quad x\in\gamma.$$

The complete solution of this nonlinear optimization problem is defined as the triple $(u^0(t,x),\vartheta^0(t,x)), V^0(t,x), J[u^0(t,x),\vartheta^0(t,x)]$ [1], where $u^0(t,x)$ is a vector distributed optimal control, $\vartheta^0(t,x)$ is a vector boundary optimal control, $V^0(t,x)$ is a optimal process, $J[u^0(t,x),\vartheta^0(t,x)]$ is a minimum value of the functional. In the article the issues are investigated of convergence of approximations to the complete solution of the tracking problem. An algorithm is developed for constructing approximations of the vector distributed and vector boundary optimal controls, resolvent and finite-dimensional approximations of the optimal process and corresponding approximations of the minimum value of the functional, and their convergences are proved in the norm of the corresponding functional spaces.

Keywords: Nonlinear optimization, tracking problem, approximation, resolvent approximation, finite-dimensional approximation, convergence of approximation.

2020 Mathematics Subject Classification: 49K20

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Principle of indefinite equations in mathematics for different systems of co-ordinates

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Abstract: We consider the analytical $z = f(p)$ functions in different coordinate systems. The centers of coordinates of the new systems are located in the $(A, 0)$ points, $A > 0$, with the r variable.

With point of view of the new coordinate system we obtain some new $y = f(x + A)$ equations for the same $y = f(x)$ graph for the only function, (the new equation of the same $(z, f(p))$ set of points for the $p = x, z = y$ complex variables). To prove the fact we can consider the $y = f(p)$ equation for the new $p = r + A$ variable with $r \equiv x$; by definition of the p and r variables $p = r + A$, (the p variable we consider in the primary coordinate system, the r variable we consider in the new coordinate system with the $(A, 0)$ center of coordinate); for all the r, x variables $r \equiv x$ by definition of the radius-vectors in both coordinate systems, (r is other designation of x). The consideration of the equations results in periodicity of the $f(p)$ function. The same result we obtain for the complex F(p) field, where $F(x+iy) = f(-x+iy)$ by definition for the $p = x + iy$ complex variable.

Keywords: Periodicity, different coordinate systems, analytical functions, fields of the complex functions, moved functions

2020 Mathematics Subject Classification: D7N20, 03H10, 11B34

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Space-dependent source identification problem for the two-dimensional neutron transport equation

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Abstract: The neutron transport equation is a critical challenge in nuclear reactor physics and radiation transport. In this study, we investigate the space dependent source identification problem for two-dimensional neutron transport equation

$$\left\{\begin{array}{l} \frac{\partial u(t,x,y)}{\partial t}=\frac{\partial u(t,x,y)}{\partial x}+\frac{\partial u(t,x,y)}{\partial y}+p\left(x,y\right)+f\left(t,x,y\right), \\ t\in\left(0,T\right),\ x,y\in\left(0,L\right), \\ u\left(0,x,y\right)=\varphi\left(x,y\right),\quad x,y\in\left[0,L\right], \\ u\left(t,0,y\right)=0,\quad u\left(t,x,0\right)=0,\quad t\in\left[0,T\right],\ x,y\in\left[0,L\right], \\ u(T_1,x,y)=\psi_1\left(x,y\right),u(T_2,x,y)=\psi_2\left(x,y\right), \\ x,y\in\left[0,L\right],\quad T_1,T_2\in\left(0,T\right). \end{array}\right. \tag{1}$$

Here, $u\left(t,x,y\right)$ and $p\left(x,y\right)$ are unknown functions, $f\left(t,x,y\right)$, $\varphi\left(x,y\right)$, $\psi_1\left(x,y\right)$, and $\psi_2\left(x,y\right)$ are given sufficiently smooth functions and all compatibility conditions are satisfied.

In the rest of paper, the theorem on the stability of differential problem (1) is established. For the approximate solution of problem (1), a first order of accuracy difference scheme is proposed. The theorem on stability of this difference scheme is established. Some results of numerical experiment are presented.

Keywords: Space-dependent source identification problem, neutron transport equation, difference scheme, differential equation, stability inequality.

2020 Mathematics Subject Classification: 65J22, 39A14, 82D75

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On some properties of generalized eigenfunctions of differential-difference operators

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Abstract: We consider a spectral problem for a differential-difference operator with Dirichlet boundary conditions. Unlike ordinary differential operators, the solutions of boundary value problems for such operators may turn out to be nonsmooth. [1] We have previously shown that the smoothness of the generalized eigenfunctions of the differential-difference operators can be violated at the inner points of the interval. [2] In this paper, conditions are obtained for preserving the smoothness of generalized eigenfunctions over the entire interval. We construct an example showing that all eigenfunctions can be nonsmooth. We consider classes of differential-difference operators for which there is an countable number of smooth generalized eigenfunctions.

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Keywords: differential–difference operator, generalized eigenfunction, smoothness

2020 Mathematics Subject Classification: 47E07, 34K08, 34K06

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Hicham Ait Mohammed [1], Mohammed El-Hadi Mezabia[1], Brahim Tellab, Analysis of existence, uniqueness, and stability for a Caputo fractional variable order ϑ-initial value problem with multi-point boundary conditions

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Abstract: This report focuses on examining the existence, uniqueness, and stability of solutions for a Caputo variable order ϑ-initial value problem (ϑ-IVP) with multi-point boundary conditions of the type:

$$\begin{cases} {}^{c}\mathcal{D}_{0+}^{r(\mathfrak{z}),\vartheta(\mathfrak{z})} z(\mathfrak{z}) = h\big(\mathfrak{z}, z(\mathfrak{z}), \mathcal{I}_{0+}^{q;\vartheta} z(\mathfrak{z})\big), \quad \mathfrak{z} \in \mathcal{O} = [0, Z], \quad \mathfrak{z} \neq Z_i, \quad i = 1, ..., n, \\ \\ z(0) = z_0, \quad z'_{\vartheta}(0) = z''_{\vartheta}(0) = ... = z_{\vartheta}^{[m-1]}(0) = 0, \Delta z\big|_{\mathfrak{z}=Z_i} = g_i\big(z(Z_i^-)\big), i = 1, ..., n, \end{cases}$$

where $m \geq 3$, $r : [0, Z] \to (m-1, m]$ is the variable order of the fractional derivative, $1 < Z < \infty$, $h : \mathcal{O} \times \mathbb{R}^2 \to \mathbb{R}$, $g_i : \mathbb{R} \to \mathbb{R}$, $i = 1, ..., n$ are continuous functions. ${}^{c}\mathcal{D}_{0+}^{r(\mathfrak{z});\vartheta(\mathfrak{z})}$ denotes the ϑ-Caputo fractional derivative depending on an increasing function ϑ of variable order $r(\mathfrak{z})$ and $\mathcal{I}_{0+}^{q;\vartheta}$ presents the ϑ-Riemann-Liouville fractional integral of constant order $q > 0$, $0 = Z_0 < Z_1 < \cdots < Z_{n+1} = Z$, $\Delta z\big|_{\mathfrak{z}=Z_i} = z\big(Z_i^+\big) - z\big(Z_i^-\big)$ where $z\big(Z_i^+\big) = \lim_{\epsilon\to 0^+} z\big(Z_i + \epsilon\big)$ and $z\big(Z_i^-\big) = \lim_{\epsilon\to 0^-} z\big(Z_i+\epsilon\big)$, and $z_0 \in \mathbb{R}$ and $z_{\vartheta}^{[i]} = \left(\frac{1}{\vartheta'(\mathfrak{z})} \frac{d}{d\mathfrak{z}}\right)^i z(\mathfrak{z})$. The proofs for uniqueness and existence leverage Sadovski's and Banach's fixed point theorems. Furthermore, we explore the Ulam-Hyers-Rassias (UHR) stability of the solution. To validate our findings, we present a numerical example.

Keywords: Caputo fractional derivative, variable order, multi-point boundary value problem, Kuratowski measure of noncompactness, fixed point theorems.

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Ulyanov inequalities in the case of limiting parameter values

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Let $L_{p_1p_2}, 1 \leq p_i \leq \infty, i = 1,2$ be the set of measurable functions of two variables $f(x_1, x_2)$, 2π - periodic in each variable, for which $\|f\|_{p_1p_2} = ||\{||f||_{p_1}\}||_{p_2} < \infty$, where

$$\|f\|_{p_i} = \left(\int\limits_0^{2\pi} |f|^{p_i} dx_i\right)^{\frac{1}{p_i}}, \text{ if } 1 \leq p_i < \infty,$$

$$\|f\|_{p_i} = \sup_{0 \leq x_i \leq 2\pi} |f|, \text{ if } p_i = \infty.$$

Let $L^0_{p_1p_2}$ be the space of functions $f \in L_{p_1p_2}$ such that $\int\limits_0^{2\pi} f(x_1, x_2)dx_1 = 0$ for almost all x_2 and $\int\limits_0^{2\pi} f(x_1, x_2)dx_2 = 0$ for almost all x_1.

For the function $f \in L_{p_1p_2}$, we define the fractional differences of positive order α_1 and α_2 with steps h_1 and h_2 respectively, by variables x_1 and x_2 as follows:

$$\Delta^{\alpha_1}_{h_1}(f) = \sum_{\nu_1=0}^{\infty} (-1)^{\nu_1} \binom{\alpha_1}{\nu_1} f(x_1 + (\alpha_1 - \nu_1)h_1, x_2),$$

$$\Delta^{\alpha_2}_{h_2}(f) = \sum_{\nu_2=0}^{\infty} (-1)^{\nu_2} \binom{\alpha_2}{\nu_2} f(x_1, x_2 + (\alpha_2 - \nu_2)h_2),$$

where $\binom{\alpha}{\nu} = 1$ for $\nu = 0$, $\binom{\alpha}{\nu} = \alpha$ for $\nu = 1$, $\binom{\alpha}{\nu} = \frac{\alpha(\alpha-1)\ldots(\alpha-\nu+1)}{\nu!}$ for $\nu \geq 2$.

Denote ([3]) by $\omega_{\alpha_1,\alpha_2}(f, \delta_1, \delta_2)_{p_1p_2}$ the mixed modulus of smoothness of positive orders α_1 and α_2, respectively, in the variables x_1 and x_2 of a function $f \in L_{p_1p_2}$, that is,

$$\omega_{\alpha_1,\alpha_2}(f, \delta_1, \delta_2)_{p_1p_2} = \sup_{|h_i| \leq \delta_i, i=1,2} ||\Delta^{\alpha_1}_{h_1}(\Delta^{\alpha_2}_{h_2}(f))||_{p_1p_2}.$$

The following (p, q)-inequality between moduli of smoothness in different metrics, nowadays called sharp Ulyanov type inequalities, is known (see [3]):

$$\omega_1(f, \delta)^{(1)}_q \ll \left(\int\limits_0^{\delta} \left(t^{-\theta}\omega_1(f,t)^{(1)}_p\right)^{q^*} \frac{dt}{t}\right)^{\frac{1}{q^*}},$$

where $1 \leq p < q \leq \infty, \theta = \frac{1}{p} - \frac{1}{q}$. For the Lebesgue spaces, Ulyanov type inequalities in the one-dimensional case have been studied for the moduli of smoothness of any positive order by many authors, in particular, Ulyanov [3], Yu. Kolomoitsev, S. Tikhonov [1], B. Simonov [2].

