Third International Conference on Analysis and Applied Mathematics – ICAAM 2016

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ICAAM 2016

THE ABSTRACT BOOK

07-10 September 2016

Institute of Mathematics
and Mathematical Modelling
Almaty, Kazakhstan

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Allaberen Ashyralyev

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The conference is organized biannually. Previous conferences were held in Gumushane, Turkey in 2012 and in Shymkent, Kazakhstan in 2014.

The aim of the International Conference on Analysis and Applied Mathematics (ICAAM) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why, we plan to found the conference series to provide a forum for researches and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics.

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Conference Sections and Minisymposiums :

- Analysis
- Applied Mathematics
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- Other Topics (Algebra, Geometry, Topology...)
- Minisymposium:Mathematical Modelling
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FOREWORD

The Organizing Committee of ICAAM and Institute of Mathematics and Mathematical Modelling are pleased to invite you to the Third International Conference on Analysis and Applied Mathematics, ICAAM 2016. The meeting will be held on September 7-10, 2016 in Almaty, Kazakhstan. This conference is dedicated to 70th birthday of Prof. Tynysbek Kalmenov.

The conference is organized biannually. Previous conferences were held in Gumushane, Turkey in 2012 and in Shymkent, Kazakhstan in 2014. The proceedings of ICAAM 2012 and ICAAM 2014 were published in AIP (American Institute of Physics) Conference Proceedings. Institute of Mathematics and Mathematical Modelling is pleased to host the third conference which is focused on various topics of analysis and its applications, applied mathematics and modeling.

The conference will consist of plenary lectures, mini symposiums and contributed oral presentations. The proceedings of ICAAM 2016 will be published in AIP (American Institute of Physics) Conference Proceedings. Selected full papers of this conference will be published in peer-reviewed international journals:

- FILOMAT (Science Citation Index),
- BOUNDARY VALUE PROBLEMS (Science Citation Index),
- CONTEMPORARY ANALYSIS AND APPLIED MATHEMATICS.

The aim of the International Conference on Analysis and Applied Mathematics (ICAAM) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why, we plan to found the conference series to provide a forum for researches and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics. The Conference Organizing Committee would like to thank our sponsors. The main organizer of the conference is Institute of Mathematics and Mathematical Modelling, Almaty, Kazakhstan. The conference is also supported by Al-Farabi Kazakh National University, Almaty and L. N. Gumilyov Eurasian National University, Astana, Kazakhstan. We would like to thank Institute of Mathematics and Mathematical Modeling, Al-Farabi Kazakh National University and L. N. Gumilyov Eurasian National University for their support. We alsa would like to thank to all invited speakers, International Organizing Committee, International Organizing Committe, and Technical Program Committee Members. With our best wishes and warm regards,

Chairs:

Prof. Allaberen Ashyralyev

Prof. Mukhtarbay Otelbaev

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TO 70TH ANNIVERSARY

May 2016 was the 70th anniversary of Tynysbek Sharipovich Kal'menov, the outstanding Kazakhstani mathematician, the academician of the National Academy of Sciences of the Republic of Kazakhstan.

He was born in the South-Kazakhstan region of the Kazakh SSR, in the village Koksaek of Lenger district, on May 5, 1946.

Tynysbek Sharipovich is a graduate of the Novosibirsk State University (1969) and a representative of the school of A.V. Bitsadze, the outstanding scientist, the member-correspondent of the Academy of sciences of the USSR. Scientific researches held by T.Sh. Kal'menov in postgraduate years (1969 - 1972) were devoted to studying the well-posedness of initial and initial-boundary value problems for degenerated hyperbolic equations of the form

$$k(y)u_{xx} - u_{yy} + au_x + bu_y + cu = f; \quad k(0) = 0; \; k(y) > 0, y \neq 0.$$

For this general equation T.Sh. Kal'menov found sufficient conditions of the well-posedness of the first and second Darboux problem including all the previously known conditions for the well-posedness. For various model cases of this equation there were first found: a criterion of the uniqueness of a solution; sufficient conditions of the well-posedness and a class of smoothness of a solution of the Darboux problem; a criterion for the continuity of a solution of the Goursat problem, and in this case it was first indicated on a different impact of characteristics for the well-posedness of a problem; a new characteristic Cauchy problem (incomplete Goursat problem) was formulated and its well-posedness in a class of analytical functions was proved.

From 1972 till 1985 T.Sh. Kal'menov worked in the Institute of Mathematics of the Academy of Sciences of the KazSSR, where he made a career from a junior researcher to the head of laboratory. In these years the field of his scientific interests belonged to theory of differential operators and he obtained significant results.

In case of boundary value problems for equations of hyperbolic and mixed types in researching problems in the weak sense there arise sufficient difficulties in clarifying the relation between generalized and classical solutions and, as a result, the lack of proof of the uniqueness of solutions, hence there is the one-sided investigation of problems. In this regard, the following question is of great importance: Under what conditions weak solutions are classical. In a more precise setting, when the generalized solution u of a boundary problem for the equation $Lu = f$ can be approximated by the sequence $u_n \in W_2^2(\Omega)$ of the classical solutions in the sense that $u_n \to u$ and $Lu_n \to f$ in some metric, as a rule, by the norm $L_2(\Omega)$. The generalized solutions having the last above-mentioned property is called *strong*.

In this direction T.Sh. Kal'menov proved the well-posedness of the semiperiodic Dirichlet problem for the degenerated equation of the mixed type

$$Lu = k(y)u_{xx} + u_{yy} + a(y)u_x + b(y)u_y + c(y)u = f(x, y);$$

and for the degenerated equation

$$Lu = signy|y|^m u_{xx} + u_{yy} = f(x, y).$$

He showed the existence of strong solutions of the Darboux and Tricomi problems.

For the Lavrent'ev–Bitsadze equation

$$Lu = -signy \; u_{xx} - u_{yy} = f(x, y),$$

T.Sh. Kal'menov proved the criterion for strong solvability of the Tricomi problem in spaces L_p, in terms of approach angles of an elliptical part of the boundary to a line changing the type. In this case, in contrast to previously known works, non-uniqueness of weak solutions was shown for the first time and criteria for the strong solvability of these problems were given.

Developing the operator approach, T.Sh. Kal'menov constructed the general theory of regular boundary value problems for equations of hyperbolic and mixed types. The classical construction of a boundary value problem is as follows: an equation and a boundary condition are given. It is necessary to investigate the solvability of this problem and properties of the solution if it exists (in the sense of belonging to some space). Beginning with the papers of J. von Neumann and M.I. Vishik, there exists another more general approach: an equation and a space are given, right-hand parts of the equation and boundary conditions, and a solution must belong to this space. It is necessary to describe all the boundary conditions, for which the problem is correctly solvable in this space.

T.Sh. Kal'menov described all the well-posed boundary value problems: for a wave equation in a characteristic triangle, for a multidimentional wave equation in a characteristic cone, for a Lavrent'ev-Bitsadze equation, for a Tricomi equation and for a multidimensional equation of mixed type. For the first time a multidimensional correct analogue of the Darboux problem was constructed, and its solution in the explicit form was given.

A large series of papers of T.Sh. Kal'menov is devoted to the spectral theory of equations of hyperbolic and mixed types. In contrast to the theory of solvability, spectral problems for the equations of hyperbolic and mixed types are poorly understood. Note that the well-known methods (in particular, the abstract spectral theory of linear operators), being a powerful tool in the study of elliptic operators, are little adapted in application to the boundary value problems for equations of hyperbolic and mixed types in the domain, part of which coincides with a characteristic cone.

For this reason, many actual problems of equations of hyperbolic and mixed types require special studies and attracting new tools and techniques. Some of such problems, in particular, are the questions of spectrum and strong solvability of local and non-local non-elliptic problems. The systematic study of spectral problems for mixed type equations started recently with the work of T.Sh. Kal'menov, E.I. Moiseev, S. Ponomarev.

T.Sh. Kal'menov formulated and proved the extremum principle for equations of mixed type, having now in literature the name "the maximum principle of Kal'menov". On the basis of this new principle of extremum he solved the problem of the existence of an eigenvalue of the Tricomi problem (this problem is not Volterra), and this fact laid the basis of a new perspective scientific direction - the spectral theory of equations of mixed type. The existence of eigenvalues of the Tricomi problem was proved for both a Lavrent'ev–Bitsadze problem and a general Gellerstedt's problem.

All these results became the basis for the doctoral thesis on "On regular boundary value problems and spectrum for equations of hyperbolic and mixed types" , defended at the Moscow State University named after M.V. Lomonosov in 1983.

From 1985 till 1991 T.Sh. Kal'menov worked as the dean of mathematical faculty of the Kazakh State University, from 1991 till 1997 he was the rector of the Kazakh Chemical-Technological University, the South-Kazakhstan Technical University. In the period from 1998 till 2003 he was the head of department of the South-Kazakhstan State University.

In these years T.Sh. Kal'menov appealed to problems in more general setting. After the appearance of the famous paper of A.V. Bitsadze and A.A.

Samarskii, nonlocal problems of mathematical physics are increasingly attracting the attention of mathematicians. Such problems are significantly nonselfadjoint and classical well-developed investigation methods are not applicable for their investigation. For the general elliptic equation

$$Lu = -\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i(x)\frac{\partial u}{\partial x_i} + a(x)u$$

in an arbitrary domain with smooth boundary, the completeness of root vectors of the main Bitsadze-Samarskii problems in general setting was proved by T.Sh. Kal'menov for the first time.

In 2004 T.Sh. Kal'menov returned to Almaty, where he became the General director of the Center of Physical-Mathematical Researches of the Ministry of Education of the Republic of Kazakhstan, reformed into the Institute of Mathematics, Informatics and Mechanics, then into the Institute of Mathematics and Mathematical Modeling in 2012. During these years T.Sh. Kal'menov obtained new significant results concerning classical problems of partial equations.

In the theory of mathematical physics, for example, in the theory of elasticity, an explicit representation of a solution of problems for an inhomogeneous biharmonic equation takes an important place. In papers of T.Sh. Kal'menov a polyharmonic Greens function of the Dirichlet problem

$$\triangle_x^m u(x) = f(x), f(x) \in L_2(\Omega_\delta),$$

$$\frac{\partial^i}{\partial n_x^i} u|_{|x|=\delta} = 0, \quad i = 0, 1, 2, ..., m-1$$

for the first time was constructed in the explicit form (in the terms of elementary functions) in the ball $\Omega_\delta \subset \mathbb{R}^n$ for the case of an arbitrary dimension of the space.

The spectral theory of non-selfadjoint differential operators is not complete yet even for a case of simplest ordinary differential equations. For partial equations and all the more for equations of the mixed type in non-selfadjoint spectral theory only some separate results have been obtained. Up to date the example of regular boundary value problem for differential equations is not found, a spectrum of this problem (except an empty set) is a finite set. T.Sh. Kal'menov was the first to establish that the spectrum of differential operators of the general form

$$Lu = \sum_{|\alpha| \leq p} a_\alpha(x) D^\alpha u = \lambda u(x),$$

generated by regular boundary conditions, is either empty, or an infinite set.

The classical volume Newton potential

$$u(x) = \varepsilon_n * f \equiv \int_\Omega \varepsilon_n(x-y) f(y) dy = L^{-1} f$$

defines the value of mass or charge distributed in Ω with density $\frac{f}{(n-2)\sigma_n}$. This potential also is often used in the theory of functions and in the embedding theorems. Since, the fundamental solution $\varepsilon_n(x-y)$ is symmetric and real-valued, and has a weak singularity, then the integral operator L^{-1} is the completely continuous self-adjoint operator in $L_2(\Omega)$ and the function $u(x) = \varepsilon_n * f(x)$ satisfies the Poisson equation $-\triangle u = f$.

In papers of T.Sh. Kal'menov the boundary condition for the volume Newton potential:

$$\frac{1}{2}u(x) = -\int_{\partial\Omega} \varepsilon_n(x-y)\frac{\partial u(y)}{\partial n_y}dS_y + \int_{\partial\Omega} \frac{\partial \varepsilon_n(x-y)}{\partial n_y}u(y)dS_y,$$

$$\text{for all } x \in \partial\Omega.$$

for the first time was found.

It turned out that these boundary conditions describe the long-known in theoretical physics effect of "transparent boundary conditions", skipping the outgoing waves and reflecting the incoming waves. The presence of such boundary conditions of the volume potential permits reducing of the problem with conditions of Sommerfeld type radiation in the infinite domain to a problem in a bounded domain and effectively apply numerical methods.

These results were further developed on a case of heat and wave potentials as well as in non-cylindric domains.

The classical Hadamard example demonstrates the ill-posedness of the Cauchy problem for a Laplace equation. The outstanding Soviet mathematicians - the academicians A.N. Tikhonov and M.M. Lavrent'ev, their disciples and followers found the conditions of well-posedness of the Cauchy problem for the Laplace equation, and of the other ill-posed problems. Also regularization methods for ill-posed problems were constructed. In papers of T.Sh. Kal'menov by the method for the expansion on eigenfunctions of a mixed Cauchy problem for the Laplace equation with a deviating argument we established necessary and sufficient well-posedness condition in the domain $\Omega = \{(x,t) : 0 < x < \pi, -1 < t < 1\}$ of the mixed Cauchy problem for the Poisson equation:

$$Lu = u_{tt}(x,t) + u_{xx}(x,t) = f(x,t), (x,t) \in \Omega,$$

$$u|_{t=-1} = \varphi_1(x), u_t|_{t=-1} = \varphi_2(x),$$

$$u|_{x=0} = 0, u|_{x=\pi} = 0.$$

Furthermore, it is shown that the solution of the Cauchy problem is unique and without setting of boundary conditions.

A characteristic feature of the mathematical creation of T.Sh. Kal'menov is the plenty of original methods and non-standard approaches to solving mathematical problems. T.Sh. Kal'menov belongs one of the few scientists managed to leave a mark almost in all fields of mathematics that he has studied. He is the worthy successor to his teacher Andrei Vasil'evich Bitsadze, the member-correspondent of AS SSSR and the bright continuer of traditions of the Soviet mathematical school.

More than 50 master's theses and 11 doctoral theses have been defended under supervision of T.Sh. Kal'menov. He has more than 150 published scientific works, a great amount of which have been published in international mathematical journals. Among them 11 works have been published in the journal "Proceedings of the USSR Academy of Sciences / Doklady Mathematics". He has published 14 works in highly rated international journals for last 5 years.

T.Sh. Kal'menov has been honored by a number of high awards and titles for his longstanding scientific, pedagogical and social organizational activity. In 1978 he was honored with the premium of Lenin Komsomol of Kazakhstan, in 1996 he became the Honored Worker of Science and Technology of the Republic of Kazakhstan. In 1989 he was elected as a member-correspondent of the Academy of Sciences of Kazakh SSR, and in 2003 he became an academician of the National Academy of Sciences of the Republic of Kazakhstan.

T.Sh. Kal'menov was honored with orders and medals of the Republic of Kazakhstan, and in 2013 he was awarded the State premium in the field of science and technology for the series of papers "To the theory of initial-boundary value problems for differential equations".

The academician T.Sh. Kal'menov as professional scientist and the head of The Institute of Mathematics and Mathematical Modeling is demanding of himself and of all scientists, that helps to keep the bar of mathematical science in Kazakhstan at a high international level. At the present moment he is one of the brighest mathematician of Kazakhstan and its principal leader.

With unflagging energy T.Sh. Kal'menov is engaged in scientific work, generously presents scientific ideas to his disciples and colleagues, realizes all new creative plans. He is full of new and original mathematical ideas.

It is wonderful that T.Sh. Kal'menov is fruitful and in the family life. He is the successful father of ten children, and now he is bringing up more than twenty five grandchildren!

ANALYSIS

Weighted estimates for the integral operator with a logarithmic singularity

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Abstract: Let $0 < q < \infty$, $1 < p < \infty$, $\frac{1}{p} + \frac{1}{p'} = 1$, $R_+ = (0, \infty)$, $U : R_+ \to R$ and $V : R_+ \to R$ are wheighted functions, i.e. non-negative measurable functions on R_+.

Since the 70-s of the last century the weighted estimate of the form

(1) $$\|UKf\|_q \le C\|Vf\|_p$$

intensively studied in the world mathematical literature for different classes of the operators K, where $\|\cdot\|_p$ is the usual norm of the space $L_p \equiv L_p(R)$. Some directions of the research of the estimate (1) until 2000 year for the integral operators are summarized in the books [1], [2], [3]. In the paper [4] the estimate (1) was studied for some classes of the operator Kf and also a full description of weights U and V was given there.

In this paper we study the estimate (1) for the singular operator in the form $Kf(x) = \int\limits_0^x \ln \frac{x}{x-s} f(s)ds$, when $V(x) = x^{1-\gamma}$, $\gamma > 0$.

Theorem. *Let $1 < p \le q < \infty$, $\gamma > \frac{1}{p}$. The inequality (1) holds if and only if*

$$A = \sup_{x>0} x^{\gamma + \frac{1}{p'}} \left(\int\limits_x^\infty U^q(t) t^{-q} dt \right)^{\frac{1}{q}} < \infty,$$

wherein $C \approx A$, where C is the best constant (1).

Keywords: integral operator with logarithmic singularity, weighted estimate.