Theorem 1. *Let $f \in L^0_{p_1p_2}, \alpha_i > 0, \delta_i \in (0, 1), i = 1, 2$. Let $1 = p_1 < q_1 < \infty$ or $1 < p_1 < q_1 = \infty$, and $1 = p_2 < q_2 < \infty$ or $1 < p_2 < q_2 = \infty$, then the following inequality*

$$\omega_{\alpha_1,\alpha_2}(f, \delta_1, \delta_2)_{q_1q_2} \ll \left(\int\limits_0^{\delta_2(\log_2 \frac{2}{\delta_2})^{\frac{1}{\alpha_2}\max(\frac{1}{q_2},1-\frac{1}{p_2})}} \left(\int\limits_0^{\delta_1(\log_2 \frac{2}{\delta_1})^{\frac{1}{\alpha_1}\max(\frac{1}{q_1},1-\frac{1}{p_1})}} \left(t_1^{-\frac{1}{p_1}+\frac{1}{q_1}} \times \right.\right.\right.$$

$$\left.\left.\left. \times t_2^{-\frac{1}{p_2}+\frac{1}{q_2}} \omega_{\alpha_1+\frac{1}{p_1}-\frac{1}{q_1},\alpha_2+\frac{1}{p_2}-\frac{1}{q_2}}(f, t_1, t_2)_{p_1p_2}\right)^{q_1^*} \frac{dt_1}{t_1}\right)^{\frac{q_2^*}{q_1^*}} \frac{dt_2}{t_2}\right)^{\frac{1}{q_2^*}} \tag{1}$$

holds.

Theorem 2. *Let $f \in L^0_{p_1p_2}, \delta_i \in (0, 1), i = 1, 2$. Let $1 = p_1 < q_1 < \infty$ or $1 < p_1 < q_1 = \infty$, and $1 = p_2 < q_2 < \infty$ or $1 < p_2 < q_2 = \infty$, $\alpha_i > 0, \rho_i \geq 0, i = 1, 2$, then the following inequality is true*

$$\omega_{\alpha_1,\alpha_2}\big(f^{(\rho_1,\rho_2)},\delta_1,\delta_2\big)_{q_1q_2} \ll \Bigg(\int\limits_0^{\delta_2(\log_2\frac{2}{\delta_2})^{\frac{1}{\alpha_2}\max(\frac{1}{q_2},1-\frac{1}{p_2})}}\Bigg(\int\limits_0^{\delta_1(\log_2\frac{2}{\delta_1})^{\frac{1}{\alpha_1}\max(\frac{1}{q_1},1-\frac{1}{p_1})}}\Big(t_1^{-\frac{1}{p_1}+\frac{1}{q_1}-\rho_1}\times$$

$$\times t_2^{-\frac{1}{p_2}+\frac{1}{q_2}-\rho_2}\omega_{\alpha_1+\rho_1+\frac{1}{p_1}-\frac{1}{q_1},\alpha_2+\rho_2+\frac{1}{p_2}-\frac{1}{q_2}}(f,t_1,t_2)_{p_1p_2}\Big)^{q_1^*}\frac{dt_1}{t_1}\Bigg)^{\frac{q_2^*}{q_1^*}}\frac{dt_2}{t_2}\Bigg)^{\frac{1}{q_2^*}}.$$

Keywords: Mixed moduli of smoothness of positive order, Ulyanov type inequality.

2020 Mathematics Subject Classification: 41A17,41A63, 46E30, 26D10.

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On the solvability of initial boundary value problems for degenerate equations

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Let $0 < T < \infty$ and $\Omega \subset \mathbf{R}^n$ be a bounded domain with the boundary $\partial\Omega \in C^2$, $Q = \Omega \times (0, T)$, $\Sigma = \partial\Omega \times (0, T)$. Consider the following initial boundary value problem for a degenerate hyperbolic equation

$$\partial_t \left(t^{\beta} \partial_t u(x,t) \right) - \Delta u(x,t) = f(x,t) \text{ in } Q, \tag{1}$$

$$u(x,t) = 0 \text{ on } \Sigma, \tag{2}$$

$$u(x,0) = 0, \quad \lim_{t \to +0} t^{\beta} \partial_t u(x,t) = 0 \text{ in } \Omega. \tag{3}$$

The work will investigate the issues of solvability, specifically, under what conditions on the right-hand side the initial-boundary value problem (1)–(3) is uniquely solvable.

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Keywords: Degenerate equation, hyperbolic equation, a priori estimates.

2020 Mathematics Subject Classification: 35L20, 35L80, 35B45

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Qualitative study of an age-structured model for the spread of Xylella Fastidiosa in olive trees

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Abstract: The main objective of this work is to analyze a mathematical model describing the dynamics of olive stands infected by the insect nicknamed Xylella Fastidiosa. The model is a system of two equations, the first one is an ordinary differential equation describing the evolution of the healthy olive trees. The second one is a partial differential equation, describing the evolution of the infected olive trees. Such models are often called hybrid mathematical models. In our analysis, we are interested in the behavior of the two types of olive trees. We study the steady states and their stability. We give some numerical simulations to illustrate our study and some conclusions and remarks at the end of this work.

Keywords: Xylella Fastidiosa, hybrid mathematical model, partial differential equation, ordinary differential equation, steady states, local and global stability, numerical simulations.

On some spectral properties of even-order ordinary differential operator with integral conditions

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Abstract: We consider ordinary differential operator of even order with non-local boundary conditions and a spectral parameter λ. The boundary conditions are set by Riemann integrals, which contain both an unknown function and derivatives of an unknown function. An equivalent norm is introduced in the Sobolev space, depending on the spectral parameter λ.

Constructive sufficient conditions for the discreteness of the spectrum and an a priori estimates of solutions of such a problem for sufficiently large values of the spectral parameter are obtained. These conditions are written in terms of determinants depending on the values of the weight functions at the boundary points of the interval.

This work is supported by the Ministry of Science and Higher Education of the Russian Federation (megagrant agreement No. 075-15-2022-1115).

Keywords: even order differential operator, nonlocal problem, integral conditions, a priori estimates

2020 Mathematics Subject Classification: 34L05

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Enhancing multi-output mchine learning regressors with kernelized PCA: a comparative analysis

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Abstract: In the research area of machine learning, enhancing the performance of multi-output regression models has gained significant attention [1]. Standart Principal Component Analysis (PCA) has been widely used for dimensionality reduction to improve model efficiency and interpretability. However, PCA's linear nature may limit its effectiveness in capturing complex, non-linear relationships within the data. This paper investigates the impact of Kernelized Principal Component Analysis (KPCA) on multi-output regression algorithms. KPCA extends PCA by utilizing kernel methods to project data into higher-dimensional feature spaces, thus enabling the capture of non-linear structures.

In this paper we compare the performance of multi-output linear regression models with and without KPCA preprocessing across classical digits dataset and a synthetic dataset which include complex relations between the features. Additionally, we explore the computational efficiency of LKF Kernel [2] in comparison to other kernels in KPCA and conventional PCA.

The findings suggest that integrating KPCA as a preprocessing step in multi-output regression can lead to substantial improvements in model performance, making it a valuable technique for complex data-driven applications.

Keywords: Multi-output regressor, KPCA, optimization

2020 Mathematics Subject Classification: 65F45, 65K10, 65F20

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Green's functions of some boundary value problems for polyharmonic operators and their correct constrictions

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Abstract: Finding general well-posed boundary value problems for differential equations is always a topical problem. The abstract theory of operator contractions and extensions originates from the work of J.von Neumann [1], in which a method for constructing self-adjoint extensions of a symmetric operator was described and a theory of extensions of symmetric operators with finite deficiency indices was developed in detail. Many problems for partial differential equations lead to operators with infinite deficiency indices.
In the early 80s of the last century, M. Otelbaev and his students [2] constructed an abstract theory that allows one to describe all well-posed restrictions of a maximal operator and, separately, all well-posed extensions of a minimal operator, in terms of the inverse operator.

Recently, interest in constructing explicit Green's functions for classical problems has been renewed. In the works of T.Sh.Kal'menov, B.D.Koshanov [3,4] the Green's function of the Dirichlet problem for a polyharmonic equation in a multidimensional ball is constructed in an explicit form.
This report will give a constructive method for constructing the Green's function of the Dirichlet problem for a polyharmonic equation in a multidimensional ball, where the method of special expansion of the fundamental solutions of the polyharmonic equation and the reflection method will be significantly used.
As an example, a general representation of real solutions of the biharmonic operator in complex form will be given.

Keywords: Polyharmonic Green function, polyharmonic Dirichlet problem, correct narrowings of the operator, n-dimensional ball

2020 Mathematics Subject Classification: 31B30, 31B20, 31B10

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A highly accurate difference method for approximating of the first derivatives of the mixed boundary value problem for Laplace's equation on a rectangle

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Abstract: In a rectangular domain, we discuss about an approximation of the first order derivatives for the solution of the mixed boundary value problem. The boundary values on the sides of the rectangle are supposed to have the $p-$th derivatives satisfying the Hölder condition where $p \in \{4, 6\}$. On the vertices, besides the continuity condition, the compatibility conditions, which result from the Laplace equation for the second and fourth derivatives of the boundary values, given on the adjacent sides, are also satisfied. Under these conditions for the approximate values of the first derivatives of the solution of mixed boundary problem on a square grid, as the solution of the constructed difference scheme a uniform error estimation of order (h^{p-1}) (h is the grid size) where $p \in \{4, 6\}$ is obtained.

Numerical experiments are illustrated to support the theoretical results.

Keywords: finite difference method, approximation of the derivatives, error estimations, mixed boundary condition, Laplace equation on rectangle

2020 Mathematics Subject Classification: 65M06, 65M12, 65M22

Inverse problem of identifying the time-dependent diffusion coefficient in the time-fractional heat equation

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Abstract: Let $\mathcal{H}$ be a separable Hilbert space and let $\mathcal{L}$ be an operator with a discrete spectrum on $\mathcal{H}$. For

$$\begin{cases} \mathcal{D}_t^{\alpha} u(t) + a(t)\mathcal{L}u(t) = f(t) \text{ in } \mathcal{H},\ 0 < t \leq T, \\ u(0) = h \text{ in } \mathcal{H}, \end{cases} \tag{1}$$

we study

Coefficient inverse problem. Given $f(t)$ and h, find a pair of functions $\{a(t), u(t)\}$ that satisfy problem (1) and the additional condition

$$F[u(t)] = E(t),\ t \in [0, T],$$

the F is the linear bounded functional.

As for this kind of inverse problem for parabolic equation, see [1]. Under suitable restrictions on the given data, we prove existence, uniqueness and continuous dependence on the data of the solution.

Keywords: Heat equation, direct problem, coefficient inverse problem, well-posedness, positive operator, Caputo fractional derivative

2020 Mathematics Subject Classification: 35R25, 35R30

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Boundary value problems in differential equations with algebraic terms

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Abstract: We consider a differential-boundary equation with algebraic terms on a finite interval $0 < x < 1$

$$\begin{aligned} l(y) \equiv \frac{d}{dx}\left(\frac{dy(x)}{dx} + \sum_{i=1}^{k} h_i(x)U_i(y) + \sum_{j=1}^{s} \lambda_j q_j(x)\right) \\ + r_1(x)\frac{dy(x)}{dx} + r_0(x)y(x) = f(x), \end{aligned} \tag{1}$$

A distinctive feature of these equations is that, alongside the function being sought, a certain number of unknown values must also be determined. This leads to the critical question of unique solvability: how many and what type of conditions need to be imposed on equation (1) to ensure that the resulting problem has a unique solution in a given space?

This kind of equations are classified as differential operator equations in [1].

Keywords: Differential-algebraic equation, differential-boundary operator, boundary value problem, inverse operator, uniqueness of solutions.

2020 Mathematics Subject Classification: 34A09, 34B05

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About a method for solving semilinear equations

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Abstract: The article addresses issues related to the development of con structive methods for solving specific classes of semilinear equations

(1) $$A(u) = f(u),$$

where $A(u)$ is a linear operator and $f(u)$ is a nonlinear operator. A method is developed that allows finding solutions for certain classes of semilinear alge braic equations, Fredholm integral equations, and differential equations. The structure of the solution is determined, sufficient conditions for the existence of a solution are established, and an algorithm for their construction is described.