2010 Mathematics Subject Classification: 47A63, 47G10

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Criterion of the boundedness of a fractional integration type operator with variable upper limit in weighted Lebesgue spaces

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Abstract: Let $0 < p, q < \infty$, $I = (a, b)$, $-\infty \leq a < b \leq \infty$, $0 < \alpha < 1$, $\frac{1}{p} + \frac{1}{p'} = 1$, $R_+ = (0, \infty)$, $W : I \to R$ is a nondecreasing, locally absolutely continuous function and $W(a) = \lim\limits_{t \to a^+} W(t) > -\infty$. $\frac{dW(t)}{dt} = w(t)$, $v : I \to R$ is a non-negative locally integrable function on I and $\varphi : I \to I$ is a strictly increasing locally absolutely continuous function with the properties $\lim\limits_{x \to a^+} \varphi(x) = a$, $\lim\limits_{x \to b^-} \varphi(x) = b$, $\varphi(x) \leq x$, $\forall x \in I$. Consider the operator in a form $K_{\alpha,\varphi} f(x) = \int\limits_a^{\varphi(x)} \frac{f(s)w(s)ds}{(W(x)-W(s))^{1-\alpha}}$, $x \in I$ from $L_{p,w} = L_{p,w}(I)$ to $L_{q,v} = L_{q,v}(I)$, where $L_{p,w}$ is a space of measurable functions $f : I \to R$ for which the functional $\|f\|_{p,w} = \left(\int\limits_a^b |f(x)|^p w(x) dx \right)^{\frac{1}{p}}$, $0 < p < \infty$ is finite. In the case $\varphi(x) \equiv x$ the operator $K_{\alpha,\varphi}$ is studied in papers [1], [2].

Theorem. The operator $K_{\alpha,\varphi}$ is bounded from $L_{p,w}$ to $L_{q,v}$ if and only if

1) $A = \sup\limits_{a<z<b} \left(\int\limits_z^b W^{q(\alpha-1)}(x) v(x) dx \right)^{\frac{1}{q}} W^{\frac{1}{p'}}(\varphi(x)) < \infty$, for $1 < p \leq q < \infty$ and $\frac{1}{p} < \alpha < 1$, where $\|K_{\alpha,\varphi}\| \approx A$;

2) $B = \left(\int\limits_a^b \left(\int\limits_t^b W^{q(\alpha-1)}(x) v(x) dx \right)^{\frac{q}{p-q}} W^{\frac{q(p-1)}{p-q}}(\varphi(t)) \frac{v(t)dt}{W^{q(1-\alpha)}(t)} \right)^{\frac{p-q}{pq}} < \infty$, when $0 < q < p < \infty$, $p > \frac{1}{\alpha}$, $0 < \alpha < 1$ and $\|K_{\alpha,\varphi}\| \approx B$, where $\|K_{\alpha,\varphi}\|$ is a norm of the operator $K_{\alpha,\varphi} : L_{p,w} \to L_{q,v}$.

Keywords: fractional integration operator, weighted estimate.

2010 Mathematics Subject Classification: 47A63, 47G10

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Common best proximity points in complex valued metric spaces

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Abstract: In this paper, we introduce the concept of common best proximity points for non-self mappings between two subsets of a complex valued metric space which is an extension of the classical metric spaces obtained by allowing the metric function to assume values from the field of complex numbers and was recently introduced by Azam et al. [2]. Let A and B be non-empty subsets of a complex valued metric space and $S, T : A \to B$ be non-self mappings. Due to the fact that S and T are non-self mappings, the equations $Sx = x$ and $Tx = x$ are likely to have no common solution, known as a common fixed point of the mappings S and T. Consequently, when there is no common solution, it is considered to determine an element x that is in close proximity to Sx and Tx in the sense of that $d(x, Sx)$ and $d(x, Tx)$ are minimum. In fact, common best proximity point theorems focus on the existence of such optimal approximate solutions, called common best proximity points, that the equations $Sx = x$ and $Tx = x$ in the case have no common solutions. As a matter of fact, given any element x in A, the distance between x and Sx and between x and Tx are at least $d(A, B)$, a common best proximity point theorem affirms global minimum of both complex valued functions $x \to d(x, Sx)$ and $x \to d(x, Tx)$ by imposing a common approximate solution of the equations $Sx = x$ and $Tx = x$ to satisfy the condition that $d(x, Sx) = d(x, Tx) = d(A, B)$. For this purpose, we extend some well-known results that was proved in classic metric space to the complex valued metric space by some new definitions. Our results supported by some examples.

Keywords: common best proximity point, complex valued metric space, common fixed points

2010 Mathematics Subject Classification: 54H25, 47H10

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On a difference scheme for nonlocal heat transfer boundary-value problem

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Abstract: In this paper, the family of boundary value problems for heat equation and finite difference schemes approximating these problems are considered. The peculiarity of the initial-boundary value problems is a special choice of the boundary conditions, which are not strongly regular. The corresponding difference schemes do not have the property of self-adjoint.

The paper proposes a new method of solving non-local problems for the heat equation

(1) $$u_t(x,t) - u_{xx}(x,t) = f(x,t),$$

satisfying the initial condition

(2) $$u(x,0) = \phi(x), \quad 0 \le x \le 1,$$

and the boundary conditions

(3)
$$\begin{cases} u_x(0,t) - u_x(1,t) + au(0,t) + bu(1,t) = 0, \quad cu(0,t) + (1+c)u(1,t) = 0. \\ cu(0,t) + (1+c)u(1,t) = 0. \end{cases}$$

with finite difference method. The main important feature of these problems is their non-self-adjointness. This non-self-adjointness causes major difficulties in their analytical and numerical solving. The problems, which boundary conditions do not possess strong regularity, are less studied. The scope of study of the paper justifies possibility of building a stable difference scheme with weights for abovementioned type of problems.

Keywords: heat equation, initial boundary problems, numerical method, difference scheme.

2010 Mathematics Subject Classification: 35P05, 35E20, 39A14, 39A05

Estimations of the best $M-$ term approximations of functions in the Lorentz space with constructive methods

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Abstract: V.N. Temlyakov [1,2] developed a constructive method of estimation of the best $n-$ term approximations functions from Nikol'skii-Besov's classes $S^{\bar{r}}_{p,\tau}B$ in the space $L_q(I^m)$, $1 < p < q < \infty$. In talk considers $L^*_{\bar{q},\bar{\theta}}(I^m)$ – the Lorentz space of periodic functions of many variables with the anisotropic norm $\|f\|^*_{\bar{q},\bar{\theta}}$. $e_n(f)_{\bar{q},\bar{\theta}}$ - the best n-term approximation of f. $e_n(F)_{\bar{q},\bar{\theta}} = \sup_{f\in F} e_n(f)_{\bar{q},\bar{\theta}}$.

Suppose $\rho(\bar{s}) = \left\{\bar{k} = (k_1, \dots, k_m) \in \mathbb{Z}^m : \quad 2^{s_j-1} \le |k_j| < 2^{s_j}, j = 1, \dots, m\right\}$, $\delta_{\bar{s}}(f, \bar{x}) = \sum\limits_{\bar{n}\in\rho(\bar{s})} a_{\bar{n}}(f)e^{i\langle\bar{n},\bar{x}\rangle}$, where $\langle\bar{y}, \bar{x}\rangle = \sum\limits_{j=1}^{m} y_j x_j$, $s_j = 1, 2, \dots$. For a function $f \in L_1(I^m)$ suppose $f_l(\bar{x}) = \sum\limits_{\bar{s}:\langle\bar{s},\bar{\gamma}'\rangle=l} \delta_{\bar{s}}(f, \bar{x})$, $\bar{x} \in I^m$, $l \in \mathbb{N}\cap\{0\} = \mathbb{N}_0$, where $\bar{\gamma}' = (\gamma_1', ..., \gamma_m')$, $\gamma_j' = (\frac{1}{q_j} - \frac{1}{p}))/(\frac{1}{q_1} - \frac{1}{p})$. We consider the class $W^{a,b}_{\bar{q},\theta} = \left\{f \in L_1(I^m) : \|f_l\|^*_{\bar{q},\theta} \le 2^{-la}\bar{l}^{(\nu-1)b}\right\}$, where $a > 0, b > 0$, $\bar{l} = \max\{1, l\}$ and let us introduce the notation $\|f\|_{W^{a,b}_{\bar{q},\theta}} = \sup\limits_{l\in\mathbb{N}_0} \|f_l\|^*_{\bar{q},\theta} 2^{la}\bar{l}^{-(\nu-1)b}$. In the case $q_1 = ... = q_m = \theta$ this class considered V.N. Temlyakov [1]. The main results:

Theorem. Let $1 < q_m \le ... \le q_{\nu+1} < q_\nu = ... = q_1 < p < 2$, $1 < \theta < p$.

1. If $a > \frac{1}{q_1} + \frac{1}{\theta} - \frac{2}{p}$, then $e_n\left(W^{a,b}_{\bar{q},\theta}\right)_p \asymp n^{-a+\frac{1}{q_1}-\frac{1}{p}}(\log n)^{(\nu-1)(a+b+\frac{2}{p}-\frac{1}{q_1}-\frac{1}{\theta})}$.
2. If $\frac{1}{q_1} - \frac{1}{p} < a < \frac{1}{q_1} + \frac{1}{\theta} - \frac{2}{p}$, then $e_n\left(W^{a,b}_{\bar{q},\theta}\right)_p \asymp n^{-a+\frac{1}{q_1}-\frac{1}{p}}(\log n)^{(\nu-1)b}$.
3. If $a = \frac{1}{q_1} + \frac{1}{\theta} - \frac{2}{p}$, then $e_n\left(W^{a,b}_{\bar{q},\theta}\right)_p \asymp n^{-(\frac{1}{\theta}-\frac{1}{p})}(\log n)^{(\nu-1)b}(\log\log n)^{\frac{1}{\theta}}$.

Keywords: Lorentz space, M-term approximations, constructive method

2010 Mathematics Subject Classification: 41A10, 41A25

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The maximum principle for the Navier-Stokes equations

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Abstract: New connections were established between extreme values of the velocity, the density of kinetic energy and the pressure of the problem for the Navier-Stokes equations in domain $Q = (0, T] \times \Omega$:

$$\frac{\partial \mathbf{U}}{\partial t} - \mu \Delta \mathbf{U} + (\mathbf{U}, \nabla)\mathbf{U} + \nabla P = \mathbf{f}, \ \mathrm{div}\mathbf{U} = 0, \tag{1}$$

$$\mathbf{U}(0, \mathbf{x}) = \mathbf{\Phi}(\mathbf{x}), \ \mathbf{x} \in \Omega \subset R_3; \ \mathbf{U}(t, \mathbf{x}) = 0, \ \mathbf{x} \in \partial\Omega. \tag{2}$$

Validity of the maximum principle was shown for problem (1, (2) using these connections:

$$\|\mathbf{U}\|_{\mathbf{C}(\bar{Q})} \leq \|\mathbf{\Phi}\|_{\mathbf{C}(\bar{\Omega})} + T\|\mathbf{f}\|_{\mathbf{C}(\bar{Q})} \equiv const, \ \forall T < \infty.$$

On the basis of which the selected space to prove the uniqueness of the weak and the existence of strong solutions to the problem Navier-Stokes equations for the whole time.

Keywords: Extreme values vector of velocity, density of kinetic energy, pressure, the maximum principle for the Navier-Stokes equations

2010 Mathematics Subject Classification: 35Q30, 35Q35

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The correctness Dirichlet problem in a cylindrical domain for three-dimensional elliptic equations with type and order extinction

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Abstract: Correct formulation of boundary value problems in the plane for elliptic equations by the method of the theory of analytic complex variable functions is well studied. In the study of similar issues, when the number of independent variables is greater than two, there are difficulties of a fundamental nature. Very attractive and convenient method of singular integral equations loses its power because of absence of any comprehensive theory of multidimensional singular integral equations. Previously correctness of the Dirichlet problems for degenerate three-dimensional elliptic equations has been established. Importance of study of boundary value problems for the elliptic equations with degeneration of order and type was noticed by A.V. Bitsadze. Let $D_\beta -$ be a cylindrical domain of the Euclidian space E_3 of points (x_1, x_2, t), bounded by the cylinder $\Gamma = \{(x,t) : |x \,|= 1\}$, the planes $t = \beta > 0$ and $t = 0$, where $|x\,| -$ is the length of the vector $x = (x_1, x_2)$. Let us denote the parts of these surfaces forming the boundary ∂D_β of the domain D_β as $\Gamma_\beta, S_\beta, S_o$ respectively. Let us consider three-dimensional hyperbolic equations with type and order confluence in the domain D_β :

$$(1) \qquad \sum_{i=1}^{2} k_i(t) u_{x_i x_i} - k_3(t) u_{tt} + \sum_{i=1}^{2} a_i(x,t) u_{x_i} + b(x,t) u_t + c(x,t) u = 0,$$

where $k_i(t) > 0$ when $t > 0$ and they vanish when $t = 0$, $k_i(t) \in C([0,\beta]) \cap C^2((0,\beta)), i = 1,2,3$. Equation (1) is elliptic if and along the plane there is a degeneration of its type and order. As a multi-dimensional Dirichlet problem, we consider

Problem 1. Find the solutions of the equation (1) in the domain D_β from the class $C(\bar{D}_\beta) \cap C^2(D_\beta)$, which satisfy the boundary value conditions

$$u|_{S_\beta} = \varphi(x), u|_{\Gamma_\beta} = \psi(x,t), u|_{S_o} = \tau(x).$$

For smooth coefficients of the equation (1) and certain conditions on the boundary data unique solvability is proved, and an explicit form of classical solution of the problem D is obtained.

Keywords: correctness, problem, equation, type and order extinction

2010 Mathematics Subject Classification: 35R12

Relation theoretic metrical fixed point theorems with an application

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Abstract: In this paper, we establish metrical relation-theoretic fixed point theorems via an implicit contractive condition which is general enough to yield a multitude of corollaries corresponding to several well known contraction conditions (e.g. Banach (Fund. Math. 3, 133-181 (1922)), Kannan (Am. Math. Mon. 76, 405-408 (1969)), Reich (Can. Math. Bull. 14, 121-124 (1971)), Bianchini (Boll. Unione Mat. Ital. 5, 103-108 (1972)), Chatterjea (C. R. Acad. Bulg. Sci. 25, 727-730 (1972)), Hardy and Rogers (Can. Math. Bull. 16, 201-206 (1973)), Ciric (Proc. Am. Math. Soc. 45, 267-273 (1974)) and several others) wherein even such corollaries are new results on their own. As simple we utilize our main results, to prove a theorem on the existence and uniqueness of the solution of an integral equation besides furnishing an illustrative example.

Keywords: Complete metric space, binary relations, implicit relations, fixed point.

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The Steklov problem and estimates for orthogonal polynomials

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Abstract: A Steklov's problem is to obtain the bounds on the sequence of orthonormal polynomials at the support of the weight of orthogonality. In 1921 [1], V.A. Steklov made a conjecture that if weight of orthogonality is strictly positive then sequence of orthonormal polynomials (at the support of the weight) is bounded.

In 1979 [2] E.A. Rakhmanov disproved this conjecture by constructing the sequence of polynomials orthonormal with respect to a positive weight which has the logarithmical rate of growth. Then the Steklov's problem becomes: to obtain the maximal possible rate of growth for these sequences. The modern version of the Steklov's problem is intimately related with the following extremal problem. For a fixed $n \in \mathbb{N}$, find

$$M_{n,\delta} = \sup_{\sigma \in S_\delta} \|\phi_n\|_{L^\infty(\mathbb{T})},$$

where $\phi_n(z)$ is the orthonormal polynomials with respect to the measure $\sigma \in S_\delta$, and S_δ is the Steklov's class of probability measures σ on the unit circle, such that $\sigma'(\theta) \geqslant \delta/(2\pi) > 0$ at every Lebesgue point of σ.

There is an elementary bound

$$M_n \lesssim \sqrt{n}.$$

In 1981 [3] E.A. Rakhmanov have proved:

$$M_n \gtrsim \sqrt{n}/(\ln n)^{\frac{3}{2}}.$$

In our joint paper with S.A. Denisov and D.N. Tulyakov [4] we have proved, that

$$M_n \gtrsim \sqrt{n},$$

i.e. the elementary bound is sharp.

In our talk we discuss the history, statement of the problem and details of the construction of a sequence of the orthonormal polynomials from the Steklov class which have the maximal possible rate of growth. We also consider the Steklov problem in L_p spaces and give new upper estimates for the polynomials orthonormal with respect Muckenhoupt weights from the Steklov's class.

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The positivity of the difference operator with periodic conditions and its applications

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Abstract: In this study, the second order of approximation of the difference operator A_h^x approximates the second order differential dependent operator A^x defined by the formula

$$A^x u = -a(x)u_{xx}(x) + \delta u(x), \delta \geq 0, a(x) = a(x+2\pi), x \in \mathbb{R}^1$$

with domain

$$D(A^x) = \left\{ u(x) : u(x), u'(x), u''(x), u(x) = u(x+2\pi), x \in \mathbb{R}^1, \int_0^{2\pi} u(x)dx = 0 \right\}$$

is presented. The Green's function of the difference operator A_h^x is constructed. The estimates for the Green's function are obtained. The positivity of operator A_h^x in the Banach space $C(\mathbb{R}_{1h})$ is established. It is proved that for any $\alpha \in (0, \frac{1}{2})$, the norms in spaces $E_\alpha = E_\alpha(C(\mathbb{R}_{1h}), A_h^x)$ and $C^{2\alpha}(\mathbb{R}_{1h})$ are equivalent uniformly with respect to h. The positivity of the operator A_h^x in Hölder spaces of $\mathring{C}^{2\alpha}(\mathbb{R}_{1h})$, $\alpha \in (0, \frac{1}{2})$ is proved. In applications, theorems on well-posedness for difference schemes of the approximate solution of local and nonlocal boundary value problems for elliptic differential equations are presented.

Keywords: positivity of difference operators, periodic boundary conditions, Green's function.

2010 Mathematics Subject Classification: 34K13, 34B09,

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On the unique solvability of a nonlocal problem with integral conditions for the system of partial differential equations of second order

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Abstract: The nonlocal boundary value problem with integral conditions for the system of hyperbolic equations second order is considered. Algorithm of finding approximate solution to researching problem is constructed and the its convergence is proved. The sufficient conditions of the unique solvability to nonlocal problem are established in the terms of initial data.

In this work, on $\Omega = [0,T]\times[0,\omega]$ the nonlocal boundary value problem with integral conditions for the system of hyperbolic equations is considered

$$\frac{\partial^2 u}{\partial x \partial t} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x), \tag{1}$$

$$P(t)u(t,\theta) + \int_0^\theta K(t,x)u(t,x)dx = \psi(t), \qquad t\in[0,T], \tag{2}$$

$$S(x)u(\eta,x) + \int_0^\eta M(t,x)u(t,x)dt = \varphi(x), \qquad x\in[0,\omega], \tag{3}$$

where $u(t,x) = col(u_1(t,x),...,u_n(t,x))$ is unknown function, the $(n\times n)$ matrices $A(t,x)$, $B(t,x)$, $C(t,x)$, n-vector function $f(t,x)$ are continuous on Ω, the $(n\times n)$ matrices $K(t,x)$, $M(t,x)$ are continuously differentiable on Ω by t, x, respectively, the $(n\times n)$ matrices $P(t)$ and $S(x)$, the n-vector functions $\psi(t)$ and $\varphi(x)$ are continuously differentiable on $[0,T]$, $[0,\omega]$, respectively, and $0\le\eta\le T$, $0\le\theta\le\omega$.

The conditions of classical solvability to problem (1)–(3) are established by method of introduction additional functional parameters [1-2].

Keywords: hyperbolic equation, nonlocal problem, integral condition

2010 Mathematics Subject Classification: 35L51, 35L53

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Asymptotical estimation of boundary value problems for singularly perturbed integro-differential equations

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Abstract: We consider the two-point boundary value problem for the singularly perturbed higher order linear integro-differential equation. The constructive formulae and asymptotic estimations for solutions and their derivatives are obtained.

Keywords: singularly perturbation, integro-differential equations, small parameter, asymptotic method, asymptotic expansion, initial jump

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Boundary value problems for loaded hyperbolic equations

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Abstract: The paper discusses some of the distinctive features, related to the formulation and study of boundary value problems for the loaded hyperbolic equations

$$u_{xx} - u_{yy} = \lambda u(x_0, y) \tag{1}$$

where λ and x_0 are given real constants. Along with the setting of the Cauchy problem we investigate domain of dependency, influence and definition of solution $u(x, y)$ for equation (1) by the Cauchy datas in the whole space, as well as in its bounded subdomain. The Goursat problem and the Darboux problem in nonlocal setting are formulated and investigated. Some of these results has already been announced in works [1] and [2] in the case of the spectrally loaded equation [3] of the form

$$u_{xx} - u_{yy} = \lambda u_{yy}(x_0, y).$$

Keywords: loaded wave equation, Goursat problem, Darboux problem

2010 Mathematics Subject Classification: 35L05, 35L20

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On the boundedness of the periodic Hilbert transform on generalized periodic Morrey spaces

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Abstract: The Hilbert transform is one of the most important operators in the field of signal theory. Mainly, the importance of the transform is due to its property to extend real functions into analytic functions. This property certainly induces a vast number of applications, especially in signal theory, and obviously the Hilbert transform is not merely of interest for mathematicians. This paper dedicated to the investigation of boundedness of periodic Hilbert transform on generalized periodic Morrey spaces. First, we give definitions of Morrey space and generalized Morrey space [1], [2], [3]. Then we prove the boundedness of periodic Hilbert transform on generalized periodic Morrey spaces. The boundedness of Hilbert transform on (nonperiodic)generalized Morrey space we refer to [4]. As a consequence, given uniform estimates of the operator norms of the partial sums of Fourier series on generalized periodic Morrey spaces.

Keywords: Hilbert transform, partial sums of Fourier series

2010 Mathematics Subject Classification: 44A15, 42A24

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Recovery of partial differential operators on classes of periodic functions with mixed smoothness

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Abstract: We consider the problem of optimal linear recovery for mixed partial differential operator $\mathcal{A}$ on the unit ball $\mathrm{SB}^r_{p\theta}(\mathbb{T}^n)$ of the Nikol'skii–Besov space of periodic functions with mixed smoothness.

We find error bounds sharp in order for optimal linear recovery of operator $\mathcal{A}$ on class $\mathrm{SB}^r_{p\theta}(\mathbb{T}^n)$. As information $I^\delta_m(f)$ about the functions f from class $\mathrm{SB}^r_{p\theta}(\mathbb{T}^n)$ we shall use Fourier coefficients with numbers from step "hyperbolic" cross.

As the linear method using the information about Fourier coefficients, we shall consider action of the mixed partial differential operator $\mathcal{A}$ on special "private" sum of decomposition on system (type as wavelets) trigonometrical polynoms.

Keywords: Nikol'skii–Besov function space, mixed smoothness, trigonometric system, recovery of differential operator

2010 Mathematics Subject Classification: 42A10

Domain of the composition of some triangles in the space of p-summable sequences, $(0 < p \leq 1)$

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Abstract: Let $0 < p \leq 1$. In the present paper, referring Ng and Lee [1] and following Altay and Başar [2, 3] and Başar [4] we introduce the space $\widetilde{\ell_p}$ as the domain of the triangle $\widetilde{B}$ which is the composition of E_1, C_1 and Δ of the Euler, Cesàro means of order one and the backward difference matrix. Furthermore, we give an inclusion relation concerning the space $\widetilde{\ell_p}$ and compute the alpha-, beta- and gamma-duals of the space $\widetilde{\ell_p}$ and construct its basis. We characterize the classes $(\widetilde{\ell_p} : \ell_\infty)$, $(\widetilde{\ell_p} : c)$ and $(\widetilde{\ell_p} : c_0)$ of infinite matrices, and give the characterization of some other classes of matrix transformations from the space $\widetilde{\ell_p}$ to the Euler, Riesz, difference, etc., sequence spaces, by means of a basic lemma.

Keywords: domain of a triangle matrix, sequence space, BK-space, the α-, β- and γ-duals, Schauder basis, matrix transformations.

2010 Mathematics Subject Classification: 46A45, 46B45, 46A35

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On some spaces of $q-$bounded double sequences

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Abstract: Let $0 < q < \infty$ and $\vartheta \in \{p, bp, r\}$. By $\mathcal{M}_u$, $\mathcal{C}_\vartheta$, $\mathcal{C}_{\vartheta 0}$ and $\mathcal{L}_q$; we denote the spaces of all bounded, $\vartheta-$convergent, $\vartheta-$null and absolutely $q-$summable double sequences, respectively (see e.g. [1–3]). In [4], Altay and Başar have defined the spaces $\mathcal{BS}$, $\mathcal{CS}_\vartheta$ and $\mathcal{CS}_{\vartheta 0}$ as the domain of four-dimensional summation matrix S in the spaces $\mathcal{M}_u$, $\mathcal{C}_\vartheta$ and $\mathcal{C}_{\vartheta 0}$; respectively. Besides these, by using four-dimensional backward difference matrix Δ they have introduced the space $\mathcal{BV}$ of all bounded variation double sequences. As a continuation of [4]; in this study, we investigate the spaces $\mathcal{LS}_q$ and $\mathcal{BV}_q$ consisting of all double sequences whose S- and Δ-transforms are in the space $\mathcal{L}_q$, respectively.

Keywords: summability theory, double sequence spaces, dual spaces, matrix transformations.

2010 Mathematics Subject Classification: 46A45, 40C05

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Fourier widths of some function classes associated with $m-$multiple Haar system

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Abstract: In the communication we obtain estimates sharp in order for Fourier widths of the Nikol'skii – Besov and Lizorkin – Triebel type classes associated with $m-$multiple Haar system in the space $L_r([0,1]^k)$ for a number of relations between the parameters of classes and space.

Keywords: m–multiple Haar system, Fourier width, the Nikol'skii – Besov type space, the Lizorkin – Triebel type space

2010 Mathematics Subject Classification: 42A10

Noncommutative Holder-type inequalities

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Abstract: We generalize some known Holder-type inequalities of matrix to the measurable operators associated with semi-finite von Neumann algebra $\mathcal{M}$ case.

In [4], Horn and Zhan have shown that if $p,\ q$ and r are positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$, then

$$\| | |A^*B|^r | \| \leq \| | |A|^{pr} | \|^{\frac{1}{p}} \| | |B|^{qr} | \|^{\frac{1}{q}} \tag{1}$$

for $A, B \in B(H)$ and for unitarily invariant norms. Hiai and Zhan [2] proved that if $A,\ B,\ C$ and D in $B(H)$, then

$$2^{\left|\frac{1}{p}-\frac{1}{2}\right|} \| | \, |C^*B + D^*B| \, | \| \leq \| | \, |A|^p + |B|^p | \|^{\frac{1}{p}} \| | \, |C|^q + |D|^q | \|^{\frac{1}{q}} \tag{2}$$

for all positive real numbers p and q such that $\frac{1}{p}+\frac{1}{q}=1$.

In this talk, we will use the concept of uniform Hardy-Littlewood majorization studied by Kalton and Sukochev [5], and their main result, to generalize the Holder type inequalities (1) and (2) for τ-measurable operators associated with semi-finite von Neumann algebra $\mathcal{M}$ and for symmetric Banach spaces norm.

Keywords: first, quadratic residue codes

2010 Mathematics Subject Classification: 94B05, 94B15

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Interpolation properties of anisotropic Nikol'skii-Besov $B^{\alpha\mathbf{q}}_{\mathbf{pr}}(\mathbb{T}^{\mathbf{d}})$ spaces and embedding theorems

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Abstract: We study interpolation properties of anisotropic of Nikol'skii–Besov spaces $B^{\alpha\mathbf{q}}_{\mathbf{pr}}(\mathbb{T}^{\mathbf{d}})$ and we obtain limit embedding theorems for anisotropic Nikol'skii-Besov and Lorentz's spaces.

In article we get interpolations theorems for anisotropic Nikol'skii–Besov $B^{\alpha\mathbf{q}}_{\mathbf{pr}}(\mathbb{T}^{\mathbf{d}})$ spaces. Apply its we proofed the embedding of the different metrics, limits theorems for anisotropic Nicol'skii–Besov spaces and the anisotropic Lorentz spaces.

Theorem 1. Let $\mathbf{1} < \mathbf{p} = (p_1, \ldots, p_n) < \mathbf{q} = (q_1, \ldots, q_n) < \infty$, $\mathbf{1} \leq \mathbf{r} = (r_1, \ldots, r_n), \tau = (\tau_1, \ldots, \tau_n) \leq \infty$. Then for $\alpha = (\mathbf{1}/\mathbf{p} - \mathbf{1}/\mathbf{q})\mathbf{d}$ the embedding holds

$$B^{\alpha\tau}_{\mathbf{pr}}(\mathbb{T}^{\mathbf{d}}) \hookrightarrow L_{\mathbf{q}\tau}(\mathbb{T}^{\mathbf{d}}).$$

Theorem 2. Let $\mathbf{1} < \mathbf{q} = (q_1, \ldots, q_n) < \mathbf{p} = (p_1, \ldots, p_n) < \infty$, $\mathbf{1} \leq \mathbf{r} = (r_1, \ldots, r_n), \tau = (\tau_1, \ldots, \tau_n) \leq \infty$. Then for $\alpha = (\mathbf{1}/\mathbf{p} - \mathbf{1}/\mathbf{q})\mathbf{d}$ takes place embedding

$$L_{\mathbf{q}\tau}(\mathbb{T}^{\mathbf{d}}) \hookrightarrow B^{\alpha\tau}_{\mathbf{pr}}(\mathbb{T}^{\mathbf{d}}).$$

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Weighted inequality of Hardy type

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Abstract: This work is an extension of the results of article. The necessary and sufficient conditions on weight functions ρ, ν, r are obtained, for which the following inequality holds true

$$\|rf\|_q = c\left(\|\rho f'\|_p + \|\nu f\|_p\right) \tag{3}$$

for $p = q = \infty$. And for the smallest constant two-sided estimates are found, which are of the same order. The criterion of compactness of embedding operator, that corresponds to (1) is provided. Let us note that (1) becomes a generalized Hardy inequality (in differential form) as $\nu \equiv 0$. For $\rho \equiv 1, \quad 1 = p \le q < \infty$, the inequality (1) was examined by K.T. Mynbayev, for $\rho \equiv 1, \quad p = q = 2$, by E.T. Sawyer, for $1 < p \le q < \infty$ by M. Otelbaev and R.Oynarov, and for $1 \le p < q < \infty, \quad 1 \le q < p < \infty$, in general situations by R. Oynarov. Similar inequalities for matrix operators are considered in the works of J. Tasmagambetova and A. Temirkhanova.

We introduce the following parameters describing the local behavior of the weighting functions:

$$\omega(x, y) = \sup\left\{d > 0 : \int_{x-d}^{x} \rho^{-1}(\tau)d\tau \le \int_{x}^{x+y} \rho^{-1}(\tau)d\tau, (x-d, x+y] \subset J\right\},$$

$$d^+(x) = \sup\left\{d > 0 : \sup_{x-\omega(x,d) \le t \le x+d} \upsilon(t) \int_{x-\omega(x,d)}^{x+d} \rho^{-1}(\tau)\, d\tau \le 1, \; [x, x+d) \subset J \right\},$$

$$d^-(x) = \omega\left(x, d^+(x)\right).$$

Keywords: additive inequality, matrix operator, bounded sequence.

2010 Mathematics Subject Classification: 42A10

On the compactness of set in generalized Morrey spaces

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Abstract: In this paper, we present sufficient conditions for strongly pre-compact sets in the generalized Morrey spaces M_p^w.

Let $1 \le p \le \infty$, $w(x)$ measurable non-negative function on $(0,\infty)$. Generalized Morrey space $M_p^{w(R)}$ is defined as the set of functions $f \in L_1^{loc}(R)$ with a finite quasi-norm: $\|f\|_{M_p^w} \equiv \sup\limits_{t \in R^n, r>0} w(r) \left(\int\limits_{B(t,r)} |f(x)|^p \, dx \right)^{\frac{1}{p}} < \infty$, where $B(t,r)$ is a ball with center at the point t and with radius r. For $f \in L_1^{loc}(R)$ and $\alpha > 0$ suppose $(M_\alpha f)(x) = \frac{1}{|B(x,\alpha)|} \int\limits_{B(x,\alpha)} f(y)dy$. We denote by Ω_p the set of all functions which are non-negative , measurable on Ω_p and for some $t_1, t_2 > 0$: $\sup\limits_{t_1 \le r < \infty} (w(r)) < \infty$, $\sup\limits_{0 < r \le t_2} \left(w(r) r^{\frac{1}{p}} \right) < \infty$.

Theorem Suppose that $1 \le p \le \infty$ and $w \in \Omega_p$. Suppose the subset S in M_p^w satisfies the following conditions:

$$\sup_{f \in S} \|f\|_{M_p^w} < \infty, \ \lim_{u \to 0} \sup_{f \in S} \|f(\cdot + u) - f(\cdot)\|_{M_p^w} = 0, \ \lim_{r \to \infty} \sup_{f \in S} \left\| f \chi_{C_{B(0,r)}} \right\|_{M_p^w} = 0.$$

Then S is strongly pre-compact set in $M_p^w(R)$.

From the proved theorem for the case of $w(r) = r^{-\lambda}$ follows a well-known result for the Morrey space M_p^λ [3] and in the case of $\lambda = 0$ this is well-known Frechet-Kolmogorov theorem.

Keywords: generalized Morrey spaces, compactness

2010 Mathematics Subject Classification: 42B20, 42B99

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A variation on lacunary statistical quasi-Cauchy sequences

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Abstract: We introduce a concept of ideal lacunary statistical quasi-Cauchyness of order α of a sequence of real numbers in the sense that a sequence (x_k) of points in $\mathbf{R}$ is called $I-$lacunary statistically quasi-Cauchy of order α, if $\{r \in \mathbf{N} : \frac{1}{h_r^\alpha}|\{k \in I_r : |\Delta x_k| \geq \varepsilon\}| \geq \delta\} \in I$ for each $\varepsilon > 0$ and for each $\delta > 0$, where an ideal I is a family of subsets of positive integers $\mathbf{N}$ which is closed under taking finite unions and subsets of its elements. The main purpose of this paper is to investigate ideal lacunary statistical ward continuity of order α, where a function f is called $I-$ lacunary statistically ward continuous of order α if it preserves I-lacunary statistically quasi-Cauchy sequences of order α, i.e. $(f(x_n))$ is a $S_\theta^\alpha(I)$-quasi-Cauchy sequence whenever (x_n) is.

Keywords: sequences, series, ideal convergence, continuity
2010 Mathematics Subject Classification: 40A05, 40A35, 26A15

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Ideal statistically quasi Cauchy sequences

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Abstract: An ideal I is a family of subsets of positive integers $\mathbf{N}$ which is closed under taking finite unions and subsets of its elements. A sequence (x_k) of real numbers is said to be $S(I)$-statistically convergent to a real number L, if for each $\varepsilon > 0$ and for each $\delta > 0$ the set $\{n \in \mathbf{N} : \frac{1}{n}|\{k \leq n : |x_k - L| \geq \varepsilon\}| \geq \delta\}$ belongs to I. We introduce $S(I)$-statistically ward compactness of a subset of $\mathbf{R}$, the set of real numbers, and $S(I)$-statistically ward continuity of a real function in the senses that a subset E of $\mathbf{R}$ is $S(I)$-statistically ward compact if any sequence of points in E has an $S(I)$-statistically quasi-Cauchy subsequence, and a real function is $S(I)$-statistically ward continuous if it preserves $S(I)$-statistically quasi-Cauchy sequences where a sequence (x_k) is called to be $S(I)$-statistically quasi-Cauchy when (Δx_k) is $S(I)$-statistically convergent to 0. We obtain results related to $S(I)$-statistically ward continuity, $S(I)$-statistically ward compactness, N_θ-ward continuity, and slowly oscillating continuity.