This article is a continuation of the work presented in (1) integral perturbation of boundary condition, characteristic determinant.

Keywords: linear operators, nonlinear operators, semilinear equations, suffi cient conditions for solvability, solution structure, solution construction algorithm.

2020 Mathematics Subject Classification: 49K20

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Analysis of a double nonlinear diffusion equation with time-weighted absorption

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Abstract: This report is devoted to the properties of solutions for a double nonlinear parabolic equation with variable density, not in divergence form with time-weighted absorption. We consider the following Cauchy problem in the domain $D = \{(x,t)|x \in R^N, t > t_0 > 0\}$

$$\rho_1(x)\, u_t = u^q \nabla\left(\rho_2(x)\, u^{m-1} \left|\nabla u^k\right|^{p-2} \nabla u^l\right) - t^n \rho_3(x)\, u^\beta, \tag{1}$$

$$u(x,t_0) = u_0(x), \qquad x \in R^N, \tag{2}$$

where $q \neq 1$, $m \geq 1$, $k > 0$, $p \geq max\{2, n_2 - n_1\}$, $l \geq 1$, $\beta > q+m+k(p-2) > 1$, $n \geq 0$, $\gamma \neq 0$, $n_1 \leq n_3$, $\rho_i(x) = |x|^{n_i}$, $i = 1,2,3$, and $u_0(x)$ is non-negative bounded and continuous function in R^N.

It is obvious to see that the equation (1) is degenerate. Therefore, it does not have classical solutions in the domain where $u = 0$, $\nabla u = 0$. Furthermore, in this case, we consider a weak solution from the class $0 \leq u(x,t)$,
$\rho_2(x)\, u^{m-1} \left|\nabla u^k\right|^{p-2} \nabla u^l \in C(D)$ and satisfying to the equation (1) in the distribution sense [2].

We denote $C_b\left(R^N\right)$ be the space of all bounded continuous functions in R^N. For $a \geq 0$, we define

$$F_a = \left\{\rho(x) \in C_b\left(R^N\right) \mid \rho(x) \geq 0 \text{ and } \lim_{|x|\to\infty} \inf |x|^a \rho(x) > 0\right\}.$$

In addition, let $a_c = \dfrac{p + n_3 - n_2}{\beta - q - m - k(p-2) - l + 1}$, $\beta > \beta_c =$
$1 + (n+1)\left[m + k(p-2) + l + 2q - 2 + \dfrac{(1-q)(p + n_1 - n_2)}{d + n_1}\right] + \dfrac{(1-q)(n_3 - n_1)}{d + n_1}$.

Theorem 2.5. *Let $0 < a < a_c$, $u_0(x) \in F_a$, $q < \frac{1}{p}$ and $\beta < \beta_c$, then the solution $u(x,t)$ of the problem (1)-(2) blows up in a finite time.*

Proof. The proof of the Theorem 1 is similar to the proof of theorems in [1]. □

Keywords: Global solvability, weak solution, critical Fujita, asymptotics

2020 Mathematics Subject Classification: 35D30, 35B44, 35B40

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On the numerical solution of hyperbolic evolution problem in a two-dimensional domain

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Abstract: Numerical investigations into two-dimensional hyperbolic and parabolic equations have garnered considerable interest across various domains. In this work, the Galerkin method is applied to construct the numerical solution of the two-dimensional telegraph equation with variable coefficients. The related equation, extensively used in engineering, was derived in 1876 by Heaviside, who addressed the charging and discharging of a finite length of cable under the effects of induction.
This study demonstrates that the Galerkin method is a highly effective and powerful tool for numerically solving such problems. Several examples of numerical experiments are provided.

Keywords: Telegraph equation, numerical solution, Galerkin method, weak solution.

2020 Mathematics Subject Classification: 65L20 · 65L12 · 37N30

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Applications of generalized Stieltjes-type integral transform

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Abstract: This study is about the applications of the generalized Stieltjes-type integral transform. Definitions for both the new integral transform and its inverse are provided in [1]. A derivative operator, the delta derivative, is included. The relationship between this derivative and the transform is obtained. A new convolution for the integral transform is defined and then the Convolution Theorem is proved.

More importantly, various problems for ordinary, partial differential and integral equations are studied. Initial value problems for differential equations, an integral equation and initial, boundary value problems for partial differential equations are considered. The solutions to these problems are obtained by applying the new integral transform.

Keywords: Integral Transform, Stieltjes Transform, Generalized Laplace-type Integral Transform, Generalized Stieltjes-type Integral Transform

2020 Mathematics Subject Classification: 44A05, 44A10, 44A15, 44A20

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On the numerical solution of a multidimensional source identification problem by a modified Crank-Nicolson difference scheme

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Abstract: The present study is allocated to the derivation of the numerical solution of a source identification problem governed by a multidimensional, linear parabolic equation subject to Dirichlet boundary conditions by a modified Crank-Nicolson difference scheme. The one-dimensional version of the problem considered in this study was investigated in paper [1], and its numerical solution was derived by the well-known Rothe and the standard Crank-Nicolson finite difference schemes. In this study, a modified Crank-Nicolson difference scheme, that is a combination of the Rothe and standard Crank-Nicolson difference schemes (see, for example, [2,3]), is employed for the numerical solution of the abovementioned multidimensional source identification problem. Moreover, some stability estimates for the employed difference scheme are established. A comprehensive numerical analysis is conducted by performing the employed difference scheme on a model problem to exhibit its advantage(s).

Keywords: Source identification, multidimensional parabolic equation, numerical solution, stability

2020 Mathematics Subject Classification: 65J22, 65M32, 65N21

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Existence and uniqueness of solutions for a fractional p-Laplacian problem

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Abstract:

This work focuses on the study of a nonlinear fractional p()-Laplacian evolution problem with variable order, arising from an equation with deviated arguments. By applying a semi-discretization in time, we establish the well-posedness of the problem, including the necessary a priori estimates. These estimates serve as the foundation for proving the convergence of a corresponding semi-discretized approximation scheme

Keywords: $p(.)-$Laplacian, semi-discretization, weak solution, Sobolev space with variable exponent.

2020 Mathematics Subject Classification: 65L20 · 65L12 · 37N30

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Numerical analysis of higher-order evolution problem

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Abstract: In physical sciences and engineering, the fourth-order partial differential evolution equation is widely employed, particularly in civil and mechanical engineering, to assess the deflection and strength of bending beams. Given the complexity of higher-order partial differential equations with intricate boundary and initial conditions, analytical solutions are often challenging to derive. Therefore, numerical techniques play a pivotal role in solving these boundary value problems.

In this study, we explore the numerical solutions of a fourth-order partial differential evolution equation. We use the finite difference and finite element methods to discretize the equation, providing a practical framework for computation. We discuss absolute error calculations, numerical stability, and the convergence properties of these methods. Our discussion is supplemented with several examples that demonstrate the efficacy of the methods.

Keywords: Higher-order partial differential equations, time dependent problem, numerical Solution, finite difference method.

2020 Mathematics Subject Classification: 65L20 · 65L12 · 37N30

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Application of Rothe's method to an evolution equation on graphs

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Abstract: This report is devoted to a time-dependent evolution equation on connected weighted graphs finished. Using the Rothe's metod, we estabilish the existent and uniqueness of a weak solution.

Keywords: Roth's method, semi-discritisation in time, apriori estimates, Finite connected Weighted Graphs.

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Existence of solutions for second order semilinear wave equation with variable cefficient

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Abstract: In this work, we study the existence of the initial and boundary value problem for equation $\dfrac{\partial^2 u}{\partial t^2} - t^{2l}\dfrac{\partial^2 u}{\partial x^2} = f(x,t,u)$. Based on priori estimates of solution we proved the existence of the weak solution in the form of Fourier series under suitable conditions. For this purpose Picard's successive approximation method was used.

Keywords: Higher order partial differential equation, semilinear partial differential equation, initial value problem, boundary value problem, existence, Picard's method, Fourier series method, the method of eigenfunction expansion

2020 Mathematics Subject Classification: 35A01, 35A02, 35C10, 35D30, 35G15, 35G30, 35J40, 35G30, 35M12

Existence of solutions for higher order semilinear parabolic equation with variable coefficient

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Abstract: In this work, we study the existence of the initial and boundary value problem for equation $\dfrac{\partial u}{\partial t} + (-1)^k t^{2l} \dfrac{\partial^{2k} u}{\partial x^{2k}} = f(x, t, u)$. Based on priori estimates of solution we proved the existence of the weak solution in the form of Fourier series under suitable conditions. For this purpose Picard's successive approximation method was used.

Keywords: Higher order partial differential equation, semilinear partial differential equation, initial value problem, boundary value problem, existence, Picard's method, Fourier series method, the method of eigenfunction expansion

2020 Mathematics Subject Classification: 35A01, 35A02, 35C10, 35D30, 35G15, 35G30, 35J40, 35G30, 35M12

An asymptotic result for linear nonautonomous mixed type differential equation

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Abstract: We consider a nonhomogeneous linear mixed type differential equation with variable coefficients and establish an asymptotic result for the solutions. Our result is obtained by the use of a solution of the so-called generalized characteristic equation of the corresponding homogeneous linear (autonomous) mixed type differential equation.

Keywords: Mixed differential equation, characteristic equation, root, asymptotic behavior

2020 Mathematics Subject Classification: 34K06, 34K12

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Existence of solutions for second order semilinear elliptic equation with variable coefficient

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Abstract: In this work, we study the existence of the boundary value problem for equation $\dfrac{\partial^2 u}{\partial t^2} + t^{2l}\dfrac{\partial^2 u}{\partial x^2} = f(x,t,u)$. Based on priori estimates of solution we proved the existence of the weak solution in the form of Fourier series under suitable conditions. For this purpose Picard's successive approximation method was used.

Keywords: Higher Order Partial Differential Equation, Semilinear Partial Differential Equation, Initial Value Problem, Boundary Value Problem, Existence, Picard's Method, Fourier Series Method, The Method of Eigenfunction Expansion

2020 Mathematics Subject Classification: 35A01, 35A02, 35C10, 35D30, 35G15, 35G30, 35J40, 35G30, 35M12

The well-posedness of elliptic problems with integral type Samarskii-Ionkin conditions

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Abstract: The present paper is devoted to the study of the abstract nonlocal boundary value problem with Samarskii–Ionkin condition for differential equation of elliptic type

$$-u''(t) + Au(t) = f(t) \quad (0 \leq t \leq T), \ u(0) = \varphi, \ u^{'}(0) = u^{'}(T) + \int_0^T \alpha(s)\, u(s) ds + \psi.$$

in an arbitrary Banach space E with the positive operator A. The well-posedness of this problem in various Banach spaces is established. In applications, theorems on the well-posedness of several nonlocal boundary value problem for elliptic equations with integral type Samarskii–Ionkin conditions are proved.

Keywords: Well-posedness, Samarskii-Ionkin conditions, coercive stability, positive operators, elliptic equation

2020 Mathematics Subject Classification: 35J25, 47E05, 34B27, 34G10.

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On the Fourier transform of functions from a Lorentz space $L_{\overline{2},\bar{r}}$ witn a mixed metric

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Abstract: Definition 1. Let $\bar{p} = (p_1, p_2)$ and $\bar{r} = (r_1, r_2)$ satisfy the following conditions: $0 < \bar{p} \le \infty, \quad 0 < \bar{r} \le \infty$. The Lorentz Space $L_{\bar{p},\bar{r}}[0,1]^2$ with a mixed metric is defined as the set of all measurable functions defined on $[0,1]^2$, for which the quantities are finite:

$$\|f\|_{L_{\bar{p},\bar{r}}} = \left\|\|f\right\|_{L_{p_1,r_1}}\Big\|_{L_{p_2,r_2}} = \left(\int_0^1 \left(t_2^{\frac{1}{p_2}} \left(\int_0^1 \left(t_1^{\frac{1}{p_1}} f^{*1}(t_1,\cdot)\right)^{r_1} \frac{dt_1}{t_1}\right)_{t_2}^{*2}\right)^{\frac{r_2}{r_1}} \frac{dt_2}{t_2}\right)^{\frac{1}{r_2}}$$

when $0 < \bar{r} < \infty$,

$$\|f\|_{L_{\bar{p},\infty}} = \sup_{t_1,t_2} t_1^{\frac{1}{p_1}} t_2^{\frac{1}{p_2}} f^{*1*2}(t_1,t_2)$$

when $\bar{r} = \infty$.