Keywords: sequences, series, ideal convergence, continuity
2010 Mathematics Subject Classification: 40A05, 40A35, 26A15

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Solvability of the Dirichlet problem for the p-Laplacian on stratified sets

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Abstract: A stratified set is defined to be a connected subset Ω of $\mathbb{R}^d$ which consists of a finite collection S of submanifolds σ_{kj}, called strata. The first subscript in σ_{kj} shows the dimension of a stratum, while the second serves to list strata of the same dimension. So

$$\Omega = \bigcup_{\sigma_{kj} \in S} \sigma_{kj}.$$

We assume each σ_{kj} to have compact closure in $\mathbb{R}^d$ and assume also that strata adhere to one another regularly like cells in a CW-complex. The set Ω is regarded as the union $\Omega_0 \bigcup \partial\Omega_0$, where Ω_0 is an open connected subset of Ω (the topology on Ω is induced by the standard topology of $\mathbb{R}^d$) composed of strata and such that the closure $\overline{\Omega}_0$ coincides with Ω. The difference $\partial\Omega_0 = \Omega \setminus \Omega_0$ is the boundary of Ω_0 in the induced topology.

A stratified measure μ on Ω is defined as

$$\mu(\omega) = \sum_{\sigma_{kj} \in S} \mu_k(\sigma_{kj} \cap \omega)$$

for naturally defined mesurable sets $\omega \subset \Omega$. Here μ_k is the measure on σ_{kj} related to the Riemannian metric on σ_{kj} induced by the Euclidean metric of $\mathbb{R}^d$.

For tangent vector fields $\vec{F}$ on Ω_0, which can be defined quite naturally, we can consider an analog $\nabla \cdot \vec{F}$ of classical divergence, defined as the density of the flux of $\vec{F}$ with respect to μ. For functions $u : \Omega \to \mathbb{R}$, we then can consider an analog of the Laplace–Beltrami operator, $\Delta u = \nabla \cdot \nabla u$, with ∇u being a specially-defined gradient.

In our talk we discuss the solvability of the Dirichlet problem for the p-Laplacian on a stratified set:

$$\nabla(|\nabla u|^{p-2}\nabla u) = 0, \tag{4}$$

$$u\big|_{\partial\Omega_0} = \phi. \tag{5}$$

Equation (4) is the Euler-Lagrange equation for the variational problem

$$\Phi(u) = \int\limits_{\Omega_0} |\nabla u|^p d\mu \to \min,$$

where the minimum is taken over all continuous functions on Ω that are continuously differentiable on every stratum of Ω_0 and whose boundary values are given by ϕ.

We show that Problem (4),(5) has a weak solution in a suitable Sobolev space $W^{1,p}_{\mu}(\Omega_0)$ determined by the norm $\|u\|_{W^{1,p}_{\mu}(\Omega_0)} = \left(\int\limits_{\Omega_0} |\nabla u|^p \, d\mu \right)^{\frac{1}{p}}$.

The key point in the proof is played by an analog of the Sobolev inequality,

$$\int\limits_{\Omega} |f|^q \, d\mu \leq C \int\limits_{\Omega_0} |\nabla f|^p \, d\mu,$$

where the parameter q depends on p and on the structure of Ω. The latter dependence is rather complicated and, rouphly speaking, shows how tightly the strata are bound to one another.

Keywords: stratified set, p-Laplacian, Sobolev inequality

2010 Mathematics Subject Classification: 35J25, 35B45

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Asymptotic estimates of the solution of the Cauchy problem for singularly perturbed integro-differential equations

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Abstract: This report is devoted to the Cauchy problem for the integro-differential equation with a small parameter in the highest derivatives:

$$\sum_{r=1}^{m} \varepsilon^r A_{n+r}(t) y^{(n+r)} + \sum_{k=0}^{n} A_k(t) y^{(k)} = F(t) + \int_0^1 \sum_{i=0}^{l+1} H_i(t,x) y^{(i)}(x,\varepsilon) dx \tag{6}$$

with the initial conditions

$$y^{(i)}(0,\varepsilon) = \alpha_{i+1}, \quad i = \overline{0, n+m-1}. \tag{7}$$

Here $\varepsilon > 0$ is small parameter and $l = \text{fix}\{0, 1, \ldots, n-1\}$.

A similar problem for differential equations considered in [1].

In this paper we consider the case when the roots $\mu_i(t)$, $i = \overline{1, m}$ of the additional characteristic equation

$$\mu^m + A_{n+m-1}(t)\mu^{m-1} + \ldots + A_{n+1}(t)\mu + A_n(t) = 0$$

have negative real parts.

Asymptotic estimates for the solution of the problem (1) and (2) and of the difference between the solutions of the original singularly perturbed problem (1) and (2) and a corresponding unperturbed problem are obtained.

Keywords: Singularly perturbations, small parameter, integro-differential equations, asymptotic estimates

2010 Mathematics Subject Classification: 34E15, 34K26, 45J05

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Weighted Sobolev spaces, quasielliptic operators and equations not solvable with respect to the highest-order derivative

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Abstract: In the paper some classes of quasielliptic operators $L(D_x)$ are considered in the whole space $\mathbb{R}^n$. In particular, they include homogeneous elliptic operators, elliptic and parabolic operators in the sense of Petrovskii, elliptic operators in the sense of Douglis – Nirenberg, operators of parabolic type with "opposite time directions" and etc. [1, 2]. We introduce special scales of weighted Sobolev spaces $W^l_{p,\sigma}$ and study their properties. We establish isomorphism theorems for the quasielliptic operators These results give us known isomorphism theorems for homogeneous elliptic operators and a number of new isomorphism theorems for matrix elliptic and parabolic operators; in particular, for the stationary Navier–Stokes operator.

Using the isomorphism theorems, we establish unique solvability of initial value problems for some classes of partial differential equations not solvable with respect to the highest-order derivative

$$L(D_x)D_t^m u + \sum_{k=0}^{m-1} L_{m-k}(D_x)D_t^k u = f(t,x).$$

In particular, we consider equations and systems of Sobolev type, pseudoparabolic and pseudohyperbolic equations [3, 4]. The author was supported by the Program of the Presidium of the Russian Academy of Sciences (project no. 0314-2015-0011).

Keywords: quasielliptic operators, weighted Sobolev spaces, isomorphism theorems, Sobolev type equations

2010 Mathematics Subject Classification: 35H30, 35K70, 35L82

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The stabilization rate of a solution to the Cauchy problem for parabolic equation with lower order coefficients

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Abstract: In the half-space $D = R^N \times (0, \infty), N \geqslant 3$, consider the Cauchy problem for a non divergent parabolic equation.

$$L_1 u \equiv Lu + (b, \nabla u) + c(x,t)u - u_t = 0, \quad (x,t) \in D, \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in R^N, \tag{2}$$

where
$$Lu = \sum_{i,k=1}^{N} a_{ik}(x,t) u_{x_i x_k}, \quad (b, \nabla u) = \sum_{i=1}^{N} b_i(x,t) u_{x_i}. \tag{3}$$

In this report we study sufficient conditions on the lower order coefficients of a parabolic equation guaranteeing the power rate of the uniform stabilization to zero of the solution to the Cauchy problem on every compact in R^N and for any bounded initial function.

We assume that following conditions are satisfied:

$$\exists k_0^2 > 0: \ k_0^2 = \inf_{D, |\xi|=1} \sum_{i,k=1}^{N} a_{ik}(x,t)\xi_i \xi_k; \ \exists k_1^2 > 0: \ k_1^2 = \sup_{D, |\xi|=1} \sum_{i,k=1}^{N} a_{ik}(x,t)\xi_i \xi_k.$$

The coefficients $b_i(x,t) (i = 1, ..., N)$ satisfy *condition* (B); i.e., there exist constant $B > 0$ such as

$$\sup_{(x,t)\in D} (1+r) \sum_{i=1}^{N} |b_i(x,t)| = B < k_0^2 N, \quad \text{where } r = \sqrt{x_1^2 + \cdots + x_N^2}.$$

The coefficient $c(x,t)$ satisfies *condition (C)*, i.e. exist $\alpha > 0$ for which

$$c(x,t) \leqslant a_\alpha(r) = -\alpha^2 \min(1, r^{-2}).$$

Let $\lambda_1(\alpha) = (2 - S + \sqrt{D_1})/2, \ D_1 = (2-S)^2 + 4\overline{\alpha}^2, \ \overline{\alpha} = \alpha / k_1.$

Theorem. *Suppose that* (B) *and* (C) *with*

$$\alpha^2 > k_1^2 (S-1), \ S = \left(k_1^2 (N-1) + k_0^2 + B\right) / k_0^2,$$

hold. Then $\lim_{t\to\infty} t^{\nu_n} u(x,t) = 0$, $\nu_n = \lambda_1(\alpha)/(2 + \frac{1}{n})$ *for any* $n = 1, 2, ...,$ *uniformly with respect to* x *on any compact* K *in* R^N.

Keywords: Cauchy problem, non-divergent equation, stabilization

2010 Mathematics Subject Classification: 35K15, 35K20, 35B40

On the solvability of a nonlocal boundary value problem for the Laplace operator with opposite flows at the part of the boundary

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Abstract: In the present paper we investigate a nonlocal boundary problem for the Laplace equation in a half-disk, with opposite flows at the part of the boundary. Our goal is to find a function $u(r,\theta) \in C^0(\bar{D}) \cap C^2(D)$ satisfying in D the equation

$$\Delta u = 0$$

with the boundary conditions

$$u(1,\theta) = f(\theta),\ 0 \leq \theta \leq \pi,$$

$$u(r,0) = 0, r \in [0,1],$$

$$\frac{\partial u}{\partial \theta}(r,0) = -\frac{\partial u}{\partial \theta}(r,\pi) + \alpha u(r,\pi), \quad r \in (0,1)$$

where $D = \{(r,\theta) : \quad 0 < r < 1, 0 < \theta < \pi\}$; $\alpha < 0$; $f(\theta) \in C^2[0,\pi]$, $f(0) = 0$, $f'(0) = -f'(\pi) + \alpha f(\pi)$.

The difference of this problem is the impossibility of direct applying of the Fourier method (separation of variables). Because the corresponding spectral problem for the ordinary differential equation has the system of eigenfunctions not forming a basis. Throughout this note we mainly use techniques from our work [1]. A special system of functions based on these eigenfunctions is constructed. This system has already formed the basis. This new basis is used for solving of the nonlocal boundary value problem. The existence and the uniqueness of the classical solution of the problem are proved.

Keywords: Laplace equation, basis, eigenfunctions, nonlocal boundary value problem

2010 Mathematics Subject Classification: 33C10, 34B30, 35J, 35P10

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About the correctness to some nonlocal boundary value problem for the equation of the mixed type of the first kind and the second order in space

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Abstract: As it is known the Dirichlet problem for the equation of the mixed type of the first sort is incorrect. Naturally there is a problem: whether it is impossible to substitute statements of the problem of Dirichlet other conditions enveloping all boundary which ensure a problem correctness? For the first time, such problems have been offered and studied in the works T.S.Kalmenov's. In the present work for the equation of the mixed type of the first kind, the second order in space unambiguously solvability and smoothness of the generalized solution, of some nonlocal boundary value problem with constant coefficients from Sobolev spaces is studied.

Let $\Omega = \prod\limits_{i=1}^{n} (\alpha_i, \beta_i) \in R^n$. $n-$ a measured parallelepiped.

In the area $Q = \Omega \times (0, T)$ we consider a differential equation of the second order

$$(1) \quad Lu = K(x)\, u_{tt} - \sum_{i,j=1}^{n} \frac{\partial}{\partial\, x_i} \left(a_{i,j}(x)\, u_{x_j} \right) + \alpha\,(x,t)\, u_t + c(x,t)\, u = f(x,t),$$

where, $x_1 K(x) > 0$ at $x_1 \neq 0$ and thus $x_1 \in (\alpha_1, \beta_1)$, $\alpha_1 < 0 < \beta_1$.

We assume, that coefficients of equation (1) -are smooth enough functions and let the condition following is satisfied: $a_{i,j}\xi_i\xi_j \geq a_0|\xi|^2$, where,

$a_{i,j}(x) = a_{j,i}(x)$; $a_0 - const > 0$, $\xi \in R^n$; $|\xi|^2 = \sum\limits_{i=1}^{n} \xi_i^2$.

Problem. To find a generalized solution of equation (1) from the Sobolev space $W{}_2^l(Q)$, ($2 \leq l$- is integer), satisfying to nonlocal boundary conditions

$$(2) \qquad \gamma D_t^p\, u|_{t=0} = D_t^p\, u|_{t=T}$$

$$(3) \qquad \eta D_{x_i}^p\, u|_{x_i=\alpha_i} = D_{x_i}^p\, u|_{x_i=\beta_i},$$

at $p = 0, 1$; here γ *and* $\eta - const \neq 0$, where $D_t^p u = \frac{\partial^{\,p} u}{\partial t^{\,p}}$ $D{}_t^0\, u = u$.

Keywords: second order mixed type equation of the first kind,nonlocal boundary value problem with constant coefficients, Sobolev spaces,unique solvability,existence of solution, smoothness of the generalized solution.

2010 Mathematics Subject Classification: 35M10

Solution of the linearized nonregular free boundary problem for the parabolic equations

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Abstract: We study linearized free boundary problem for the system of the parabolic equations with two small parameters at the principle terms in the boundaries conditions. The original nonlinear free boundary problem describes the process of phase transition (melting, solidification) of substance with admixture of unknown concentration.

There are proved in the Hölder spaces the existence, uniqueness and uniform with respect to the small parameters coercive estimates of the solution to the linearized free boundary problem.

These results are on the basis of the solution of the nonlinear singular perturbed free boundary problem for the system of the parabolic equations.

Keywords: Hölder space, parabolic equation, boundary value problem, small parameters, existence, uniqueness, coercive estimates

2010 Mathematics Subject Classification: 35K51, 35B20, 35B65

Properties of an isolated solution to a nonlinear boundary value problem with parameters

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Abstract: A nonlinear two-point boundary value problem for ordinary differential equation with parameters is studied. Necessary and sufficient conditions for the existence of an isolated solution are obtained. The conditions of continuity dependence of isolated solution are established.

We consider the nonlinear two-point boundary value problem with parameter for system of ordinary differential equations

$$\frac{dx}{dt} = f(t, x, \mu), \qquad t \in (0, T), \qquad x \in R^n, \qquad \mu \in R^m \tag{1}$$

$$g(\mu, x(0), x(T)) = 0, \tag{2}$$

where $f : [0, T] \times R^n \times R^m \to R^n$, $\quad g : R^m \times R^n \times R^n \to R^{n+m}$ are continuous.

The essentially nonlinear problems usually have several, moreover, infinitely many solutions. Isolation is one of the most important properties of solutions to nonlinear problems. Continuous dependence of solution on the change of considered differential equations and boundary conditions is very important in applications for the adequate describing of processes studied. This property is also necessary while constructing the approximate and numerical methods for finding the solutions. However, in general, the isolated solution considered as an isolated element of the set of solutions to problem (1), (2) does not have that property. Therefore, it is considered an "isolated" in a narrow sense solutions to problem (1), (2).

Keywords: nonlinear boundary value problem with parameters, "isolated" solution.

2010 Mathematics Subject Classification: 34A34, 34B08

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Unique solvability of an inverse problem for a semilinear equation with final overdetermination

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Abstract: This study is devoted to the investigation of unique solvability of the inverse problem governed by a semilinear equation subject to a final overdetermination

$$\left\{\begin{array}{l} \frac{du}{dt}+Au\left(t\right)=f\left(t,u(t)\right)+p,\ 0<t<T, \\ \\ u\left(0\right)=\varphi,\ u\left(T\right)=\psi \end{array}\right. \tag{4}$$

in an arbitrary Banach space E. Here, A is a linear operator acting in E, with domain $D\left(A\right)$. Assume that $-A$ is the generator of the analytic semigroup $\exp\{-tA\}$ with an exponentially decreasing norm

$$\left\|e^{-tA}\right\|_{E\rightarrow E}\leq Me^{-\delta t},\ t\left\|Ae^{-tA}\right\|_{E\rightarrow E}\leq M,\ t\geq 0. \tag{5}$$

For the numerical solution of this problem the first order of accuracy Rothe difference scheme is proposed. The existence and uniqueness results for this difference scheme is given. As an application, an inverse problem governed by a semilinear parabolic equation subject to a final overdetermination is considered. Throughout this note we mainly use techniques from [1, 2].

Keywords: Existence and uniqueness, semilinear equation, inverse problem, final overdetermination, difference scheme

2010 Mathematics Subject Classification: 65L12, 31A25, 35K58, 34A12

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Green's function of a heat problem with a periodic boundary condition

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Abstract: In the paper a non-local initial-boundary value problem for a non-homogeneous one-dimensional heat equation is considered. The domain under consideration is a rectangle. The classical initial condition with respect to t is put. A non-local periodic boundary condition with respect to a spatial variable x is put. It is well-known that a solution of problem can be constructed in the form of convergent orthonormal series according to eigenfunctions of a spectral problem for an operator of multiple differentiation with periodic boundary conditions. Therefore Green's function can be also written in the form of an infinite series with respect to trigonometric functions (Fourier series). For classical first and second initial-boundary value problems there also exists a second representation of the Green's function by Jacobi function. In this paper we find the representation of the Green's function of the non-local initial-boundary value problem with periodic boundary conditions in the form of series according to exponents.

Throughout this note we mainly use techniques from [3-5].