Definition 2. Let $f \in L_1(\mathbb{R}^2)$. Then be determined by its two-dimensional Fourier transform

$$\hat{f}(\xi_1\xi_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,x_2)\, e^{-2\pi i(x_1\xi_1+x_2\xi_2)} dx_1 dx_2$$

Theorem A. [1]. Let $\{\varphi_n\}_{n=1}^{\infty}$ be an orthonormal system of complex-valued functions on $[0,1]$, such that

$$\|\varphi_n\| \le M, n = 1, 2, \ldots,$$

and let $f \in L_{2,r}, \quad 2 < \bar{r} \le \infty$. Then the inequality

$$\sup_{n\in N} \frac{1}{|n|^{\frac{1}{2}} (\log(n+1))^{\frac{1}{2}-\frac{1}{r}}} \sum_{m=1}^{n} a_m^* \le C\|f\|_{L_{2,r}}$$

holds, where a_n are the Fourier coefficients of the system $\{\varphi_n\}_{n=1}^{\infty}$. In 2015 we had an analog of Bochkarev's theorem for the Fourier transform of the function in $L_{2,r}(\mathbb{R})$.

Theorem 1 [2]. Let $\Re_N = \left\{A = \bigcup_{i=1}^N A_i\right.$, where A_i are segments in $\mathbb{R}\}$, then there be an inequality for any function: $f \in L_{2,r}(\mathbb{R}), 2 < r < \infty$

$$\sup_{N\ge 8 A\subset\Re_N} \frac{1}{|A|^{\frac{1}{2}} (\ln(N+1))^{\frac{1}{2}-\frac{1}{r}}} \left|\int_A \hat{f}(\xi)d\xi\right| \le 23\|f\|_{L_{2,r}}.$$

The aim of this article is to obtain a two-dimensional analog of the Bochkarev type theorem for the Fourier transform. Lemma 1. Let $\frac{4}{3} < q_1, q_2 < 2$ and $f \in L_{\bar{q},\overline{2}}(\mathbb{R}^2)$. Then for any measurable sets A_1 and A_2 of finite measure of $\Re_N$, the inequality

$$\sup_{A_1\subset\Re_N A_2\subset\Re_N} \frac{1}{|A_1|^{\frac{1}{q_1}} |A_2|^{\frac{1}{q_2}}} \left|\hat{f}(\xi_1,\xi_2)\right| d\xi_1 d\xi_2 \le$$

$$\le C\left(\frac{q_1}{2(q_1-1)}\right)^{\left(\frac{1}{q_1}-\frac{1}{2}\right)} \left(\frac{q_2}{2(q_2-1)}\right)^{\left(\frac{1}{q_2}-\frac{1}{2}\right)} \|f\|_{L_{\bar{q},\overline{2}}}$$

holds.

Theorem 2. Let $\Phi_{m_1,m_2}(x_1,x_2) = \varphi_{m_1}(x_1)\cdot\psi_{m_2}(x_2), m_1, m_2 \in \mathbb{N}$ be an orthonormal system of functions. Then $f \in L_{\overline{2},\bar{r}}[0,1]$ and $2 < r_1, r_2 < \infty$ we have the inequality:

$$\sup_{|A_1|\ge 8 |A_2|\ge 8}\ \sup_{A_1\subset N A_2\subset N} \frac{1}{|A_1|^{\frac{1}{2}} |A_2|^{\frac{1}{2}} (\log_2(|A_1|+1))^{\frac{1}{2}-\frac{1}{r_1}} (\log_2(|A_2|+1))^{\frac{1}{2}-\frac{1}{r_2}}} \times$$

$$\int_{A_1}\int_{A_2} \left|\hat{f}(\xi_1,\xi_2)\right| d\xi_1 d\xi_2 \le \|f\|_{L_{\overline{2},\bar{r}}}.$$

Keywords: The Lorentz Space, Fourier transform, Bochkarev's theorem

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On the critical behavior for inhomogeneous semilinear biharmonic heat equations on exterior domains

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Abstract: This report is devoted to the existence and nonexistence of global weak solutions to the inhomogeneous semilinear heat equations with forcing terms on exterior domains

$$\begin{cases} u_t + \Delta^2 u = |u|^p + f(x), & \text{in } D^c \times (0,\infty), \\ u(x,0) = u_0(x), & \text{in } D^c, \end{cases} \tag{1}$$

where $p > 1$ is a constant, Δ is the Laplace operator, and $D = \overline{B_1}$ is the closed unit ball, and D^c is its complement in $\mathbb{R}^N$.

We investigate the critical behavior of solutions to the semilinear biharmonic heat equation with forcing term $f(x)$, under six homogeneous boundary conditions. By employing a method of test function, we derive the critical exponents p_{Crit} in the sense of Fujita. Moreover, we show that $p_{Crit} = \infty$ if $N = 2, 3, 4$ and $p_{Crit} = \frac{N}{N-4}$ if $N \geq 5$. The impact of the forcing term on the critical behavior of the problem is also of interest, and thus a second critical exponent in the sense of Lee-Ni, depending on the forcing term is introduced. We also discuss the case $f \equiv 0$, and present the finite-time blow-up results for the subcritical and critical cases.

Keywords: Exterior problem, Fujita critical exponent, nonexistence, existence

2020 Mathematics Subject Classification: 35A01, 35B09, 35B44

Perturbation and uniform mean ergodicity of α-times integrated semigroups

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When considering the generator of a C_0-semigroup, the theory surrounding the asymptotic behavior of semigroups is well-established. In the early 1980s, a more general concept called integrated semigroups was introduced and has been extensively studied by many authors. This concept generalizes strongly continuous semigroups, cosine operator functions, and exponentially bounded distribution semigroups. Later, for $\alpha \geq 0$, the notion of α-times integrated semigroups was introduced; for convenience, we refer to a C_0-semigroup as a 0-times integrated semigroup and an integrated semigroup as a 1-times integrated semigroup. A natural first question is the convergence of means, specifically ergodic theorems. On the other hand, perturbation theory for operator semigroups is a crucial tool in applications, making it a well-developed field. Most of these perturbation theorems assume the relative boundedness of the perturbation operator.

In this talk, we will explore the (uniform) Cesaro ergodicity and (uniform) Abel ergodicity of α-times integrated semigroups under additional assumptions and discuss some counterexamples.

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Keywords: integrated semigroups, perturbations, Cesaro means, Abel averages, uniform Abel ergodic, uniform Cesaro ergodic.

2020 Mathematics Subject Classification: 47D62 47D06, 47A55, 37A30.

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On the semi-linear delay parabolic differential equations

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Abstract: In the present work, an initial-boundary value problem for the semi-linear delay differential equation in a Banach space with unbounded positive operators is studied. Theorem on the existence and uniqueness of a bounded solution of this problem is established. The application of the main theorem to some different semi-linear delay parabolic differential equations is presented. The first and second-order accuracy difference schemes for the solution of a one-dimensional semi-linear time-delay parabolic equation are considered. The new desired numerical results of this paper and their discussion are presented.

A comparison principle for a nonlinear fractional diffusion equation

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Abstract: The comparison principle of the nonlinear space fractional diffusion equation [1] is the subject of our work. In a bounded domain $\Omega \subset \mathbb{R}^N$, the problem is studied with homogeneous Dirichlet-type and Neumann-type boundary conditions.

We use elementary algebraic relations for several parameter sets to clearly illustrate the comparison principle for the given situation. The asymptotic behavior of global solutions, the existence of global weak solutions, and the blow-up phenomena have all been classified using the comparison principle.

Keywords: fractional differential equation, comparison principle, blow-up, global solution.

2020 Mathematics Subject Classification: 35R11, 35A01, 35B51, 35K55.

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Spectrum of the Hilbert transform on Lorentz spaces $L_{p,q}$

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Let f be a complex-valued locally integrable function on Ω, where Ω is either $\mathbb{R}_+$ or $\mathbb{R}$. The Hilbert transform of the function f on Ω is defined by the following singular integral

$$(Hf)(t) :\overset{\text{p.v.}}{=} \frac{1}{\pi i} \int\limits_{\Omega} \frac{f(x)}{t-x} dx$$

where p.v. means the Cauchy principal value of the integral.

DEFINITION.[1. Definition 4.1] Let $0 < p \leq \infty$, $0 < q \leq \infty$. Then the Lorentz $L_{p,q}(\mathbb{R}_+)$ space is the set of all Lebesgue measurable functions f such that the functional $\|f\|_{L_{p,q}} < \infty$, where

$$\|f\|_{L_{p,q}} = \begin{cases} \left(\int\limits_0^\infty \left(t^{\frac{1}{p}} f^*(t) \right)^q \frac{dt}{t} \right)^{\frac{1}{q}} & \text{if } 1 < p < \infty,\ 1 \leq q < \infty, \\ \sup\limits_{t>0} t^{\frac{1}{p}} f^*(t) & \text{if } 0 < p \leq \infty,\ q = \infty. \end{cases}$$

The Lorentz space $L_{p,q}$ is the generalization of the Lebesgue space L_p. If we take $p = q$, $L_{p,q}$ coincides with L_p and

$$\|f\|_{L_{p,p}} = \|f\|_{L_p}, \quad (f \in L_p).$$

REMARK. The Hilbert transform is bounded from $L_{p,q}(\Omega)$ to $L_{p,q}(\Omega)$ [2].

Theorem 1. *Let $p \in (1, \infty)$, $q \in (1, \infty]$. For the Hilbert transform H on $L_{p,q}(\mathbb{R}_+)$ we have*

$$\sigma(H) = \left\{ \lambda : \lambda = \pm 1 \quad \text{or} \quad \frac{1}{2\pi} \arg \frac{\lambda+1}{\lambda-1} = \frac{1}{p} \right\}$$

and, for $\lambda \in \rho(H)$, one can define its resolvent $R(\lambda, S_{p,q})$ as follows

$$R(\lambda, H)f(x) = (\lambda^2 - 1)^{-1} \left\{ \lambda f(x) + \frac{1}{\pi i} \int\limits_0^\infty \left(\frac{y}{x} \right)^{w(\lambda)} (y-x)^{-1} f(y) dy \right\},$$

where $w(\lambda) = \dfrac{1}{2\pi i} \log \dfrac{\lambda+1}{\lambda-1}$ is a holomorphic function and $w(\infty) = 0$.

Theorem 2. *Let $L_{p,q}(\mathbb{R})$ be a Lorentz space over $\mathbb{R}$ with $1 < p < \infty$ and $1 \leq q \leq \infty$ and let H be the Hilbert transform on $L_{p,q}(\mathbb{R})$. Then,*

$$\sigma(H) = \sigma_p(H) = \{\pm 1\},$$

and for $\lambda \in \rho(\lambda) = \mathbb{C} \setminus \{\pm 1\}$ we have

$$R(\lambda, H) = \frac{1}{2}(\lambda+1)^{-1}(I - H) + \frac{1}{2}(\lambda-1)^{-1}(I + H).$$

Key words: Hilbert transform, the spectrum, a resolvent, the Lorentz spaces $L_{p,q}$, Banach algebra.

2010 Mathematics Subject Classification: 47A10, 47A75, 46E30

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$L_p - L_q$ Fourier multipliers on non-commutative torus and its applications

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Abstract: In this talk,we present a short and a simple proof of $L^p - L^q$ boundedness of Fourier multipliers on noncommutative torus. We also obtain the Paley type inequality with the same approach. As applications of these results, we obtain local well-posedness in time of nonlinearpartial differential equations in the noncommutative torus.

Keywords: non-commutative torus, non-commutative L^p spaces, Fourier series, Fourier multiplier

2020 Mathematics Subject Classification: 46L52, 47L25, 42B05, 42A16, 42B15

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Analyzing stability and numerical approaches for boundary value problems of third-order partial differential equations

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Abstract: This study investigate the stability of a boundary value problem for a third-order partial differential equation within a Hilbert space, with a self-adjoint, positive definite operator. It introduces a theorem of stability estimates for this problem. Additionally, the applicability of this theorem is illustrated through three examples of third-order partial differential equations. In numerical analysis, the research proposes first and second-order difference schemes for one-dimensional third-order partial differential equations, with the second-order scheme aimed at improving accuracy. . The study also includes error analyses for both the first and second-order approximation schemes.

Keywords: Self-adjoint operator, Positive definite operator, Stability analysis, Hilbert spaces, Boundary value problems, Third-order partial differential equations, Numerical methods, Approximation schemes.

2020 Mathematics Subject Classification: 35G15; 47A62; 65M06; 65M12; 65N12

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Stability study for a multipoint nonlocal third-order partial differential equation

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Abstract: This study examines the stability of a boundary value problem associated with a third-order partial differential equation in a Hilbert space, which is regulated by a self-adjoint, positive definite operator. A new stability theorem is introduced, offering precise estimates for this context. The relevance and effectiveness of this theorem are further highlighted through its application to two specific instances of third-order partial differential equations.

Keywords: Self-adjoint operator, Positive definite operator, Stability analysis, Hilbert spaces, Boundary value problems, Third-order partial differential equations.

2020 Mathematics Subject Classification: 35G15; 47A62; 65M06.

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The asymptotic solution formulas and uniform high order difference schemes for hyperbolic perturbation problems

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Abstract: The study investigates asymptotic solution formulas for problems with a ε parameter. It also examines high-order of accuracy two-step uniform difference schemes for perturbation problems with ε^2 parameter in the highest derivative. The numerical implementation of ε uniform difference schemes is presented for the first time. Moreover, a symbolic differentiation algorithm is proposed for solving multidimensional hyperbolic boundary value perturbation problems numerically. The symbolic differentiation algorithms are developed using Matlab, and the results from numerical experiments and analyses are included.

Grants: This research was supported by Research Fund of Yıldız Technical University by grant No. FYL-2024-6281.

Keywords: Perturbation theory, Asymptotic solutions, Difference Schemes, Boundary value problems

2020 Mathematics Subject Classification: 47A55, 34E10, 39A30, 35G15

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A note on the coupled system of hyperbolic differential equations

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Abstract: This paper is concerned with a system of second order linear hyperbolic differential equations. The existence of a solution of Cauchy's initial problem is proved. Moreover, it is established that the solution is differentiable in a certain sense if the right-hand side of the equation and its coefficients have appropriate differentiability properties.

Keywords: System of hyperbolic equations, existence, Cauchy's initial problem, boundedness

2020 Mathematics Subject Classification: 35G46, 35N05

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On a new combination of orthogonal polynomials sequences

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Abstract: In this paper, we are interested in the following inverse problem. We assume that $\{Pn\}_{n\geqslant 0}$ is a monic orthogonal polynomials sequence with respect to a quasi-definite linear functional u and we analyze the existence of a sequence of orthogonal polynomials $\{Qn\}_{n\geqslant 0}$ such that we have a following decomposition

$$Q_n(x) + r_n Q_{n-1}(x,) = P_n(x) + s_n P_{n-1}(x) + t_n P_{n-2}(x) + v_n P_{n-3}(x), n > 0$$

when $v_n r_n \ngeqslant 0$, for every $n > 4$. Moreover, we show that the orthogonality of the sequence $\{Qn\}_{n\geqslant 0}$ can be also characterized by the existence of sequences depending on the parameters rn, sn, tn, vn and the recurrence coefficients which remain constants. Furthermore, we show that the relation between the corresponding linear functionals is

$$k(x-c)u = (x3 + ax2 + bx + d)v$$

where $c, a, b, d \in C$ and $k \in C\backslash\{0\}$. We also study some subcases in which the parameters r_n, s_n, t_n and v_n can be computed more easily. We end by giving an illustration for a special example of the above type relation.

Keywords: orthogonal polynomials, linear functionals, inverse problem, Chebyshev polynomials.

AMS Subject Classification: 33C45, 42C05.

On the theory of integro-differential operators of distributed order

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Abstract: The report discusses the properties of the distributed order integro-differential operators

$$D_{0x}^{[\mu]}u(x) = \int_{\mathbb{R}} D_{0x}^{t}u(x)\,\mu\,(dt) \tag{1}$$

and

$$\partial_{0x}^{[\mu]}u(x) = \int_{\mathbb{R}} \partial_{0x}^{t}u(x)\,\mu\,(dt)\,, \tag{2}$$

where D_{0x}^{t} and ∂_{0x}^{t} are the Riemann–Liouville and Gerasimov–Caputo fractional derivatives (integrals) of order t with respect to x; μ is a signed Borel measure on $\mathbb{R}$.

The operators (1) and (2) belong to the class of fractional integro-differentiation operators of distributed order [1].

For the operators (1) and (2), we construct the convolution representations, find Sonin pairs [2, 3] for the kernels, prove inversion formulas, composition laws and the Newton-Leibniz formulas.

Keywords: fractional derivative, distributed order differentiation operator, convolution, Sonin pair, Newton-Leibniz formula.

2020 Mathematics Subject Classification: 26A33, 34A08, 45E10

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New eighth-order difference schemes for approximation of the fourth-order differential equations

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Abstract Two types compact four-step difference schemes of eighth-order of approximation for the numerical solutions of local and nonlocal boundary value problems for fourth-order differential equations with dependent coefficients are studied. These these difference schemes are generated by a new Taylor decomposition on five points. The theoretical statements for the solution of these difference schemes are supported by the results of numerical experiments.

Mathematics Subject Classification (2020): 34B10, 35K10, 49K40

Keywords: Taylor's decomposition on five points, local and nonlocal problems, difference schemes, approximation, numerical experiments

Anisotropic Morrey-type spaces and their interpolation properties

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Abstract: In this paper, we consider anisotropic local Morrey spaces $LM^{\bar{\lambda}}_{\bar{p}\bar{q}}$ and generalized anisotropic Morrey-type spaces $M^{\bar{\lambda}}_{\bar{p}\bar{q}}$. We study the embedding properties of the defined spaces and their interpolation properties.

Let $k \in \mathbb{Z}$, denote by G_k the set of all cubes in $\mathbb{R}^n$ of the form $[0, 2^k)^n + 2^k m,\ m \in \mathbb{Z}^n$. Consider the set $\mathbb{G} = \bigcup\limits_{\neg\in\mathbb{Z}} \mathbb{G}_\neg$. Let $Q \in G_k$. Then the set of mutually disjoint cubes $\mathbb{T} = \{\mathbb{Q}\} \subset \mathbb{G}$ is called a local partition of the space $\mathbb{R}^n$ if $\mathbb{R}^n = \overline{\bigsqcup\limits_{Q\in\mathbb{T}} Q}$ and $|\mathbb{T} \cap \mathbb{G}_\neg| < \infty$.

Now let $\bar{n} = (n_1, ..., n_d)$: $n_i \in \mathbb{N}$, $|n| = n_1 + \cdots + n_d$, $\bar{k} = (k_1, ..., k_d)$: $k_i \in \mathbb{Z}$. Denote $G_{\bar{k}} = \{Q = Q_1 \times \cdots \times Q_d : \ Q_i \subset G_{k_i}, i = 1, ..., d\}$. Mutually disjoint cubes $\mathbb{T}_i = \{Q_i\} \subset G_{k_i}$ are called local partitions of the space $\mathbb{R}^{n_i}$, the set $\mathbb{T}_1, \ldots, \mathbb{T}_d$ - local partitions of the spaces $\mathbb{R}^{n_1}, \ldots, \mathbb{R}^{n_d}$ respectively. Family of mutually non-intersecting parallelepipeds $\mathbb{T} = \mathbb{T}_{\nVdash} \times \cdots \times \mathbb{T} = \{\mathbb{Q} = \mathbb{Q}_{\nVdash} \times \cdots \times \mathbb{Q} : \ \mathbb{Q}_\beth \subset \mathbb{T}_\beth, \ \beth = \nVdash, ..., \}$ will be called a local partition of the space $\mathbb{R}^{|\bar{n}|}$.

Let $\bar{p} = (p_1, ..., p_d),\ \bar{q} = (q_1, ..., q_d),\ \bar{\lambda} = (\lambda_1, ..., \lambda_d)$ such that $0 < p_i \leq \infty,\ 0 < q_i \leq \infty$.

For $-\infty < \lambda_i < \infty$ we define an anisotropic local Morrey space $LM^{\bar{\lambda}}_{\bar{p},\bar{q}}(\mathbb{T})$ as the set of measurable functions f for which

$$\|f\|_{LM^{\bar{\lambda}}_{\bar{p},\bar{q}}(\mathbb{T})} = \left(\sum_{k_d\in\mathbb{Z}} \cdots \left(\sum_{k_1\in\mathbb{Z}} \left(2^{-\langle \bar{k},\bar{\lambda}\rangle} \sum_{Q\in\mathbb{T}_\neg} \|f\|_{L_{\bar{p}}(Q)} \right)^{q_1} \right)^{\frac{q_2}{q_1}} \cdots \right)^{\frac{1}{q_d}} < \infty.$$

For $0 < \lambda_i < \infty$ we define the generalized anisotropic Morrey-type spaces $M^{\overline{\lambda}}_{\overline{p},\overline{q}}$ as the set of all Lebesgue measurable functions $f \in L^{loc}_{\overline{p}}(\mathbb{R}^{\overline{n}})$ for which the following norm is finite

$$\|f\|_{M^{\overline{\lambda}}_{\overline{p},\overline{q}}} = \left(\sum_{k_d\in\mathbb{Z}} \cdots \left(\sum_{k_1\in\mathbb{Z}} \left(2^{-\langle \overline{k},\overline{\lambda}\rangle} \sup_{Q\in G_{\overline{k}}} \|f\|_{L_{\overline{p}}(Q)} \right)^{q_1} \right)^{\frac{q_2}{q_1}} \cdots \right)^{\frac{1}{q_d}} < \infty,$$

where $G_{\overline{k}} = \{Q = Q_1 \times \cdots \times Q_d : \ Q_i \subset G_{k_i}, \ i = \overline{1, d}\}$.

The following theorems consider the embedding properties of these spaces and their interpolation properties.