Keywords: heat equation, initial-boundary value problems, periodic boundary condition, Green's function

2010 Mathematics Subject Classification: 5C15, 35E15, 35K05, 35K20

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On the properties of operators with the semi-power type lacunas

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Abstract: For linear matrix operators

$$u_N(f,\Lambda,X)=\frac{a_0}{2}+\sum_{k=1}^{N}\lambda_k^{(n)}(a_k\cos kx+b_k\sin kx),$$

($a_k,\ b_k$ are Fourier coefficients of $f(x)$), constructed with respect to subsequences of the Fourier sums S_{n_k} with the lacunas $n_k=[\frac{k^\gamma}{\ln^\beta(k+1)}],\ k=1,2,...,N$, a sufficient condition for boundedness of norms $\|u_N\|_{C_{2\pi}}$

$$(1)\qquad \|u_N\|\le c\{n_N^{\frac{1}{\gamma}}\sum_{k=0}^{N}(\Delta\lambda_k)^2\ln^{\beta/\gamma}(k+1)[1+(N-k)^{1-1/\gamma}k^{1/\gamma-1}]\},$$

is given. N is the number of non-zero differences $\Delta\lambda_k=\lambda_{n_k}-\lambda_{n_k+1},\ c>0,\ c=const.$ $[y]$ is the integer part of y. $\beta\ge0,\ \gamma\ge1$. $\lambda_k^n\equiv\lambda_k$ are limited multipliers of summation.

Using (1) approximative properties of the discrete analog of biharmonic operators $D(f,r,x)$ are investigated. In particular the precise estimate

$$(2)\quad \sup_{f\in C^1}\|f(x)-D(f,r,x)\|_{C_{2\pi}}=O\big((\gamma-\beta)\sqrt{(1-r)\ln^{\beta/\gamma}\frac{1}{1-r}}\big),\ r\to1-.$$

is obtained in the class of differentiable functions C^1.

If $\gamma=1,\ \beta=0$ then (2) coincides with the estimate of the work [1].

Keywords: semi-power type lacunas, biharmonic operator

2010 Mathematics Subject Classification: 42A24

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Initial value problem for a class of fractional order inhomogeneous equations in Banach spaces

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Abstract: The fractional order linear inhomogeneous equation in Banach space with the Riemann–Liouville derivative, and with a bounded operator at the unknown function endowed by initial conditions for fractional derivatives of a solution is studied. Theorem on unique solvability of the problem is proved. As application of the obtained result an initial boundary value problem for Barenblatt–Zheltov–Kochina equation with the Riemann–Liouville fractional derivative with respect to time is considered. Unique solvability conditions and a form of the problem solution are found in terms of the Fourier series with respect to the eigenfunctions of the Laplace operator.

Famous works (see [1] and others) concerning related problems do not contain similar results. Note papers [2, 3], where analogous problems are solved in the case of the Caputo fractional derivative, correspondently, with initial conditions for the integer derivatives of a solution.

Keywords: Riemann–Liouville fractional derivative, Riemann–Liouville fractional integral, Mittag-Leffler function, initial boundary value problem, partial differential equation

2010 Mathematics Subject Classification: 35R11, 34A08

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On an illposed model of oscillations of a flat plate with a variety of mounts on opposite sides

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Abstract: We consider a model case of stationary vibrations of a thin flat plate, one side of which is embedded, the opposite side is free, and the sides are freely leaned. In mathematical modeling there is a local boundary value problem for the biharmonic equation in a rectangular domain. Boundary conditions are given on all boundary of the domain.

Problem C. *Find in* $D = \{(x, y) : 0 < x < \pi, 0 < y < l\}$ *a solution to the biharmonic equation*

$$\triangle^2 u \equiv u_{xxxx}(x, y) + 2u_{xxyy}(x, y) + u_{yyyy}(x, y) = 0, \quad (x, y) \in D,$$

satisfying boundary conditions in the first spatial variable x:

$$u|_{x=0} = 0,\ \triangle u|_{x=0} = 0; \quad u|_{x=\pi} = 0,\ \triangle u|_{x=\pi} = 0; \quad 0 \le y \le l;$$

and boundary conditions in the second spatial variable y:

$$u|_{y=0} = \varphi_1(x),\ \frac{\partial u}{\partial y}\Big|_{y=0} = \varphi_2(x), \quad 0 \le x \le \pi;$$

$$\triangle u|_{y=l} = \psi_1(x), \quad \frac{\partial \triangle u}{\partial y}\Big|_{y=l} = \psi_2(x),\ \ 0 \le x \le \pi.$$

We show that the considered problem is self-adjoint. Herewith the problem is ill-posed. We show that the stability of solution to the problem is disturbed. Necessary and sufficient conditions of existence of the problem solution are found. Spaces of the ill-posedness of the considered problem are constructed.

Throughout this note we mainly use techniques from our works [1-2].

Keywords: oscillations, thin flat plate, biharmonic equation, boundary value problem, ill-posed problem

2010 Mathematics Subject Classification: 35J40, 35P20

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Approximation properties of some summation methods in the Smirnov classes with variable exponent

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Abstract: Let $G \subset \mathbb{C}$ be a finite domain in the complex plane, bounded by a rectifiable Jordan curve Γ. The variable exponent Lebesgue spaces $L^{p(\cdot)}(\Gamma)$ for a given Lebesgue measurable variable exponent $p(z) \geq 1$ on Γ we define as the set of Lebesgue measurable functions f, such that $\int_\Gamma |f(z)|^{p(z)} |dz| < \infty$. The function

$$\|f\|_{L^{p(\cdot)}(\Gamma)} := \inf \left\{ \lambda > 0 : \int\limits_\Gamma |f(z)/\lambda|^{p(z)} |dz| \leq 1 \right\}$$

defines a norm on $L^{p(\cdot)}(\Gamma)$.

Given Lebesgue measurable function $p(\cdot) : \Gamma \to [1, \infty)$ we define the variable exponent Smirnov classes of analytic functions in G as $E^{p(\cdot)}(G) := \{ f \in E^1(G) : f \in L^{p(\cdot)}(\Gamma) \}$.

Each function $f \in E^{p(\cdot)}(G)$, has the non-tangential limits almost everywhere $(a.e)$ on Γ and hence if we define $\|f\|_{E^{p(\cdot)}(G} := \|f\|_{L^{p(\cdot)}(\Gamma)}$, then the space $E^{p(\cdot)}(G)$ is also a normed space of analytic functions in G.

In this work we continue our investigations [1,2], on the approximation problems in the variable exponent Smirnov classes $E^{p(\cdot)}(G)$. Namely, we study the approximation properties of the different approximation aggregates and obtain the appropriate estimations in term of the higher modulus of smoothness for a given function $f \in E^{p(\cdot)}(G)$.

Keywords: Variable exponent Smirnov classes, Faber series, De Vallée Poussin means, Jackson means

2010 Mathematics Subject Classification: 30E10, 41A10

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Algorithm of finding of a solution bounded on stripe of linear hyperbolic equation with the mixed derivative

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Abstract: A semiperiodic boundary value problem is examined for linear hyperbolic equation with two independent variables. Euler's modification broken method is offered for construction of a solution bounded on a stripe. The conditions of existence of a solution bounded on a stripe are received.

We consider a linear hyperbolic equation with two independent variables

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u + f(x,t), \tag{1}$$

where $A(x,t), B(x,t), C(x,t)$ are functions continuous on $\overline{\Omega}$, $f : \overline{\Omega} \times R^3 \to R$, is function continuous on $t \in \mathbb{R}$ at fixed $x \in [0,\omega]$ and uniformly relatively $t \in \mathbb{R}$ continuous on $x \in [0,\omega]$.

The solution of equation (1) is investigated satisfying to the following conditions

$$u(0,t) = \psi(t), \quad t \in \mathbb{R} = (-\infty, \infty), \quad \frac{\partial u(x,t)}{\partial x} \in C^*(\overline{\Omega}, \mathbb{R}), \tag{2}$$

where $C^*(\overline{\Omega}, \mathbb{R})$ a space of bounded functions, the continuously differentiable function $\psi(t)$ is bounded on $\mathbb{R}$ together with the derivative $\dot{\psi}(t)$. The problem (1), (2) we will reduce to the equivalent problem, entering new unknown functions

$$v(x,t) = \frac{\partial u(x,t)}{\partial x}, \; w(x,t) = \frac{\partial u(x,t)}{\partial t}.$$

For finding bounded on a stripe solution we apply Euler's modification broken method [1] and we get a problem for the systems ordinary differential equations. Unlocal boundary value problems are built for equation (1) on a finite area, allowing with set to approximate narrowing exactness on this area of bounded on the stripe solution.

Keywords: linear hyperbolic equation, boundary value problem, Euler's modification broken method, solution bounded on a stripe

2010 Mathematics Subject Classification: 34A30, 34B05, 35L20

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On boundary value problem for the fourth order mixed type equation with fractional derivative

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Abstract: In the present work we investigate nonlocal problem for the fourth order mixed type equations.

Let $\Omega = \{(x,t) : 0 < x < 1, -p < t < q\}, \Omega^+ = \Omega \cap (t > 0), \Omega^- = \Omega \cap (t < 0)$, where $p, q > 0$. In the domain Ω we consider equation

(1) $$Lu(x,t) = f(x,t)$$

where

$$Lu(x,t) = \begin{cases} \frac{\partial^4 u}{\partial x^4} + {}_C D_{0t}^{\alpha} u & t > 0, \\ \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2}, & t < 0, \end{cases}$$

${}_C D_{0t}^{\alpha} u$ is the Caputo fractional operator of the order $\alpha \in (0,1]$

Problem A. Find a function $u(x,t)$, which is:

1) continuous in $\overline{\Omega}$, together with derivatives appearing in boundary conditions;

2) satisfies equation (1) in domains $\Omega^+ \cup \Omega^-$;

3) satisfies boundary conditions

(2)
$$u(0,t) = u_{xx}(1,t) = 0, u_x(1,t) = u_x(0,t), u_{xxx}(1,t) = u_{xxx}(0,t), -p \le t \le q,$$

(3) $$u(x,-p) = 0, 0 \le x \le 1;$$

4) satisfies gluing condition

(4) $${}_C D_{0t}^{\alpha} u(x,+0) = \frac{\partial u(x,-0)}{\partial t}, 0 < x < 1.$$

The main result of this study is the following theorem.

Theorem. Let $f(x,t) \in C_{x,t}^{4,0}(\bar{\Omega})$, $\frac{\partial^5 f(x,t)}{\partial x^5} \in L_2(\Omega)$,

$$f(0,t) = f_{xx}(1,t) = f_{xxx}(0,t) = 0, f_x(1,t) = f_x(0,t), f_{xxx}(1,t) = f_{xxx}(0,t),$$

and $\Delta_n = \lambda_n^2 \sin \lambda_n^2 p + \cos \lambda_n^2 p \neq 0$ at $n = 1, 2, ...$

Then the solution of Problem A exists and unique.

On boundary value problem for the fourth order mixed type equation with fractional derivative

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Abstract: Many problems in diffusion and dynamical processes, electrochemistry, biosciences, signal processing, system control theory lead to differential equations of fractional order. In the present work we investigate nonlocal problem for the fourth order mixed type equations.

Let $\Omega = \{(x,t) : 0 < x < 1, -p < t < q\}, \Omega^+ = \Omega \cap (t > 0), \Omega^- = \Omega \cap (t < 0)$, where $p, q > 0$. In the domain Ω we consider equation

$$Lu(x,t) = f(x,t) \tag{1}$$

where

$$Lu(x,t) = \begin{cases} \frac{\partial^4 u}{\partial x^4} +_C D_{0t}^{\alpha} u & t > 0, \\ \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2}, & t < 0, \end{cases}$$

${}_C D_{0t}^{\alpha} u$ is the Caputo fractional operator of the order $\alpha \in (0, 1]$.

Problem A. Find a function $u(x,t)$, which is:
1) continuous in $\overline{\Omega}$, together with derivatives appearing in boundary conditions;
2) satisfies equation (1) in domains $\Omega^+ \cup \Omega^-$;
3) satisfies boundary conditions
(2)
$$u(0,t) = u_{xx}(1,t) = 0, u_x(1,t) = u_x(0,t), u_{xxx}(1,t) = u_{xxx}(0,t), -p \le t \le q,$$

$$u(x, -p) = 0, 0 \le x \le 1; \tag{3}$$

4) satisfies gluing condition

$${}_C D_{0t}^{\alpha} u(x, +0) = \frac{\partial u(x, -0)}{\partial t}, 0 < x < 1. \tag{4}$$

The main result of this study is the following theorem.

Theorem 1. *Let* $f(x,t) \in C_{x,t}^{4,0}(\bar{\Omega})$, $\frac{\partial^5 f(x,t)}{\partial x^5} \in L_2(\Omega)$,

$$f(0,t) = f_{xx}(1,t) = f_{xxx}(0,t) = 0,$$

$$f_x(1,t) = f_x(0,t), f_{xxx}(1,t) = f_{xxx}(0,t),$$

and $\Delta_n = \lambda_n^2 \sin \lambda_n^2 p + \cos \lambda_n^2 p \neq 0$ *at* $n = 1, 2, ...$ *Then the solution of Problem A exists and unique.*

Keywords: mixed type equation, Caputo fractional derivative, Mittag-Leftler function.

2010 Mathematics Subject Classification: 35R11, 35M12, 35M32

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Asymptotics solutions of a singularly perturbed integro-differential problem with rapidly oscillating coefficients

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Abstract: In this paper we consider the Cauchy problem for a singularly perturbed integro-differential systems with rapidly oscillating coefficients:
(1)

$$\varepsilon\dot{z} - A(t)z - \varepsilon\varphi(t)cos\frac{2\beta(t)}{\varepsilon}Bz - \int\limits_{t_0}^{t} k(s)z(s,\varepsilon)ds = h(t),\ z(t_0,\varepsilon) = z^0,\ t \in [0,T],$$

where $z(t,\varepsilon) = \{u(t,\varepsilon), \vartheta(t,\varepsilon)\}$ is an unknown vector-function, $z^0 = \{u^0, \vartheta^0\}$ is a known constant vector, $h(t) = \{h_1(t), h_2(t)\}$ are known vector-function, $A(t)$, $k(t)$, B are given matrices, $\varphi(t)$, $\beta(t) > 0$ are known functions, $\varepsilon > 0$ is a small parameter. It is required to construct the main term of asymptotics of a solution of (1) at $\varepsilon \to +0$.

A particular case of problem (1) (with $\beta(t) = \beta \cdot t, \beta = const, h(t) \equiv 0, k(t) \equiv 0$), describing the phenomenon of parametric amplification, been considered in [1–3]. We will illustrate the application of the regularization method [4, 5] for obtaining the principal term of the asymptotic behavior of the solution of problem (1).

Keywords: Integro-differential system, small parameter, asymptotic solution

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On a problem of the Frankl type for an equation of the mixed parabolic-hyperbolic type

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Abstract: In the paper a new non-local boundary value problem for an equation of the parabolic-hyperbolic type is formulated. This equation is of the first kind, that is, the line of type change is not a characteristic of the equation. The suggested new non-local condition binds points on boundaries of the parabolic and hyperbolic parts of the domain with each other. This problem is generalization of the well-known problems of Frankl type. Unlike the existing publications of the other authors related to the theme, in the suggested formulation of the problem the hyperbolic part of the domain coincides with a characteristic triangle. Unique solvability of the formulated problem is proved in the sense of classical and strong solutions.

Let $\Omega \subset R^2$ be a finite domain bounded for $y > 0$ by the segments AA_0, A_0B_0, B_0B, $A = (0,1)$, $B_0 = (1,1)$, $B = (1,0)$ and for $y < 0$ by the characteristics $AC: \ x + y = 0$ and $BC: \ x - y = 1$ of an equation of the mixed parabolic-hyperbolic type

(1) $$Lu = \left\{ \begin{array}{c} u_x - u_{yy},\ y > 0 \\ \\ u_{xx} - u_{yy},\ y < 0 \end{array} \right\} = f(x,y).$$

Problem F. *Find a solution to Eq.*(1) *satisfying classical boundary conditions*

(2) $$u|_{AA_0} = 0, \quad u_y|_{A_0B_0} = 0,$$

and a non-local boundary condition

(3) $$\alpha u\left(\theta_0\left(t\right)\right) + \beta u\left(\theta_1\left(t\right)\right) = \gamma u\left(\theta\left(t\right)\right), \quad 0 \le t \le 1,$$

where $\theta\left(t\right) = (t,1)$, $\theta_0\left(t\right) = \left(\frac{t}{2}, -\frac{t}{2}\right)$, $\theta_1\left(t\right) = \left(\frac{t+1}{2}, \frac{t-1}{2}\right)$; α, β *and* γ *are given numbers.*

Keywords: non-local boundary conditions, equation of the parabolic-hyperbolic type, Green's function, Frankl problem

2010 Mathematics Subject Classification: 35M10, 35M12

Weighted multiplicative estimate for norm of discrete Hardy operator

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Abstract: Let $f \geq 0$ be a sequence of real numbers $f = \{f_i\}_{i=1}^{\infty}$ with non-negative terms.

Let $v > 0$, $u \geq 0$, and $w \geq 0$ be weight sequences. Let A be a matrix operator in the form:

$$(Af)_i = \sum_{j=1}^{i} a_{i,j} f_j, \quad i \geq 1,$$

where $a_{i,j} \geq 0$ for $i \geq j \geq 1$ and $a_{i,j} = 0$ for $i < j$.

We consider the following weighted multiplicative inequality:

$$\|uPf\|_q \leq C \|vf\|_p^{\alpha} \, \|wAf\|_r^{1-\alpha}, \quad f \geq 0, \quad 0 \leq \alpha \leq 1, \tag{4}$$

where $(Pf)_i = \sum\limits_{j=1}^{i} f_j$ is the discrete Hardy operator and $\|\cdot\|_q$ is the standard norm of the space l_q.

When $\alpha = 1$ inequality (1) is a weighted estimate of the discrete Hardy operator, which is well-studied for all values of the parameters $0 < p, q < \infty$ and has numerous applications (see [1,2]). Multiplicative inequalities are also of great importance and have applications to different problems of Analysis. Some of applications can be found in [3].

When $\alpha = 0$ from (1) we get the inequality that is of independent interest.

Keywords: multiplicative inequality, Hardy-type inequality, matrix operator, sequence

2010 Mathematics Subject Classification: 26D10, 26D15, 39B82

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Weighted additive estimate for norm of discrete Hardy operator

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Abstract: Let $f \geq 0$ be a sequence of real numbers $f = \{f_i\}_{i=1}^{\infty}$ with non-negative terms.

Let $v > 0$, $u \geq 0$, and $w \geq 0$ be weight sequences. Let P be a discrete Hardy operator $(Pf)_i = \sum\limits_{j=1}^{i} f_j$, and A be a matrix operator in the form $(Af)_i = \sum\limits_{j=1}^{i} a_{i,j} f_j$, $i \geq 1$, where $a_{i,j} \geq 0$ for $i \geq j \geq 1$ and $a_{i,j} = 0$ for $i < j$.

We consider the following weighted additive estimate:

$$\|uPf\|_q \leq C\left(\|vf\|_p + \|wAf\|_r\right), \quad f \geq 0, \tag{5}$$

where $\|\cdot\|_q$ is the standard norm of the space l_q.

In papers [1–3] under some conditions on the elements $(a_{i,j})$ the authors have found necessary and sufficient conditions for the validity of the inequality:

$$\|uAf\|_q \leq C\left(\|vf\|_p + \|wPf\|_p\right), \quad f \geq 0,$$

where $1 < p, q < \infty$.

Moreover, continuous analogue of inequality (1) has been studied in paper [**?**] for $A \equiv P$, $r = p$ and $1 < p \leq q < \infty$.

Keywords: additive inequality, Hardy-type inequality, matrix operator, sequence

2010 Mathematics Subject Classification: 26D10, 26D15, 39B82

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Variations on strongly lacunary quasi Cauchy sequences

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Abstract: In this paper $N_\theta^\alpha(p)$-ward continuity is introduced in the sense that a real valued function f defined on a subset A of $\mathbf{R}$, the set of real numbers is $N_\theta^\alpha(p)$-ward continuous if $(f(x_k))$ is $N_\theta^\alpha(p)$-quasi-Cauchy whenever (x_k) is $N_\theta^\alpha(p)$-quasi-Cauchy sequence of points in A, where a sequence (x_k) is called $N_\theta^\alpha(p)$-quasi-Cauchy if $\lim_{r\to\infty}\frac{1}{h_r^\alpha}\sum_{k\in I_r}|\Delta x_k|^p=0$, where $\Delta x_k = x_{k+1}-x_k$ for each positive integer k, p is a constant positive integer, α is a constant in $]0,1]$, $I_r=(k_{r-1},k_r]$, and $\theta=(k_r)$ is a lacunary sequence, which is an increasing sequence of positive integers such that $k_0\neq 0$, and $h_r : k_r - k_{r-1}\to\infty$.

Keywords: Sequences, series, strongly lacunary convergence, continuity

2010 Mathematics Subject Classification: 40A05, 40A35, 26A15

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On an Uniqueness and Correct Solvability of the Biharmonic Boundary Value Problem

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Abstract: In this paper a generalized third boundary value problem for the biharmonic equation:

$$\Delta^2 u = f(x), \quad x \in S,$$

in the unit ball $S = \{x \in \mathbb{R}^n : |x| < 1\}$ with boundary operators up to third order containing normal derivatives: $\frac{\partial}{\partial \nu}$ and Laplacian: Δ

$$\begin{cases} a_{00}u + a_{01}\frac{\partial}{\partial \nu}u + a_{02}\Delta u\Big|_{\partial S} = \varphi_1(s), \quad s \in \partial S, \\ \\ a_{11}\frac{\partial}{\partial \nu}u + a_{12}\Delta u + a_{13}\frac{\partial}{\partial \nu}\Delta u\Big|_{\partial S} = \varphi_2(s), \quad s \in \partial S, \end{cases}$$

is investigated.

The existence and uniqueness theorems are proved.

Some particular cases of general problem are considered.

Keywords: biharmonic equation, boundary value problems, Laplace operator, harmonic polynomials

2010 Mathematics Subject Classification: 35J05, 35J25, 26A33.

On Generalized Quasi-Convex Bounded Sequences

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Abstract: The space of all sequences $a = (a_k)$ for which $||a||_q = \sum\limits_k k|\Delta^2 a_k| + \sup_k |a_k| < \infty$ is denoted by q [2]. Here, $\Delta a_k = a_k - a_{k+1}$ and $\Delta^m a_k = \Delta(\Delta^{m-1} a_k) = \Delta^{m-1} a_k - \Delta^{m-1} a_{k+1}$ with $\Delta^0 a_k = a_k$, $m \geq 1$. If $a = (a_k) \in q$ then $k\Delta a_k \to 0 \ \ (k \to \infty)$ and $q \subset bv$, the space of all sequences of bounded-variation, since $\sum\limits_k |\Delta a_k| \leq \sum\limits_k k|\Delta^2 a_k|$. In the study we shall give a generalization of quasi-convex bounded sequences.

Keywords: FK spaces, $\beta-$ and $\gamma-$ duals, topological sequence spaces.

2010 Mathematics Subject Classification: 46A45, 40G05.

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On $AK(S)$ and $AB(S)$ Properties of a K-Space

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Abstract: A typical sum s on a K- space S has often the representation,

$$s(z) = \lim_{\gamma \in \Gamma} \sum_k u_{\gamma k} z_k, \ z = (z_k) \in S \tag{6}$$

where Γ is a directed index set and $u_\gamma = (u_{\gamma k}) \in \phi$, the space of finitely non-zero sequences, for each $\gamma \in \Gamma$. Let a K-space S be equipped with a sum (6). Then, for each $x = (x_k)$ and $\gamma \in \Gamma$, the sequence $P_\gamma(x) = \sum_k u_{\gamma k} x_k \delta^k, \quad (\gamma \in \Gamma)$ is called the γth S-section of x [2]. Here δ^k is the sequence whose kth component is 1 all the others are 0.

If $\lambda \supset \phi$ is a K space, then Boos and Leiger defined the spaces $\lambda_{AB(S)}$ and $\lambda_{AK(S)}$ in [2] as

$$\lambda_{AB(S)} = \{x \in \omega | (P_\gamma(x))_{\gamma \in \Gamma} \text{ is a bounded net in } \lambda\},$$

and

$$\lambda_{AK(S)} = \{x \in (\lambda_{AB(S)} \cap \lambda) | \lim_\gamma P_\gamma(x) \text{ exists in } \lambda\}.$$

In this work, we investigate some properties of these spaces and give some theorems related to the duals.

Keywords: K- spaces, n-th section of a sequence, β-, γ-, f- duality.

2010 Mathematics Subject Classification: Primary: 40H05, 46A45, Secondary: 40G99, 40C05.

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On q^λ and q_0^λ Invariant Sequence Spaces

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Abstract: Invariant sequence spaces are very helpful for investigations of the duality of sequence spaces. For instance, if the sequence space X satisfies the condition $\ell_\infty.X = X$ then its $\alpha-$, β- and γ- duals are same [4]. Garling [1] investigated B- and B_0- invariant sequence spaces and Buntinas [2] introduced and investigated q- and q_0- invariant sequence spaces and recently, Grosse- Erdmann [3] studied on ℓ_1 invariant sequence spaces.

In this work, we define q^λ and q_0^λ invariant sequence spaces, X with $q^\lambda.X = X$ and $q_0^\lambda.X = X$, respectively. and give some related theorems.

Keywords: K- spaces, λ-boundedness and λ-convergence of a sequence, β-, γ-, f- duality.

2010 Mathematics Subject Classification: Primary: 40H05, 46A45, Secondary: 40G99, 40C05.

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Solution of a Cauchy problem for iterated generalized two-axially symmetric equation of hyperbolic type

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Abstract: In the work an explicit solution of Cauchy problem for iterated generalized two-axially symmetric equations of hyperbolic type was constructed.

In this work, we considered an analogue of the Cauchy problem in a domain $\Omega = \{(x, y) :\ 0 < y < x\}$ finding a classical solution $u(x, y) \in C^{2m-1}(\bar{\Omega})$ of the equation

$$(1) \qquad [h^{\lambda}_{\beta,\alpha}]^m(u) \equiv \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} - \frac{2\alpha}{x}\frac{\partial}{\partial x} + \frac{2\beta}{y}\frac{\partial}{\partial y} + \lambda^2\right)^m u(x, y) = 0,$$

satisfying the initial conditions

$$(2) \qquad \left.\frac{\partial^{2k} u}{\partial y^{2k}}\right|_{y=0} = \varphi_k(x),\ x \geqslant 0; \quad \left.\frac{\partial^{2k+1} u}{\partial y^{2k+1}}\right|_{y=0} = 0,\ x \geqslant 0,\ \ k = \overline{0, m-1};$$

where $\alpha, \beta, \lambda \in R,\ m \in N$, and $0 < \alpha < 1,\ \ 0 < \beta < 1/2$, $\varphi_k(x)$, $(k = \overline{0, m-1})$ are given smooth functions.

By applying two-dimensional generalized Erdelyi-Kober operator [1], the explicit formula of a solution of the problem $\{(1), (2)\}$ was received

$$u(x, y) = \sum_{n=0}^{m-1} \frac{2^{-2n} y^{1-2\beta}}{n!\Gamma(\beta + n)} \int\limits_{x-y}^{x+y} f_n(s) \left(\frac{s}{x}\right)^{\alpha} \frac{\Xi_2(\alpha, 1-\alpha; \beta + n; \sigma, \omega)}{[y^2 - (x-s)^2]^{1-\beta-n}} ds,$$

where $f_n(x) = \sum\limits_{j=0}^{n} (-1)^j C_n^j [B^x_{\alpha-1/2}]^j \tilde{\varphi}_{n-j}(x)$, $\tilde{\varphi}_n(x) = \sum\limits_{j=0}^{n} \gamma_n C_n^j \lambda^{2(n-j)} \varphi_j(x)$, $\gamma_n = \Gamma((2n+1)/2+\beta)/\Gamma((2n+1)/2)$, $\sigma = [y^2 - (s-x)^2]/(4xs)$, $\omega = -(\lambda^2/4)[y^2 - (s-x)^2]$, $\Xi_2(\alpha, 1-\alpha; \beta + n; \sigma, \omega)$ is Humbert hypergeometric function.

Keywords: Erdélyi-Kober operator, two-axially symmetric equations

2010 Mathematics Subject Classification: 26A33, 35L10

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On the sufficient condition for embedding of generalized Lizorkin-Triebel spaces

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Abstract: In this paper the sufficient conditions for the embedding of generalized Lizorkin–Triebel spaces $F^{\varphi}_{p\theta}$ are given.

Denote by U the set of all continuous, increasing on $[0, 2\pi]$ functions such that $\varphi(0) = 0$ and for some $\alpha > 0$ $\varphi(t)t^{-\alpha}$ almost increases. Let $1 \leq p, \theta \leq \infty$ and $\varphi \in U$. We say that a function f belongs to generalized Lizorkin–Triebel spaces $F^{\varphi}_{p\theta}$ if $f \in L^p(R)$ and

$$\|f\|_{F^{\varphi}_{p,\theta}} = \|f\|_p + \left\| \left\{ \sum_{k=0}^{\infty} \left[\varphi^{-1}(2^{-k}) \left| S_{2^{k+1}} f - S_{2^k} f \right| \right]^{\theta} \right\}^{\frac{1}{\theta}} \right\|_{L_p} < \infty,$$

where $S_k f(x)$ partial sum Fourier series. If $\varphi(x) = x^r,\ r > 0$ this space coincides with the space $F^r_{p\theta}$ [1].

Theorem 2. *Let $1 \leq p \leq p' \leq \infty$, $1 \leq \theta, \theta' < \infty$. Let $\varphi, \psi \in U$.*

$$1)\ \textit{for } \theta' < \theta \textit{ if } \sum_{s=0}^{\infty} \left[\varphi(2^{-s}) \psi^{-1}(2^{-s}) 2^{\frac{1}{p'}} \right]^{\frac{\theta' \theta}{\theta - \theta'}} < \infty,$$

$$2)\ \textit{for } \theta' \geq \theta \textit{ if } \sup_{0<t\leq 1} \left[\varphi(t)\psi(t)^{-1} t^{\kappa} \right] < \infty,$$

where $\kappa = \frac{1}{p} - \frac{1}{p'}$. Then the embedding $F^{\varphi}_{p\theta} \hookrightarrow F^{\psi}_{p'\theta'}$ holds.

In the case of generalized Nikol'skii – Besov spaces similar embedding was considered in [2].

Keywords: Lizorkin-Triebel spaces, embedding

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About one special boundary value problem for multidimensional parabolic integro-differential equation

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Abstract: In this paper we consider a special boundary value problem for multidimensional parabolic integro-differential equation with boundary conditions that contains as a boundary condition containing derivatives of order higher than the order of the equation [1,2,4]. The solution is sought in the form of a thermal potential of a double layer. Shows lemma of finding the limits of the derivatives of the unknown function in the neighborhood of the hyperplane. Using the boundary condition and lemma obtained integral-differential equation (IDE) of parabolic operators, whre an unknown function under the integral contains higher-order space variables derivatives. IDE is reduced to a singular integral equation (SIE), when an unknown function in the spatial variables satisfies the Holder.The characteristic part is solved in the class of distribution function using method of transformation of Fourier-Laplace. Found an algebraic condition for the transition to the classical generalized solution. Integral equation of the resolvent for the characteristic part of SIE is obtained. Integro-differential equation is reduced to the Volterra-Fredholm type integral equation of the second kind by method of regularization. It is shown that the solution of SIE is a solution of IDE. Obtain a theorem on the solvability of the boundary value problem of multidimensional parabolic integro-differential equation, when a known function of the spatial variables belongs to the Holder class and satisfies the solvability conditions.

Keywords: Multidimensional parabolic integro-differential equation, singular integral equation, regularization

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About solvability of some boundary value problems for Poisson equation in the ball

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Abstract: In the paper we study properties of some integral - differential operators of fractional order. As an application of the properties of these operators for Poisson equation we examine questions on solvability of a fractional analogue of Neumann problem and analogues of periodic boundary value problems for circular domains. The exact conditions for solvability of these problems are found

Keywords: fractional derivative, Hadamard operator, Poisson equation, Neumann problem, periodic problem

2010 Mathematics Subject Classification: 35J15, 35J25

On weak solutions of the Dirichlet problem for the Tricomi equation

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Abstract: Let $Tu \equiv yu_{xx} + u_{yy}$ — formal differential operator. Δ — curvilinear triangle bounded by segment AB of x-axis, arc BC of the characteristic $x_B = x + \frac{2}{3}|y|^{3/2}$ and smooth curve AC which have not characteristic directions. Also Δ is convex with respect to characteristics of T.

Consider operator Λ' defined on functions $u \in C^2(\overline{\Delta})$, $u|_{AB \cup AC} = 0$ by equality: $\Lambda' u = Tu$. Let Λ is closure of Λ' in $L_2(\Delta)$. Weak solution of the Dirichlet problem for the Tricomi equation $Tv = g$, $g \in L_2$ is $v \in L_2$ such that $\langle v, \Lambda u\rangle = \langle g, u\rangle$ $\forall u \in D(\Lambda)$. From the definition follow that v is the weak solution of the Dirichlet problem for the Tricomi equation $Tv = g$ if and only if $v \in D(\Lambda^*)$.

Theorem. Any weak solution of the Dirichlet problem belongs to $W_2^1(\Delta \backslash O_A)$ where O_A any neighbourhood of point A. Moreover there is orthogonal decomposition of $L_2(\Delta)$ to the sum $H_1 \oplus H_2$ such that for $g \in H_1$ corresponding $v \in W_2^1(\Delta)$ and for $g \in H_2$ $v \notin W_2^1(\Delta)$.

Keywords: Dirichlet boundary problem, Tricomi equation

2010 Mathematics Subject Classification: 94B05, 94B15

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Global solvability in some models related to equations of Sobolev type

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Abstract: In the report there are results on global solvability of initial-boundary value problems for

— higher-order Sobolev-type strongly nonlinear equations (pseudoparabolic equations) appearing in viscoelasticity;

— nonlinear analogs of the Boussinesq equations of long waves appearing in modeling a line of transmissions containing nonlinear elements (for example, diodes, capacity of which is a nonlinear function of voltage);

— linear and nonlinear integro-differential equations of viscoelasticity containing degenerate equations.

Some properties of solutions are also additionally studied.

Keywords: Global solvability, pseudoparabolic equation, equations of Sobolev type, Boussinesq equation, integro-differential equations.

2010 Mathematics Subject Classification: 35S05, 35K55

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Oscillation and nonoscillation of half-linear second order differential equation with alternating potential

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Abstract: In this work we study the questions of oscillation and nonoscillation of the following half–linear second order differential equation

$$(HL) \qquad \Big(\rho(t)\left|y'(t)\right|^{p-2} y'(t)\Big)' + q(t)\left|y(t)\right|^{p-2} y(t) = 0, \;\; t \geq 0,$$

on the interval $I = (a, \infty)$, $-\infty \leq a < \infty$, when $1 < p < \infty$, $p \neq 2$. Here $\rho(t) > 0$ and $q(t)$ continuous functions on I.

The results of investigations of the properties and methods for the equation (HL) up to year 2005 are exposed in the book by Došlý and Řehák [1].

For researching properties of equation (HL) we use the variational method, and we establish the conditions of oscillation and nonoscillation of the equation (HL), when the function $q(t)$ changes the sign on the interval I.