Theorem 1. (i) Let the vectors $\overline{n} = (n_1, ..., n_d),\ \overline{p}_0 = (p_1^0, \ldots, p_d^0),\ \overline{p}_1 = (p_1^1, \ldots, p_d^1)$ è $\overline{q} = (q_1, \ldots, q_d)$ such that $0 < p_i^0 < p_i^1 < \infty, 0 < q_i \leq \infty,\ i = \overline{1, d}$. Then

$$LM^{\overline{\lambda_1}}_{\overline{p}_1,\overline{q}}(\mathbb{T}) \hookrightarrow LM^{\overline{\lambda_0}}_{\overline{p}_0,\overline{q}}(\mathbb{T}), \ \ M^{\overline{\lambda}_1}_{\overline{p}_1,\overline{q}} \hookrightarrow M^{\overline{\lambda}_0}_{\overline{p}_0,\overline{q}},$$

where $\overline{\lambda}_0 = (\lambda_1^0, \ldots \lambda_d^0),\ \overline{\lambda}_1 = (\lambda_1^1, \ldots \lambda_d^1)$ such that $\lambda_i^0 = \lambda_i^1 - \frac{n_i}{p_i^1} + \frac{n_i}{p_i^0},\ i = \overline{1, d}$;

(ii) Let the vectors $\overline{p} = (p_1, \ldots, p_d),\ \overline{q}_0 = (q_1^0, \ldots, q_d^0),\ \overline{q}_1 = (q_1^1, \ldots, q_d^1)$ such that $0 < q_i^0 < q_i^1 \leq \infty,\ i = \overline{1, d}$. Then

$$LM^{\overline{\lambda}}_{\overline{p},\overline{q}_0}(\mathbb{T}) \hookrightarrow LM^{\overline{\lambda}}_{\overline{p},\overline{q}_1}(\mathbb{T}), \ \ M^{\overline{\lambda}}_{\overline{p},\overline{q}_0}(\mathbb{T}) \hookrightarrow \mathbb{M}^{\overline{\lambda}}_{\urcorner,\shortparallel_{\nVdash}}(\mathbb{T}).$$

The following theorem shows the interpolation properties of the spaces mentioned.

Let the vectors be given $\overline{p} = (p_1, \ldots, p_d), \quad \overline{q} = (q_1, \ldots, q_d), \quad \overline{q}_0 = (q_1^0, \ldots, p_d^0), \quad \overline{q}_1 = (q_1^1, \ldots, p_d^1), \quad \overline{\lambda}_0 = (\lambda_1^0, \ldots, \lambda_d^0), \quad \overline{\lambda}_1 = (\lambda_1^1, \ldots, \lambda_d^1), \quad \overline{\theta} = (\theta_1, \ldots, \theta_d)$.

Theorem 2. Let $\lambda_i^0 \neq \lambda_i^1$, $0 < p_i \leq \infty$, $0 < q_i, q_i^0, q_i^1 \leq \infty$, $\theta_i \in (0,1)$ and $\overline{\lambda} = (\lambda_1, \ldots, \lambda_d):\ \lambda_i = (1-\theta_i)\lambda_i^0 + \theta_i \lambda_i^1$. Then
for $-\infty < \lambda_i^0 < \lambda_i^1 < +\infty$

$$\left(LM^{\overline{\lambda}_0}_{\overline{p},\overline{q}_0}(\mathbb{T}), LM^{\overline{\lambda}_1}_{\overline{p},\overline{q}_1}(\mathbb{T})\right)_{\overline{\theta},\overline{q}} = LM^{\overline{\lambda}}_{\overline{p},\overline{q}}(\mathbb{T}),$$

for $0 < \lambda_i^0 < \lambda_i^1 < +\infty$

$$\left(M^{\overline{\lambda}_0}_{\overline{p},\overline{q}_0}, M^{\overline{\lambda}_1}_{\overline{p},\overline{q}_1}\right)_{\overline{\theta},\overline{q}} \hookrightarrow M^{\overline{\lambda}}_{\overline{p},\overline{q}}.$$

Keywords: anisotropic local Morrey-type spaces, generalized anisotropic Morrey-type spaces, embedding properties of anisotropic Morrey spaces, interpolation properties of anisotropic Morrey spaces

Mathematics Subject Classification: 46Axx

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A stable difference scheme for the solution of a source identification problem for telegraph-parabolic equations

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Abstract: We construct a first order of accuracy difference scheme for the approximate solution of the inverse problem for telegraph-parabolic equations with an unknown spacewise dependent source term

$$u''(t) + \alpha u'(t) + Au(t) = p + f(t),\ 0 < t < 1, \tag{1}$$

$$u'(t) + Au(t) = p + g(t),\ -1 < t < 0, \tag{2}$$

$$u(0+) = u(0-),\ u'(0+) = u'(0-),\ u(-1) = \varphi,\ u(\lambda) = \psi,\ -1 < \lambda \leq 1 \tag{3}$$

in a Hilbert space H with a self-adjoint positive definite operator A satisfying $A \geq \delta I$, where $\delta > \frac{\alpha^2}{4}$ and $\alpha \geq 0$. The unique solvability of constructed difference scheme and the stability estimates for its solution are obtained. The proofs are based on the spectral representation of the self-adjoint positive definite operator in a Hilbert space. In applications, this abstract result permits us to obtain the stability estimates for the solutions of first order of accuracy difference schemes for approximate solutions of two source identification problems for telegraph-parabolic equations. We discuss the numerical procedure for implementation of these difference schemes and provide error analysis for numerical results of simple test problems.

Keywords: Source identification problem, telegraph-parabolic equation, stable difference scheme

2010 Mathematics Subject Classification: 35M10, 35R30, 65M06, 65M32

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Boussinesq convolution equations in Fourier type spaces and applications

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Abstract: In this talk, the Cauchy problem for linear and nonlinear nonlocal Boussinesq equations is studied. The equation involves convolution terms including abstract operator functions in Fourier type Banach space E. Here, assuming enough smoothness on the initial data and the growth assumptions on operator functions, the local, global existence, uniqueness and regularity properties of solutions are established in terms of fractional powers of given sectorial operator. We can obtain a different classes of nonlocal Boussinesq equations by choosing the space E and linear operator, which occur in a wide variety of physical systems.

The aim here, is to study the existence, uniqueness and regularity properties of solutions of the Cauchy problem for convolution abstract Boussinesq equation (BE)

$$(1) \qquad (1-\Delta_x)\, u_{tt} + a * \Delta u - A * u = [B * f(u)]\,,\ x, t \in \mathbb{R}^n \times (0,T)\,,$$

$$u(x,0) = \varphi(x)\,,\ u_t(x,0) = \psi(x) \ \text{ for a.e. } x \in \mathbb{R}^n,$$

where $A = A(x)$ is a linear, $B = B(x)$, $f(u)$ are nonlinear operator functions defined in a Banach space E, a is a complex valued function, $\upsilon * u$ denotes the convolution of $\upsilon = \upsilon(x)$, $u = u(x)$ and $T \in (0,\ \infty]$. Here, Δ_x denotes the Laplace operator with respect to $x \in \mathbb{R}^n$, $\varphi(x)$ and $\psi(x)$ are the given E-valued initial functions.

First, we consider Cauchy problem for the corresponding linearized problem

$$(1-\Delta_x)\, u_{tt} + a * \Delta u - A * u = g(x,t)\,,\ t \in (0,T)\,,$$

$$(2) \qquad u(x,0) = \varphi(x)\,,\ u_t(x,0) = \psi(x) \ \text{ for a.e. } x \in \mathbb{R}^n,$$

where $g(x,t)$ is a given E-valued function.

We prove first, the well-posedness of the linear problem (2) in E-valued L^p spaces. The L^p-well-posedness of the Cauchy problem (1) depends crucially on the presence of a suitable kernel. This fact can be used to obtain the local, global existence, uniqueness and regularity properties of nonlinear problem (1).

Keywords: Nonlocal wave equations, Fourier type Banach spaces, Boussinesq equations, abstract differential equations, Fourier multiplier

The Hardy-Littlewood theorem for double Fourier-Haar series from mixed metric Lebesgue $L_{\bar{p}}[0,1]^2$ and net $N_{\bar{p},\bar{q}}(M)$ spaces

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Abstract: We obtain a criterion given in terms of the Fourier-Haar coefficients for the function $f(x_1,x_2)$ to belong to the net space $N_{\bar{p},\bar{q}}(M)$ and to the Lebesgue space $L_{\bar{p}}[0,1]^2$ with mixed metric, where $1<\bar{p}<\infty$, $0<\bar{q}\le\infty$, $\bar{p}=(p_1,p_2)$, $\bar{q}=(q_1,q_2)$ and M is the set of all rectangles in $\mathbb{R}^2$. We prove the Hardy-Littlewood theorem for multiple Fourier-Haar series.

Let us consider the Fourier-Haar series for a function $f(x_1,x_2)\in L_1[0,1]^2$

$$f(x_1,x_2)\sim\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{j_1=1}^{2^{k_1}}\sum_{j_2=1}^{2^{k_2}} a_{k_1,k_2}^{j_1,j_2}(f)\chi_{k_1}^{j_1}(x_1)\chi_{k_2}^{j_2}(x_2),$$

where $a_{k_1,k_2}^{j_1,j_2}(f)$ are the Fourier-Haar coefficients of f, that is

$$a_{k_1,k_2}^{j_1,j_2}(f)=\int_0^1\int_0^1 f(x_1,x_2)\chi_{k_1}^{j_1}(x_1)\chi_{k_2}^{j_2}(x_2)dx_1dx_2.$$

For $\bar{\sigma}=(\sigma_1,\sigma_2)\in\mathbb{R}^2$, $1\le\bar{q}=(q_1,q_2)\le\infty$ we define the space $l_{\bar{q}}^{\bar{\sigma}}(l_\infty)$ as the set of all sequences $a=\{a_{k_1k_2}^{j_1j_2}: k_i\in\mathbb{Z}, k_i\ge 0, 1\le j_i\le 2^{k_i}, i=1,2\}$, for which

$$\|a\|_{l_{\bar{q}}^{\bar{\sigma}}(l_\infty)}=\left(\sum_{k_2=0}^{\infty}\left(\sum_{k_1=0}^{\infty}\left(2^{\sigma_1k_1+\sigma_2k_2}\sup_{\substack{1\le j_i\le 2^{k_i}\\ i=1,2}}|a_{k_1k_2}^{j_1j_2}|\right)^{q_1}\right)^{\frac{q_2}{q_1}}\right)^{\frac{1}{q_2}}<\infty,$$

where the expression $\left(\sum\limits_{k=0}^{\infty}b_k{}^q\right)^{\frac{1}{q}}$ with $q=\infty$ is understood as $\sup_{k\ge0}b_k$. Let M be the set of all rectangles $Q=Q_1\times Q_2$ from $\mathbb{R}^2$. For a function $f(x_1,x_2)$ integrable on each set $Q\in M$, we define

$$\bar{f}(t_1,t_2;M)=\sup_{|Q_i|\ge t_i}\frac{1}{|Q_1||Q_2|}\left|\int_{Q_1}\int_{Q_2}f(x_1,x_2)dx_1dx_2\right|,\quad t_i>0,$$

where $|Q_i|$ is the length of a segment Q_i.

Let $0<\bar{p}=(p_1,p_2)<\infty$, $0<\bar{q}=(q_1,q_2)\le\infty$. By $N_{\bar{p},\bar{q}}(M)$ denote the set of all functions $f(x_1,x_2)$, for which

$$\|f\|_{N_{\bar{p},\bar{q}}(M)}=\left(\int_0^\infty\left(\int_0^\infty\left(t_1^{\frac{1}{p_1}}t_2^{\frac{1}{p_2}}\bar{f}(t_1,t_2;M)\right)^{q_1}\frac{dt_1}{t_1}\right)^{\frac{q_2}{q_1}}\frac{dt_2}{t_2}\right)^{\frac{1}{q_2}}<\infty,$$

where if $q=\infty$, the expression $\left(\int_0^\infty(\varphi(t))^q\,\frac{dt}{t}\right)^{\frac{1}{q}}$ is understood as $\sup_{t>0}\varphi(t)$.

Theorem 1. Let $1<\bar{p}=(p_1,p_2)<\infty$, $0<\bar{q}=(q_1,q_2)\le\infty$, $\sigma_i=\frac{1}{2}-\frac{1}{p_i}$, $i=1,2$, M be the set of all rectangles in $[0,1]^2$. Let $f(x_1,x_2)\sim\sum\limits_{k_1=0}^{\infty}\sum\limits_{k_2=0}^{\infty}\sum\limits_{j_1=1}^{2^{k_1}}\sum\limits_{j_2=1}^{2^{k_2}}a_{k_1,k_2}^{j_1,j_2}(f)\chi_{k_1}^{j_1}(x_1)\chi_{k_2}^{j_2}(x_2)$ be the Fourier-Haar series for a function $f(x_1;x_2)$. Then, for $f\in N_{\bar{p},\bar{q}}(M)$ it is necessary and sufficient that the sequence of its Fourier-Haar coefficients $a=\{a_{k_1k_2}^{j_1j_2}: k_i\in\mathbb{Z}, k_i\ge0, 1\le j_i\le 2^{k_i}, i=1,2\}$ belongs to the space $l_{\bar{q}}^{\bar{\sigma}}(l_\infty)$.