Keywords: oscillation, nonoscillation, half-linear equation, alternating potential, variational method

2010 Mathematics Subject Classification: 34C10

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Soliton deformations of some minimal surfaces

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Abstract: Modified Veselov-Novikov equation was introduced in [1]. A method for constructing exact solutions of this equation by Mutard transformations was proposed in [2] and in [3] was obtained a geometric interpretation of Mutard transformations. It is given by the solution of the Dirac equation and three real constants. Any solution of the Dirac equation defines a surface in three-dimensional Euclidean space, given up to shifts by Weierstrass representation[4]. Fixing the three constants, we completely fix the surface. On this surface are given a conformal parameter, and the potential of the Dirac operator U is the potential representation of the surface. Applying inversion to this surface with its center at the origin, we obtain a new surface with the same conformal parameter and new potential. In this paper applied this transformations for mVN equation on the examples of higher order Enneper's surface, catenoid and helicoid and constructed soliton deformations [5] of this surfaces.

Keywords: Dirac operator, modified Veselov-Novikov equation, Mutard transformations, Enneper's surface,catenoid, helicoid,deformation of surface, inversion.

2010 Mathematics Subject Classification: 35Q51, 49Q05

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Necessary and sufficient condition that function belong to the Nikolskii-Morrey space $H^{(r)}M^{\lambda}_{p\,x_1}$

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Abstract: In this paper there were established the necessary and sufficient conditions that the function belongs to the Nikol'skii–Morrey space $H^{(r)}M^{\lambda}_{p\,x_1}$.

Definition 1. *Let $0 < p \le +\infty$, $0 \le \lambda \le \frac{n}{p}$. Denote by $M^{\lambda}_p(\mathbb{R}^n)$ the space of all real-valued function f, measurable on $\mathbb{R}^n$, such that*

$$\|f\|_{M^{\lambda}_p(\mathbb{R}^n)} = \sup_{\substack{x\in\mathbb{R}^n\\ \rho>0}} \|\rho^{-\lambda}\|f\|_{L_p(B(x,\rho))}\|_{L_\infty(0,\infty)} < \infty.$$

Definition 2. *We say measurable function $f(x_1,\dots,x_n)$ belongs to the class $H^{(r)}M^{\lambda}_{p\,x_1}(B)$ $(r>0,\ 1\le p\le +\infty)$, if it satisfies the following condition.*

Assume that $r=\rho+\alpha$, where ρ is integer and $0<\alpha\le 1$. Let for almost all $x_2,\dots,x_n$ exists the partial derivative $\frac{\partial^\rho f}{\partial x_1^\rho}$ defined almost everywhere on $\mathbb{R}^n$ that belong to the Morrey space $M^{\lambda}_p(\mathbb{R}^n)$ and such that for all h

$$\left\|\frac{\partial^\rho f(x_1+h,x_2,\dots,x_n)}{\partial x_1^\rho} + 2\frac{\partial^\rho f(x_1,\dots,x_n)}{\partial x_1^\rho} + \frac{\partial^\rho f(x_1-h,x_2,\dots,x_n)}{\partial x_1^\rho}\right\|_{M^{\lambda}_p(\mathbb{R}^n)} \le B|h|^\alpha.$$

Definition 3. *Denote by $A_{\nu\,x_1}(f;M^{\lambda}_p(\mathbb{R}^n))$ the best approximation of function $f\in M^{\lambda}_p(\mathbb{R}^n)$ by entire functions g_ν of exponential type ν in metrics of the space $M^{\lambda}_p(\mathbb{R}^n)$, i.e.*

$$A_{\nu\,x_1}(f;M^{\lambda}_p(\mathbb{R}^n)) = \inf_{g_\nu}\|f-g_\nu\|_{M^{\lambda}_p(\mathbb{R}^n)}.$$

Theorem 4. *Function $f\in M^{\lambda}_p(\mathbb{R}^n)$ belongs to the class $H^{(r)}M^{\lambda}_{p\,x_1}$ (with some constant B), if and only if there exists the constant K such that*

$$A_{\nu,x_1}(f;M^{\lambda}_p(\mathbb{R}^n)) < \frac{K}{\nu^r},$$

for all $\nu\ge 1$ or for all ν that run over geometric increasing progression $\nu=a^k$ $(a>1,\quad k=0,1,\dots)$.

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Keywords: Morrey space, Nikolskii-Morrey space, best approximation, entire functions of exponential type

2010 Mathematics Subject Classification: 41A50, 46E35

Cauchy problem for a nonlinear degenerate parabolic system not in divergence form

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Abstract: In this paper, we consider the following Cauchy problem to a nonlinear degenerate parabolic system not in divergence form:

$$\left.\begin{array}{l}\frac{\partial u}{\partial t}=v^{m_1}\nabla\left(\left|\nabla u^k\right|^{p-2}\nabla u^n\right),\\ \frac{\partial v}{\partial t}=u^{m_2}\nabla\left(\left|\nabla v^k\right|^{p-2}\nabla v^n\right),\end{array}\right. \quad t>0,\ x\in R^N,$$

$$u|_{t=0}=u_0(x)\geq 0,\quad v|_{t=0}=v_0(x)\geq 0,\ \ \forall x\in R^N$$

where $p,\ m_1,\ m_2,\ k,\ n$ - are positive constants, $N\geq 1$.

This system can be used to describe the development of multiple groups in the dynamics of biological groups, where $u,\ v$ are the densities of the different groups (see [1]). In paper [1], Gao et al. considered parabolic systems not in divergence form in the case $n=1$ and $p=2$, with source and null Dirichlet boundary conditions. The local existence and uniqueness of classical solution are proved. Moreover, it is proved that all solutions exist globally with homogeneous Dirichlet boundary condition. Porous medium equations with local sources or with nonlocal sources subjected to nonlocal boundary conditions were studied (see [3, 4]). They discussed the conditions of existence and blow-up.

In this paper, by using the comparison principle [2] and the standard equation method [2], we will obtain a global solvability of solutions of Cauchy problem and using the method of nonlinear splitting, we can construct an asymptotic representation of the solution as $t\to\infty$. We obtain the estimates of the solution of Cauchy problem. Results of computational experiments shows, that the self - similar solutions are very appropriate approximation and the iterative method basing on the method Picard is effective for the solution of nonlinear problems and leads to the nonlinear effects if we will use as initial approximation the solutions of self similar equations constructed by the method of nonlinear splitting and by the method of standard equation.

Keywords: nonlinear system, non-divergence form, global solutions, asymptotic behavior of solutions

2010 Mathematics Subject Classification: 35B40, 35B51, 35C06

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On separability of a differential operator of non-classical type in an unbounded domain

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Abstract: Review of literature on the equations of nonclassical type can be found in [1–3]. Properties of the semi periodic Dirichlet problem for degenerate equations were studied in [4–6].

In these studies we investigated the solvability and smoothness of solutions of boundary value problems for nonclassical equations without degeneration

$$Lu = -\frac{\partial^2 u}{\partial y^2} - \sum_{k=0}^{s} R_k(y)\frac{\partial^{2k+1}u}{\partial x^{2k+1}} + \sum_{k=0}^{m}(-1)^k C_k(y)\frac{\partial^{2k}u}{\partial x^{2k}}$$

defined on a set $C_o^\infty(\Omega)$, where $\Omega = \{(x,y) : -\infty < x < +\infty, 0 < y < 1\}$, $C_o^\infty(\Omega)$ is the set of infinitely differentiable functions satisfying the condition:

$$u(x,0) = u(x,1) = 0,$$

and compactly supported with respect to x.

The conditions existence of a solution and the separability of the operator were found during the study.

Keywords: non-classical type operator, resolvent, separability, an unbounded domain

2010 Mathematics Subject Classification: 02.30.Jr, 02.60.Lj

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On spectral properties of a Shrödinger operator with a negative parameter

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Abstract: In the paper a Shrödinger operator with a negative parameter

(1) $$-\Delta + (-t^2 + ita(x) + c(x))$$

is studied in the space $L_2(\mathbb{R}^n)$, where t is a parameter and $-\infty < t < \infty, i^2 = -1$. $\mathbb{R}^n$ is an n-dimensional Euclidean space $x \in \mathbb{R}^n$ $(x = (x_1, x_2, ..., x_n))$.

It is well known that when $t = 0$ the spectral properties of the Schrödinger operator $\Delta + c(x)$ are strongly dependent on the behavior of $c(x)$ on infinity. In this case, the spectral characteristics of the Schrödinger operator are well studied.

The discreteness of the spectrum and the estimates of approximate numbers (s-numbers) of a Schrödinger operator

$$-\Delta + q_1(x) + iq_2(x), (q_1(x) \geq 0, q_2(x) \geq 0)$$

have been studied.

In this paper such questions as:

a) the existence of the resolvent; b) the discreteness of spectrum; c) the estimates of the distribution function of singular values (s-values) will be studied for the operator (1).

Review of the literature shows that these questions for the Schrödinger operator with a negative parameter are insufficiently studied.

Keywords: resolvent, spectrum, singular values, Shrödinger operator

2010 Mathematics Subject Classification: 02.30.Jr, 02.30.Sa

On solvability of a class of nonlinear two-point boundary value problems

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Abstract: We study the solvability of a class of nonlinear two-point boundary value problems for systems of ordinary second-order differential equations on the plane. On the basis of properties of the leading nonlinear terms, we prove a criterion for the solvability of boundary value problems under arbitrary perturbations in a given set by using methods for the computation of the winding number of vector fields.

In this work, we study the solvability of nonlinear two-point boundary value problems of the form

$$z''(t) = \overline{z'(t)}^{m} + f(t, z(t), z'(t)), \quad z = x + iy \in \mathbf{C}, \quad 0 < t < 1, \tag{1}$$

$$z'(0) = A_0(z(0)) + h_0(z), \quad z'(1) = A_1(z(0)) + h_1(z). \tag{2}$$

Here m is an integer larger than unity, $\mathbf{C}$ is the complex plane, $\overline{z} = x - iy$, and the mappings $f : [0,1] \times \mathbf{C}^2 \mapsto \mathbf{C}$ and $A_0, A_1 : \mathbf{C} \mapsto \mathbf{C}$ are continuous and satisfy the conditions:

$$A_j(\lambda z) \equiv \lambda A_j(z) \quad \text{for any } \lambda \geq 0, j = 0, 1, \tag{3}$$

$$\max_{0 \leq t \leq 1} |f(t, z, w|(|z| + |w|)^{-m} \to 0 \quad \text{as } |z| + |w| \to \infty. \tag{4}$$

The mappings h_0 and h_1 act continuously from $C^1([0,1]; \mathbf{C})$ into $\mathbf{C}$, where $C^1([0,1]; \mathbf{C})$ is the space of complex-valued functions continuously differentiable on the interval $[0,1]$, and satisfy the conditions

$$|h_j(z)|/||z||_{C^1} \to 0 \quad \text{as } ||z||_{C^1} \to \infty, j = 0, 1. \tag{5}$$

For this class of boundary value problems, we study the solvability on the basis of an a priori estimate of solutions with the use of methods for the computation of the winding number of vector fields [1]. This problem was earlier considered in [2, 3, 4] and is of interest from the viewpoint of the application of methods of nonlinear analysis and the development of investigation methods for nonlinear two-point boundary value problems.

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Keywords: nonlinear two-point boundary value problem, the winding number of vector fields.

2010 Mathematics Subject Classification: 34B15

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Convergence of kernel density estimators on compact sets

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Abstract: In many statistical applications, one has to estimate the rate of convergence in probability of a kernel density estimator. The usual estimate on compact sets is of order $\left(\frac{\log n}{nh}\right)^{1/2}$ where n is the sample size, $h > 0$ is a bandwidth that depends on n and $h \to 0$, $nh \to \infty$ [1]. We improve this estimate by replacing $\log n$ with a function of n which grows slower than $\log n$. For example, one can take the kth iterated log, for any natural k. The idea is illustrated in estimating a density in the one-dimensional case and can be used in many other setups.

Let K be a kernel, that is a function on R satisfying $\int_R K(t)dt = 1$. By $X_1, ..., X_n$ we denote independent observations from a density f. A kernel density estimate of $f(x)$ is defined by $\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^n K\left(\frac{x-X_i}{h}\right)$.

Theorem 1. Let the kernel K satisfy the Lipschitz condition on R, be bounded and let $\int_R K^2(t)(1+|t|)dt < \infty$. Assume that the density f also satisfies the Lipschitz condition on R and is bounded. Fix a compact $G \subset R$ and let f be positive on G. Suppose a function $\psi(n)$ is such that $\psi(n) \to \infty$ as $n \to \infty$ and $\limsup_{n\to\infty} \frac{n^2}{\psi(n)} < \infty$, $\liminf \frac{nh}{\psi(n)} > 0$. Then

$$\sup_{x\in G}\left|\hat{f}(x) - E\hat{f}(x)\right| = O_p\left(\left(\frac{\psi(n)}{nh}\right)^{1/2}\right).$$

Keywords: kernel density estimator, convergence in probability.

2010 Mathematics Subject Classification: 62F12, 62G07, 62G20

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Existence and nonexistence of positive solutions for singular nth-order three-point nonhomogeneous boundary value problem

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Abstract: In this article, we consider the boundary value problem $u^{(n)}(t)+f(t,u(t))=0$, $0<t<1$, subject to the boundary conditions $u(0)=u'(0)=0,.....,u^{(n-3)}(0)=u^{(n-2)}(0)=0$ and $u^{(n-2)}(1)-\alpha u^{(n-2)}(\eta)=\lambda$. In the setting, $0<\eta<1$ and $\alpha\in[0,\frac{1}{\eta})$ are constants and $\lambda\in[0,+\infty)$ is parameter. By placing certain restrictions on the nonlinear term f, we prof the existence and nonexistence of at least one positive solution to the boundary value problem with the use of the Krasnosel'skii fixed point theorem. The novelty in our setting lies in the fact that $f(t,u)$ may be singular at $t=0$ and $t=1$. We conclude with examples illustrating our results obtained in this paper.

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Riesz Lemma and Bernstein Polynomials deformations of some minimal surfaces

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Abstract: The talk focuses on the possibility to organize the taking of non-negative trigonometric roots of polynomials converging to a piecewise-analytic function to save convergence. The most interesting for the author is the case when the trigonometric polynomials are determined using Bernstein polynomials approximating a piecewise linear function. This issue is related to the construction of compactly supported wavelets that preserves the localization with increasing smoothness.

Summability of the Fourier coefficients of functions from anisotropic Lorentz space

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Abstract: Let $1 < p < \infty$ and $\Phi = \{\phi\}_{k=1}^{\infty}$ be the orthonormal system bounded in total, $f \in L_p[0,1], f \overset{\text{a.e.}}{=} \sum_{k=1}^{\infty} a_k \phi_k(x)$. In a case of $1 < p < 2$ the task about properties of summability of Fourier coefficients $\{a_k\}_{k=0}^{\infty}$ can be solved by classical Hardy-Littlewood-Polya inequality [1]

$$\sum_{k=1}^{\infty} k^{p-2} |a_k|^p \leq c \|f\|_{L_p}^p. \tag{6}$$

For the case when orthonormal system $\Phi = \{\phi\}_{k=1}^{\infty}$ is regular and $2 \leq p < \infty$, according to inequality, proved by E.D. Nursultanov ([2]), it follows

$$\sum_{k=1}^{\infty} k^{p-2} |\bar{a_k}|^p \leq c \|f\|_{L_p}^p, \tag{7}$$

here $\bar{a_k} = \frac{1}{k} \left| \sum_{m=1}^{k} a_m \right|$.

These inequalities are also valid in the multidimensional case (see e.g. [3]). For function from anisotropic spaces with a vector parameter $\mathbf{p} = (p_1, ..., p_n)$, it is necessary to consider a problem of summability of Fourier coefficients when some parameters p_i more than 2, and others less or equal to 2.This article is devoted to the problem.

Keywords: Anisotropic Lorentz space, Summability of the Fourier coefficients, Hardy-Littlewood-Polya inequality

2010 Mathematics Subject Classification: 42B08, 42C10

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Integral operators with two variable integration limits on the cone of monotone functions

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Abstract: The work is done together with A.A. Kalybay.

Let $I=(a,b)$, $-\infty \le a < b \le \infty$, and $1<p,q<\infty$. Let ω and v be non-negative and measurable functions almost everywhere finite on I such that $\omega^{-q'}$, ω^q, $v^{p'}$ and $v^{-p'}$ are locally summable on I.

Let $M\downarrow$ and $M\uparrow$ be sets of functions non-increasing and non-decreasing on I, respectively.

For the integral operators

$$K_-f(x)=\int\limits_{\alpha(x)}^{\beta(x)} K(s,x)f(s)ds \quad \text{and} \quad K_+f(x)=\int\limits_{\alpha(x)}^{\beta(x)} K(x,s)f(s)ds$$

we consider the inequalities

$$\|\omega K_-f\|_q \le C\|vf\|_p, \quad f\in M\downarrow, \tag{1}$$

$$\|\omega K_+f\|_q \le C\|vf\|_p, \quad f\in M\uparrow, \tag{2}$$

where $\|\cdot\|_p$ is the standard norm of the space $L_p(I)$. Moreover, the boundary functions α and β satisfy the following conditions:

(i) $\alpha(x)$ and $\beta(x)$ are functions differentiable and strictly increasing on I;

(ii) $\alpha(x)<\beta(x)$ for any $x\in I$; $\lim\limits_{x\to a^+}\alpha(x)=\lim\limits_{x\to a^+}\beta(x)=a$ and $\lim\limits_{x\to b^-}\alpha(x)=\lim\limits_{x\to b^-}\beta(x)=b$.

In the case when $K(s,x)\equiv 1$ the operator $K_-f(x)$ is called the Hardy-Steklov operator. Weighted inequalities of the form (1) and (2) for the Hardy-Steklov operators have been investigated with success [1]. The similar problem for operators with kernels has remained unsolved up to now. The aim of this work is to get necessary and sufficient conditions for the validity of inequalities (1) and (2) when the kernels $K(\cdot,\cdot)$ are from a large class.

2010 Mathematics Subject Classification: 26D10, 39B62

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Additive and multiplicative weighted estimates of intermediate integral operator

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Abstract: The work is done together with A.A. Kalybay.