Furthermore

$$\|f\|_{N_{\bar{p},\bar{q}}(M)} \asymp \|a\|_{l^{\bar{\sigma}}_{\bar{q}}(l_\infty)}. \tag{1}$$

Let $0 < \bar{p} \leq \infty$. The space $L_{\bar{p}}[0,1]^2$ is defined as the set of measurable functions f on $[0,1]^2$ such that

$$\|f\|_{L_{\bar{p}}[0,1]^2} := \left(\int_0^1 \left(\int_0^1 |f(x_1,x_2)|^{p_1}\, dx_1 \right)^{\frac{p_2}{p_1}} dx_2 \right)^{\frac{1}{p_2}}$$

is finite and it is called the Lebesgue space with a mixed metric. The function $f(x_1,x_2)$ is called monotonically non-increasing in each variable, if for $0 \leq y_1 \leq x_1$ and $0 \leq y_2 \leq x_2$ the following inequality holds

$$f(x_1,x_2) \leq f(y_1,y_2).$$

Theorem 2. Let $1 < \bar{p} = (p_1,p_2) < \infty$, $\sigma_i = \frac{1}{2} - \frac{1}{p_i}$, $i = 1,2$, $f(x_1,x_2)$ be non-negative, monotonic function in each variable. Then $f \in L_{\bar{p}}[0,1]^2$ if and only if the sequence of its Fourier-Haar coefficients $a = \{a^{j_1 j_2}_{k_1 k_2} : k_i \in \mathbb{Z}, k_i \geq 0, 1 \leq j_i \leq 2^{k_i}, i = 1,2\}$ belong to the space $l^{\bar{\sigma}}_{\bar{p}}(l_\infty)$, and the following relation holds

$$\|f\|_{L_{\bar{p}}[0,1]^2} \asymp \|a\|_{l^{\bar{\sigma}}_{\bar{p}}(l_\infty)}.$$

Keywords: net space, Lebesgue space, anisotropic space, Fourier series, Haar system.

2020 Mathematics Subject Classification: 42B05, 42B35, 46B70

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Modern internet environment in educational technologies of the higher educational institutions

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Abstract: This report is devoted to a spectral problem for a multiple differentiation operator with an integral perturbation of boundary conditions of one type which are regular, but not strongly regular: Electronic education is almost ideal for organizing distance learning, as well as for implementing the educational process in university branches. e-Learning can be used to organize training with permanent teaching, i.e. mixed learning. Its advantages are undeniable: Internet access to electronic courses from any location; effective feedback from the teacher to the student; the possibility of training without interruption from work; high quality of educational materials; technological training – use of innovative information and telecommunication technologies in the educational process; ability of the student to regulate the speed of studying the material; reducing the material costs of training, since the exchange of information in electronic form via the Internet reduces the cost of purchasing textbooks and writing materials.

It should be noted that these advantages were appreciated by our students specializing in applied mathematics and computer science. The Turkmen market of electronic education is developing rapidly, and the number of universities implementing these technologies in the educational process is constantly increasing.

In this study, an analysis of works on the specialization of applied mathematics and computer science was carried out.

Keywords:E-Learning, learning process, MOODLE, distance learning, higher educational institution.

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Unsupervised and supervised machine learning methods for customer personality analysis

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Abstract: The main goals of research to increase the marketing strategies and understanding of consumer behavior by incorporating clustering and classification algorithms into personality analysis, this study used unsupervised learning approaches such as K-Means, DBSCAN, and Affinity Propagation to segment consumer data based on purchasing habits and these approaches were compared using the first Silhouette Coefficient, the second Davies Bouldin Score, and the third Calinski Harabasz Index. at the same time, the first Logistic Regression, the second Naive Bayes, and the third Decision Tree, and the Fourth Random Forest algorithms were used to predict consumer behaviors, which were measured by accuracy, precision, recall, and F1. These results reveal that K Means outperforms other clustering algorithms in terms of well-defined clusters and optimal grouping and the Random Forest has been found to be the best supervised model, with an effective balance of accuracy and complexity that is suitable for extensive customer insights, the inclusion of these tools has considerably improved consumer engagement and loyalty by allowing more precise segmentation and behavior prediction [1].

Keywords: Customer Personality Analysis, K-Means Clustering, Logistic regression, Random Forest ,naive bayes.

2020 Mathematics Subject Classification: 68T99, 62P99

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Machine learning-based approach for malignant mesothelioma diagnosis

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Abstract: Malignant mesothelioma is a kind of cancer that is known for its sneaky nature and high death rate [1]. The occurrence rate of Malignant Mesothelioma is higher in rural areas especially where soil composition contains asbestos such as “white soils”, “corak” and “luto soils” leading to the development of MM. The paper aims at establishing whether machine learning algorithms can be effectively employed to accurately diagnose MM. A dataset of 324 MM patient records based on research conducted at the Faculty of Medicine, Dicle University, Turkey was used to conduct a comparative evaluation of six machine learning models, including Naive Bayes, K-Neighbors, Random Forest, Decision Tree, Gradient Boosting and SVM. The Random Forest model had an accuracy rate of 99.5% which was better than the others according to our findings. On this account, it can be claimed that machine learning approaches have a potential for this purpose because they are capable of finding patterns and associations within MM data useful for strategies focused on early recognition and prevention.

Keywords: Decision Tree, Gradient Boosting, K-Neighbors, Malignant Mesothelioma, Random Forest, SVM

2020 Mathematics Subject Classification: 68T99, 62P99

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Evaluating machine learning algorithms for detecting cervical cancer

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Abstract: One of the significant diseases in women's public health is cervical cancer. This cancer is preventable, and it is necessary to provide accurate public awareness to the community. Treatment for this kind of cancer is substantially aided by early detection. This study aims to find acceptable machine-learning models for the detection of cervical cancer by comparing various classifiers. Cross-validation approaches were used to determine the classification performances of six models: XGBoost, Gradient Boosting Machine (GBM), Random Forest (RF), Naive Bayes (NB), Support Vector Machine (SVM), and Decision Tree (DT). A voting classifier was also constructed using the top-performing individual models. The model performance results show that the accuracy rates of the RF and SVM models are as high as 91.97% and 89.11%, respectively. With an accuracy of 93.34%, the voting classifier showed superior performance fort the detection of cervical cancer.

Keywords: Cervical Cancer, Machine Learning, Voting Classifier

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Smote and adasyn technology to compare light GBM, decision tree, and logistic regression in credit card fraud identification

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Abstract: This paper aims to compare logistic regression, decision tree, and LightGBM to determine the effectiveness of each model in credit card fraud identification using data imbalance handling techniques such as SMOTE and ADASYN. According to our analysis, LightGBM demonstrated superior detection performance in terms of accuracy, specificity, and F1-score when using data imbalance handling techniques SMOTE and ADASYN [1] .

2020 Mathematics Subject Classification: 68T99, 62P99

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A comparison between naive bayes and random forest for SMS spam detection

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Abstract: This study aims how two popular algorithms Naive Bayes and Random Forest goes well when the requested SMS message is classified as spam or ham. including recall, precision, and overall accuracy. This demonstrates its ability to detect and filter advanced spam messages which is ideal for applications that require high accuracy but increase computing resources. These spam detection methods will run efficiently protecting customers from fakes, and maintain trust in these important communication channels.This study shows Naive Bayes has better performance in the terms of classification measures for SMS spam detection.

Keywords: naïve bayes, spam filter, random forest, sms spam

2020 Mathematics Subject Classification: 68T99, 62P99

Project development and management

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Abstract: The article investigates the characteristics analyzed in the project solution: technological feasibility, long-term viability, economic efficiency, political, social and economic potential, organizational and management capabilities. Changes in the value of monetary assets over time are determined by the following conditions: devaluation, income to be received in the form of interest (alternatively), default (in the event of default by the principal or borrower). In this regard, the project analysis uses an adjustment for changes in the above conditions of the current costs of the project (discounting method). As a result of using the method, it is possible to calculate the future value of the released funds and the present value of future funds received. Project management is a synthetic science and includes specialized knowledge that requires unique knowledge beyond specialized knowledge. Specialized knowledge reflects the specifics of the types of work involved in projects (construction, innovation, environmental, analytical, organizational). In addition, project management itself has become a true science because of the methods and tools successfully applied to various projects, thanks to the knowledge gained from the study of general principles characteristic of projects in all business sectors. Project management is the management of the entire work (life cycle) of a project as a result of using modern methods and system of management technologies in order to achieve the results of the structure and scope of specified work. Project, cost, time, quality and satisfaction of project participants is also the ability to coordinate human and material resources.

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Some properties of the solution of the nonlinear multidimensional Volterra integro-functional equation

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Abstract: In the domain $D=\prod_{i=1}^{p}\left[-1,1+a_i\right],\ a_i\geq 0\ \ (i=\overline{1,n}),$ we investigate the multidimensional Volterra integro-functional equation

$$x(t_1,t_2,\ldots,t_n)=f(t_1,t_2,\ldots,t_n)+\int_{a_1-t_1}^{t_1}K_1(s_1,t_2,\ldots,t_n,x\left(s_1,t_2,\ldots,t_n\right))ds_1$$

$$+\int_{a_1-t_1}^{t_1}\int_{a_2-t_2}^{t_2}K_2(s_1,s_2,t_3,\ldots,t_n,x\left(s_1,s_2,t_3,\ldots,t_n\right))ds_1ds_2+\ldots$$

$$+\int_{a_1-t_1}^{t_1}\cdots\int_{a_n-t_n}^{t_n}K_n(s_1,s_2,\ldots,s_n,x\left(s_1,s_2,\ldots,s_n\right))ds_1ds_2...ds_n.$$

Here, $f(t_1,t_2,\ldots,t_n)$ and $K_i(s_1,s_2,\ldots,s_n,x\left(s_1,s_2,\ldots,s_n\right)),i=\overline{1,n}$ are given smooth functions. The main theorem on the existence and uniqueness of a continuous solution of this equation is proved. Specifically, it shows that there exists a continuous solution to this equation for the given data. Moreover, we also study the continuously dependence on parameters of the solution of the following multidimensional Volterra integro-functional equation

$$x(t_1,t_2,\ldots,t_n)=f(t_1,t_2,\ldots,t_n)+\int_{a_1-t_1}^{t_1}K_1(s_1,t_2,\ldots,t_n,x\left(s_1,t_2,\ldots,t_n,\mu_1\right))ds_1$$

$$+\int_{a_1-t_1}^{t_1}\int_{a_2-t_2}^{t_2}K_2(s_1,s_2,t_3,\ldots,t_n,x\left(s_1,s_2,t_3,\ldots,t_n\right),\mu_2)ds_1ds_2+\ldots$$

$$+\int_{a_1-t_1}^{t_1}\cdots\int_{a_n-t_n}^{t_n}K_n(s_1,s_2,\ldots,s_n,x\left(s_1,s_2,\ldots,s_n\right),\mu_n)ds_1ds_2...ds_n.$$

Here, $\mu_i\in R^1(i=\overline{1,n})$ are the parameters.

Keywords: integro-differential equation, multidimensional, existence and uniqueness, Volterra integral equation with partial integrals, classical solution

2020 Mathematics Subject Classification: 34A34, 34A12, 34A30

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On the solution of hyperbolic Volterra integro-functional equation

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Abstract: In this work, we study initial boundary value problem for hyperbolic Volterra integrofunctional equation

(1) $$u_{xt}(x,t) = \Phi(x,t,u(x,t),u_x(x,t),\int_0^x\int_0^t K(x,t,\tau,s,u(\tau,s)),u_\tau(\tau,s)dsd\tau,\\ (x,t)\in\ [0,a]\times[0,T]=D$$

(2) $$u(x,0)+\sum_{i=1}^{p}\alpha_i u\left(x,t_i\right)=\varphi(x),\quad x\ \in[0;a],$$

(3) $$u(0,t)+\sum_{i=1}^{q}\beta_i u\left(x_i,t\right)=\psi(t),\quad t\ \in[0;T],$$

(4) $$\varphi(0)-\psi(0)=\sum_{i=1}^{p}\alpha_i u\left(0,t_i\right)-\sum_{i=1}^{q}\beta_i u\left(x_i,0\right).$$

Here

$$t_i\ \in[0;T]\ \left(i=\overline{1,p}\right),\quad x_i\in[0;a]\ \left(i=\overline{1,q}\right),\\ 1+\textstyle\sum_{i=1}^{p}\alpha_i=A\neq0,\quad 1+\sum_{i=1}^{q}\beta_i=B\neq0.$$

We prove existence and uniqueness theorem for problem (1)-(4) under some condition. Moreover, we also study the continuously dependence of the solution of this equation on the parameters.

Keywords: integro-differential equation, hyperbolic, existence and uniqueness, Volterra integral equation with partial integrals, classical solution

2020 Mathematics Subject Classification: 34A34, 34A12, 34A30

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A remark on positivity of differential operator with Samarskii-Ionkin type condition

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Abstract: The present report considers the operator

$$A^x u = \left(-\frac{d^2}{dx^2} + \mathrm{I}\right) u, \tag{1}$$

where I denotes the identity operator and domain of A^x is

$$\mathcal{D}(A^x) = \left\{ u \in C^{(2)}[0,1] : u(0) = 0,\ u'(0) = u'(1) + \mu u(1) \right\} \tag{2}$$

with $\mu > 0$.

It establishes the positivity of the differential operator A^x in $C[0,1]$. Furthermore, it investigates the structure of interpolation spaces $E_\alpha(C[0,1], A^x)$. Moreover, it proves for each $0 < \alpha < 1/2$ that the topological equivalence of the interpolation space $E_\alpha(C[0,1], A^x)$ and the Hölder space $\overset{\circ}{C}{}^{2\alpha}[0,1]$ and hence it establishes the positivity of A^x in $\overset{\circ}{C}{}^{2\alpha}[0,1]$. It also applies the theoretical results to obtain noval coercive inequalities for the solution of a nonlocal boundary value problem for parabolic equation.

Keywords: Positive differential operators, Interpolation space generated by a positive differential operator, Green's fuction, Samarskii-Ionkin type condition, coercive inequalities

2020 Mathematics Subject Classification: 35J08, 47B65

Hörmander multiplier theorem on the noncommutative Euclidean space and its applications

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Abstract: To explore applications of the L^p-L^q boundedness of Fourier multipliers [1] in the context of partial differential equations (PDEs) on noncommutative Euclidean spaces. Specifically, we derive L^p-L^q norm estimates for solutions to heat, wave, and Schrödinger-type equations for the range $1 < p \leq 2 \leq q < \infty$.

2020 Mathematics Subject Classification: 46L51, 46L52, 47L25, 11M55, 46E35, 42B15, 42B05, 43A50, 42A16, 32W30

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On numerical solution of the Schrödinger-parabolic equation with nonlocal boundary condition

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Abstract: In the present paper, the nonlocal boundary value problem for the following Schrödinger-parabolic equation with the nonlocal boundary condition

$$(1) \qquad \begin{cases} \frac{du(t)}{dt} + Au\,(t) = f\,(t)\,(0 \le t \le 1)\,, \\ i\frac{du(t)}{dt} + Au\,(t) = g\,(t)\,(-1 \le t \le 0)\,, \\ u\,(-1) = \sum\limits_{j=1}^{N} \alpha_j u\,(\mu_j) + \varphi, \ \sum\limits_{j=1}^{N} |\alpha_j| \le 1, 0 < \mu_j \le 1 \end{cases}$$

in a Hilbert space H with self-adjoint positive definite operator A is considered. The stability estimates for the solution of the given problem is established. The first and second order of difference schemes are presented for approximately solving a specific nonlocal boundary problem. The theoretical statements for the solution of these difference schemes are supported by the result of numerical examples.

Keywords: partial differential equation, nonlocal boundary value problem, stabilty.

2020 Mathematics Subject Classification: 65L10, 34B10, 65M12

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On Sobolev inequality on the quantum Euclidean space

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Abstract: In this talk we study the Sobolev inequality on the quantum Euclidean space and discuss its proof in noncommutative setting.Moreover, we show that the logarithmic Sobolev inequality is equivalent to the Nash inequality for possibly different constants on the quantum Euclidean space by completing the list in noncommutative Varopoulos's theorem. This is a joint work with Michael Ruzhansky and Serikbol Shaimardan.

Keywords: Noncommutative or quantum Euclidean space, Sobolev inequality, Nash inequality, Logarithmic Sobolev inequality

2020 Mathematics Subject Classification: 46L51, 47L25, 46E35, 42B15, 35L76, 35L05

Applications of a new series approach for solving Volterra integro-differential equations

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Abstract: This paper explores the applicability and efficiency of a numerical procedure for solving some practical examples of differential equations of Volterra type based on a novel approach namely variational iteration method (VIM), Laplace transform and Pad450 approximation which will reduce the time used and the effort expended in calculations resulting from the high order of approximation limits. Through this research, we present a comprehensive framework for the proposed approach and how to apply it to give better and more accurate results that are often identical to the exact solution. To illustrate the effectiveness and feasibility of our approach, we provide several practical examples. The results are displayed in tables and figures, and our approach is compared to the exact solutions. This study not only contributes to this type of equation but also offers a powerful and efficient way to a wide range of other types of differential equations with increased accuracy and reduced computational effor

Keywords: Integral equations, series expansion, VIM technique, Laplace Transform, Pade approximant.

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Approximation formula for a propagator in symmetric operator ideals

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Abstract: Let H be a separable Hilbert space and A and $B(t)$, $t \geq 0$ be the non-negative self-adjoint operators on H. We consider the following abstract non-autonomous evolution equation on a compact interval $[0, 1]$

$$\begin{cases} \frac{du(t)}{dt} = -(A + B(t))u(t), \\ u(s) = u_s \in H, \end{cases} \qquad 0 \leq s \leq t \leq 1. \tag{1}$$

The main problem of solving the non-autonomous evolution equation (1), is to find a so-called propagator (solution operator) $\{U(t,s)\}_{0 \leq s \leq t \leq 1}$ such that $u(t) = U(t,s)u_s$ is a solution of (1) in certain sense for an appropriate set of initial data u_s.

Assume that an operator A is the generator of a strongly continuous semigroup $\{e^{-tA}\}_{t \geq 0}$ such that $\{e^{-tA}\}_{t>0} \subset \mathcal{I}(H)$, where $\mathcal{I}(H)$ is an arbitrary symmetric Banach ideal.

In this talk, assuming further several other classical conditions, we prove that there exists a propagator $U(\cdot, \cdot)$ which is continuous (in the norm of $\mathcal{I}(H)$) in both variables and is continuously differentiable (in the norm of $\mathcal{I}(H)$) in the first variable and which "solves" the non-autonomous evolution equation (1). Furthermore, we prove an approximation formula for this propagator in the norm of $\mathcal{I}(H)$.

For more detailed discussion of the given problem, we refer the reader to [1].

Keywords: Evolution equation, propagator formula, symmetrically normed ideals.

2020 Mathematics Subject Classification: 47D06, 47B25, 47L20

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An accurate numerical method for solving fourth-order partial differential equation

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Abstract: In this work, an efficient numerical method is proposed for solving time-dependent problems governed by a fourth-order partial differential equation, known as the Euler-Bernoulli Beam model. The proposed method is based on compact finite difference techniques. Through rigorous analysis, we demonstrate the efficiency of the new scheme by solving various test problems. The numerical results highlight the accuracy of the proposed method, showing that it outperforms existing approaches. Several examples of numerical experiments are provided to validate its effectiveness.

Keywords: Beam equation, numerical solution, compact finite scheme, Stability analysis.

2020 Mathematics Subject Classification: 65L20 · 65L12 · 37N30

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Classification of points of two-dimensional surfaces in $^1R_5$

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Abstract: The study of curves and surfaces of maximum dimension in multidimensional pseudo-Euclidean spaces was developed by B.A. Rosenfeld [1]. When the codimension of surfaces is greater than one, there is no general theory of surfaces even in the Euclidean multidimensional space R_n [2], [3]. Of vectors $\overrightarrow{r}_u, \overrightarrow{r}_v, \overrightarrow{N}_1, \overrightarrow{N}_2, \overrightarrow{N}_3$ following applies:

Theorem 1. Vectors $[\overrightarrow{r}_u, \overrightarrow{r}_v, \overrightarrow{N}_1, \overrightarrow{N}_2, \overrightarrow{N}_3]$ forms a linearly independent basis point $M \in F_2$, and vectors $[\overrightarrow{N}_1, \overrightarrow{N}_2, \overrightarrow{N}_3]$ the basis of the normal space W_3.

We present a method for determining the internal geometry of a two-dimensional tangent plane using tangent $\overrightarrow{r}_u, \overrightarrow{r}_v$ vectors.

Theorem 2. When the condition is satisfied for a function from equations (3):

1. $z'_u \cdot z'_v > 0$ and $[x'_1(v)]^2 > [y'_1(v)]^2$ - geometry in the tangent plane of Euclidean.
2. $z'_u z'_v > 0$ and $\overline{r}_v^2 < 0$ are the Minkowski plane.
3. $[x'_1(v)]^2 = [y'_1(v)]^2$ is the Galiliev plane.

It is not advisable to study two-dimensional surfaces when they belong to the same hyperplane of dimension four. Although two-dimensional surfaces in four-dimensional space have been studied, relatively little has been done [4].

Definition. The point $M \in F_2$ in $^1R_5$ is called Euclidean, pseudo-Euclidean and Gallium in accordance with the geometry on the tangent plane. We are interested in the existence of entire two-dimensional surfaces having only one type of two-dimensional tangent planes.

Theorem 3. When the requirement of Theorem 2 is fulfilled in all values $(u, v) \in D$ and $z(u, v)$ are not constant, $x(u), y(u), x_1(v), z_1(v)$ take values other than zero in some interval there is an entire two-dimensional surface having one type of two-dimensional tangent planes.

Keywords: : Pseudo-Euclidean spaces, orthogonality, isotropic vector, tangent two-dimensional plane, normal space, isotropic cone, Minkowski plane, Galileo plane.

2020 Mathematics Subject Classification: 51F20,51F25

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Numerical solution of one-phase fluids flows in fractal media

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Abstract: In the article, the finite difference method is applied to find the approximate solution of the first boundary value problem for the partial differential equation with the order $1 + a$, $0 < a < 1$, which describes the flow of a one-phase fluid in a fractal medium. An auxiliary problem whose smoothness of the solution is higher than the smoothness of the solution of the main problem is included, and using its advantages, effective finite difference algorithms are proposed for finding the approximate solution of the main problem. Thus, the considered differential problem is brought to a system of three-diagonal algebraic equations, which automatically satisfies the robustness of the driving method, and the solution of this system can be easily calculated.

Keywords: Flow in a fractal medium, fractional differential equation, finite difference equivalents of differentiation and integration in the Riemann-Liouville sense.

2020 Mathematics Subject Classification: 35R11, 65M06

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