We consider the inequalities

$$\|uK_\beta f\|_q \le C\left(\|\rho f\|_p + \|vKf\|_r\right), \quad f \ge 0, \tag{1}$$

$$\|uK_\beta f\|_q \le C\|\rho f\|_p^\alpha \|vKf\|_r^{1-\alpha}, \quad f \ge 0, \tag{2}$$

where $0 \le \alpha, \beta \le 1$ and $\|\cdot\|_p$ is the standard norm of the space $L_p(0,\infty)$. Moreover, K and K_β are integral operators of the form

$$Kf(x) = \int\limits_0^\infty K(x,s)f(s)ds, \quad K_\beta f(x) = \int\limits_0^\infty K^\beta(x,s)f(s)ds$$

with the kernel $K(\cdot,\cdot) \ge 0$ satisfying the condition

$$\exists D \ge 1 : D^{-1}\Big(K(x,t) + K(t,s)\Big) \le K(x,s) \le D\Big(K(x,t) + K(t,s)\Big)$$

for $x \ge t \ge s > 0$.

We give necessary and sufficient conditions for the validity of inequalities (1) and (2) for $1 \le \max\{p,r\} \le q \le \infty$ and $\frac{\alpha}{p} + \frac{1-\alpha}{r} \ge \frac{1}{q}$, respectively.

As an application, we obtain criteria for the inequalities

$$\|uf^{(k)}\|_q \le C\left(\|\rho f^{(n)}\|_p + \|vf\|_r\right)$$

and

$$\|uf^{(k)}\|_q \le C\|\rho f^{(n)}\|_p^\alpha \|vf\|_r^{1-\alpha}$$

to hold on the class of n-convex functions absolutely monotone on the interval $(0,\infty)$; in particular, when $(n = 2)$ on the class of convex and twice differentiable functions, where $0 \le k < n$.

Keywords: multiplicative inequality, additive inequality, integral operator, kernel

2010 Mathematics Subject Classification: 26D10, 39B62

Inverse coefficient problems for one-dimensional heat transfer with a preservation of medium temperature condition

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Abstract: We examine mathematical model models describing the process of heat transfer in a homogeneous bar with a prescribed law of changes of the average temperature. So there is an inverse problem for the heat equation in which together with finding the solution of the equation it is required to find unknown coefficient depending only on the time variable.

In the domain $\Omega = \{(x,t) : 0 < x < 1, 0 < t < T\}$ consider a problem on finding unknown coefficient $p(t)$ of the heat equation

$$u_t = u_{xx}(x,t) - p(t)u(x,t) + f(x,t)$$

and its solution satisfying the initial condition

$$u(x,0) = \varphi(x),\ 0 \leq x \leq 1,$$

the nonlocal boundary condition

$$u_x(0,t) = u_x(1,t) - \alpha u(1,t),\ \ u(0,t) = 0,\ \ 0 \leq t \leq T,$$

and the overdetermination conditions

$$\int_0^1 u(x,t)dx = E(t),\ E(t) \neq 0, 0 \leq t \leq T,$$

where $E(t) \in W_2^1(0,T)$. Here the parameter α is any positive number, and $f(x), \varphi(x)$ and $E(t)$ are given functions.

In [1] (for $\alpha < 0$) some of these conditions were removed and it is shown that the inverse problem has the unique generalized solution. This work is a continuation of these studies. We consider the case $\alpha > 0$.

Keywords: heat transfer, law of changes of the average temperature, inverse problem, heat equation

2010 Mathematics Subject Classification: 33C10, 34B30, 35J, 35P10

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Construction of mathematical models of extraction processes with non-local conditions by a spatial variable

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Abstract: In this paper we consider issues of constructing mathematical models of extraction processes from solid polydisperse porous materials considering the porosity of structure of particles, taking into account the connection of the residence time of fractions with particle size in the extractant, based on inverse problems of recovery of coefficients of diffusion processes under various variants of boundary conditions by a spatial variable.

We consider a mathematical model which models the extraction process of a target component from the polydispersed porous material. The suggested model is demonstrated by the example of a at solid material with bidispersed pores of different size in the form of a system of channels of macropores with micropores facing their walls. The macropores and the micropores in the material have homogeneous size. We model a case when micropores of the solid material (dispersed medium) are initially filled with an oil (dispersion phase), which is our target component. And the macropores are filled in with a pure solvent. In the process of extraction the oil diffuses from the micropore to the macropore, and then from the micropores to the external solvent volume, wherein the ratio of concentrations in the macropore and the micropore is taken in accordance with the linear law of adsorption. The well-posedness of the formulated mathematical model has been justified.

The theoretical mathematical science has deep enough advanced in solving inverse problems for diffusion processes. And besides, as a rule, the problems are researched under simplest selfadjoint boundary conditions by a spatial variable. Unlike the mentioned works we propose to consider the problems with more general boundary conditions by a spatial variable. The selfadjointness of the boundary conditions is not assumed, only requirement of their regularity by Birkhoff is sufficient. The inverse problems researched by us are directly obtained from mathematical models of technological processes.

Keywords: extraction processes, non-local conditions, inverse problems

2010 Mathematics Subject Classification: 35K15, 35P10, 35R30

Nonlocal estimates for solutions of a singular higher order differential equation

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Abstract: We consider the linear differential equation of even order

$$L_0 y := y^{(2n)} + a_1 y^{(2n-1)} + a_2 y^{(2n-2)} + ... + a_{2n} y = f(x), \tag{1}$$

where $x \in \mathbf{R} = (-\infty, +\infty)$, and $a_k = a_k(x)$ $(k = 1, 2, ..., 2n)$ and $f \in L_2 = L_2(\mathbf{R})$ are given functions.

In this paper we investigate the equation (1) and give some sufficient conditions for its unique and coercive solvability. Similar questions were studied in many papers (see for instance the monographs [1], [2] and [3]). However, their results were obtained under the following conditions: the intermediate coefficients $a_s(\, s = 1, 2, \ldots, 2n-1)$ of equation (1) are constant, or their growths are bounded above by some degrees of the function $|a_{2n}|$ at infinity. In the case $n = 1$, the equation (1) has been investigated in [4].

We give an estimate of norms of solution and its derivatives up to the order $2n$ of the equation (1), in the case a_s $(\, s = 1, 2, \ldots, 2n-1)$ are not bounded and a_{2n} can be equal to zero.

Keywords: even order differential equation, coercive solvability, intermediate coefficient

2010 Mathematics Subject Classification: 34B40

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Coercive solvability of degenerate system of second order difference equations

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Abstract: We consider the following infinite system of second order difference equations:

$$l_0 y = -\Delta^{(2)} y + R\Delta y = F, \tag{1}$$

where $y = \{y_j\}_{j=-\infty}^{+\infty}$, $\Delta y = \{\Delta y_j\}_{j=-\infty}^{+\infty} = \{y_{j+1} - y_j\}_{j=-\infty}^{+\infty}$,
$\Delta^{(2)} y = \{\Delta(\Delta y_j)\}_{j=-\infty}^{+\infty} = \{y_{j+1} - 2y_j + y_{j-1}\}_{j=-\infty}^{+\infty}$, $F = \{F_j\}_{j=-\infty}^{+\infty}$ and $R = (r_{i,j})_{i,j=-\infty}^{+\infty}$ is a real matrix.

Coercive solvability of the diagonal system

$$-\Delta^{(2)} y_j + \hat{R}\Delta y_j + Q y_j = F_j (\ j \in \mathbf{Z}),$$

where $\hat{R} = diag\{r_j, \ j \in \mathbf{Z}\}$, $Q = diag\{q_j, \ j \in \mathbf{Z}\}$ was discussed in [1]. In the case where oscillation of the sequence $\{r_j\}_{j=-\infty}^{j=+\infty}$ satisfies some conditions, we proved that this system is unique and coercive solvable.

In this paper, for difference system (1), we obtain some sufficient conditions of the solvability in the space of all square-summable real sequences. We also will establish the coercive estimate for its solution.

Keywords: difference system, solvability, coercive estimates of solutions

2010 Mathematics Subject Classification: 39A06

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Approximation in time of fractional equations

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Abstract: In this talk we continue our investigations [1] on discretization of differential equations of fractional order in time.

Recently, in [2] and [3] were considered the relation between well-posed Cauchy problems

$$v'(t) = A^l v(t) + g(t), t > 0, v(0) = u^0,$$

and

$$(\mathbf{D}_t^{1/l} u)(t) = Au(t), t > 0, u(0) = u^0.$$

Moreover, they have shown that for such kind problems with the operator A which generates analytic C_0-semigroup one has $v(t) \equiv u(t)$ for any $t > 0$ as soon as $l = 2$ and special choice of $g(t)$.

In this talk, we would like to use such kind of relations for discretization of differential equations of fractional order in abstract spaces.

Acknowledgement.

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Keywords: fractional differential equations, approximation

2010 Mathematics Subject Classification: 65N40, 35-99

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Sobolev type equations of time-fractional order with periodical boundary conditions

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Abstract: The main purpose of the paper is studying of the unique solvability for initial boundary value problem to the fractional differential equation

$$D_t^\alpha Lx(t) = Mx(t) + N(t, x(t), x^{(1)}(t), \ldots, x^{(m-1)}(t)), \tag{1}$$

where the operators L and M are differential operators with respect to the spatial variables, N is a nonlinear operator, $\alpha > 0$, $m \in \mathbb{N}$, $m - 1 < \alpha \leq m$, D_t^α is the Gerasimov–Caputo derivative. Initial conditions have the Cauchy form and boundary conditions are periodical with respect to every spatial variable on a parallelepiped. Such problems arise in many engineering and scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, aerodynamics, electrodynamics of complex medium, polymer rheology [1]. Using some results on fractional differential equations in Banach spaces from [2] the local unique solvability conditions are found for the initial boundary value problem to equation (1). General results are applied to the research of the time-fractional order Benjamin–Bona–Mahony–Burgers and Allair partial differential equations.

Equations of form (1) not solved with respect to the time derivative are called Sobolev type equations [3]. Some classes of Sobolev type fractional equations were studied in [4].

Keywords: Caputo fractional derivative, Sobolev type equation, nonlinear equation, initial boundary value problem, periodical boundary condition

2010 Mathematics Subject Classification: 35R11,34A08

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Pohozhaev identities and applications for quasilinear equations of mixed elliptic-hyperbolic type

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Dedicated to 70th birthday of Prof. Tynysbek Kalmenov.

Abstract: It is well known result of Pohozhaev (1965), that the homogeneous Dirichlet problem for semilinear elliptic equations in a bounded subset Ω of $\mathbb{R}^n$, with $n > 2$, permits only the trivial solution if the domain is star-shaped, the solution is sufficiently regular, and the power of nonlinearity $p > 2^*(n) := 2n/(n-2)$, where the latter quantity is the critical exponent in the Sobolev embedding of $H_0^1(\Omega)$ into $L^p(\Omega)$ for $p < 2^*(n)$. To the opposite of this fact, in the case $2 < p < 2^*(n)$ there exist nontrivial solutions. In the last 50 years the Pohozhaev identities and results have been used and extended for a large class of elliptic problems. Let us mention now that in [1], [2] it has been shown that the nonexistence principle in supercritical case also holds for certain two dimensional problems for the mixed elliptic-hyperbolic Gellersted operator L (instead of Δ), with some appropriate boundary conditions. It is also valid for a large class of such problems even in higher dimensions [3]. In dimension 2, such operators have a long-standing connection with transonic fluid flow. Of course, the critical Sobolev embedding in this case is for a suitable weighted version of $H_0^1(\Omega)$ into $L^p(\Omega)$. As usual, in the BVP for such mixed elliptic-hyperbolic Gellersted operator L, the boundary data are given only on the proper subset of the boundary $\partial\Omega$. To compensate the lack of a boundary condition on a part of boundary, a sharp Hardy-Sobolev inequality is used, as was first done in [1], [2] and later in [3], [4]. Some further results, already published or in progress, prepared jointly with colleagues from Italy and Norway will be also discussed.

Keywords: quasilinear equation, equation of mixed elliptic-hyperbolic type, Pohozhaev identities.

2010 Mathematics Subject Classification: 35B33, 35L80, 35M10

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To the theory of linear ordinary differential equations of fractional order with constant coefficients

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Abstract: Consider the equation

$$\sum_{k=1}^{m} \lambda_k \frac{d^{\sigma_k}}{dx^{\sigma_k}} u(x) = f(x),$$

where $d^{\sigma_k}/dx^{\sigma_k}$ is a fractional derivative of order σ_k ($\sigma_k \geq 0$), $\lambda_k \in \mathbb{R}$. The fractional differentiation is given by the Dzhrbashyan-Nersesyan operator [1], associated with the sequence $\{\gamma_0^k, \gamma_1^k, \cdots, \gamma_{n(k)}^k\}$.

In the work we discuss the questions concerning the construction of the fundamental solution, the representation of the general solution and the correct form of initial conditions for the equation under consideration.

Keywords: fractional derivative, Dzhrbashyan-Nersesyan operator, fractional order differential equation

2010 Mathematics Subject Classification: 34A08

References:

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Some properties of linear operators in locally convex cones

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Abstract: A cone is defined to be a commutative monoid $\mathcal{P}$ together with a scalar multiplication by nonnegative real numbers satisfying the same axioms as for vector spaces. A locally convex cone is a generalization of topological vector space. The theory of locally convex cones uses an order theoretical concept or a convex quasiuniform structure to introduce a topological structure on a cone (see [2] and [3]). We verify some proprties of linear operators between two locally convex cones [1].

Keywords: locally convex cones, quasiuniform structures

2010 Mathematics Subject Classification: 46A03

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Some problems with nonlocal conditions for hyperbolic equations

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Abstract: In this report we present some new results concerning with nonlocal problems for hyperbolic equations. Namely, we consider a problem with nonlocal integral condition and present some methods to study it. It is well known that classical methods most widely used to prove solvability of initial-boundary problems break down as a rule when applied to nonlocal problems. In this talk, we focus on spatial nonlocal integral condition of the second kind and consider a problem: find a solution of

$$\mathcal{L}u \equiv u_{tt} - (a(x,t)u_x)_x + c(x,t)u = f(x,t) \tag{2}$$

in $Q_T = (0,l) \times (0,T)$, $\ l, T < \infty$, satisfying initial data

$$u(x,0) = 0, \quad u_t(x,0) = 0, \tag{3}$$

boundary condition

$$u_x(0,t) = 0, \tag{4}$$

and nonlocal condition

$$\alpha(t)u(l,t) + \int_0^l K(x)u(x,t)dx = 0. \tag{5}$$

Two methods to prove solvability of the problem will be presented in the talk. The choice of a method depends on properties of the function $\alpha(t)$ in (5). We propose a new approach which anables to prove a unique solvability of the nonlocal problem with degenerate integral condition and modification of certain known one for the case $\alpha(t) = 1$.

Keywords: nonlocal problem, hyperbolic equation, integral conditions dynamical boundary conditions, degenerate integral condition

2010 Mathematics Subject Classification: 35L10, 35L20, 35L99

References:

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To theory one class of integral equation by tube domain

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Abstract: Let Ω denote the tube $\Omega = \{(t, z) : a < t < b, |z| < R\}$. Lower ground this cylinder denote by $D = \{t = a,\ |z| < R\}$ and lateral surface denote by $= \{a < t < b,\ |z| = R\}$, $z = x + iy$. In Ω we shall consider the following integral equation

$$\varphi(t,z) + \lambda \int_a^x \frac{\varphi(\tau,z)}{\tau - a} d\tau - \frac{\mu}{\pi} \iint_D \frac{exp\,[i\theta]\,\varphi(t,s)}{(R-\rho)^\beta\,(s-z)} ds -$$

$$\frac{\delta}{\pi} \int_a^x \frac{d\tau}{\tau - a} \iint_D \frac{exp\,[i\theta]\,\varphi(\tau,s)}{(R-\rho)^\beta\,(s-z)} ds = f(t,z), \tag{1}$$

where λ, μ, δ – are given real constants, $f(t,z)$ – are given function, $\theta = \arg s$, $s = \xi + i\eta$, $ds = d\xi d\eta$, $\varphi(t,z)$ – unknoun function, $\rho^2 = \xi^2 + \eta^2$.

The solution to this equation is sought in the class of function $\varphi(t,z) \in C\left(\overline{\Omega}\right)$, $\varphi(a,z) = 0$, $\varphi(t, Re^{i\theta}) = 0$, $\theta = \arg z$ and its asymptotic behavior at $t \to a$, $r \to R$ is given by formulas

$\varphi(t,z) = 0\left[(t-a)^\varepsilon\right]$, $\varepsilon > 0$ at $t \to a$, $\varphi(t,z) = 0[(R-r)^{\delta_1}]$, $\delta_1 > \beta - 1$, at $t > a$.

For integral equation (1) found condition to parameters, present in kernels, at fulfilment which, the problem found solution this type integral equation reduce, to problem found two splitting system integral equation, theory which detailed investigation in author works. In this case solution integral equation (1) found in explicit form. In those cases, when general solution contain arbitrary function, found inverse formula, that is arbitrary functions found by valued solution integral equation in surface cylinder. In the case, when constants present in kernels no among themselves, the solution integral equation(1) found in form absolutely and uniformly converges generalized power series by power $(t-a)$, by infinite numbers analytic functions variable point ground this cylinder. Found invers formula, that is arbitrary analytic functions founded by valued solution integral equation and its derivative at $t = 0$. Obtained integral representation and its inverse formula use for stand and investigate Dirichlet type boundary value problems.

Keywords: integral equation, tube domain, two splitting system integral equation

2010 Mathematics Subject Classification: 45E05, 45B05

About solutions properties of Tricomi problem for Gellerstedt equation

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Abstract: Tricomi problem research has a fundamental importance for all theory of mixed type equations and various areas of application. This problem for Tricomi equation $y\,u_{xx} + u_{yy} = f(x,y)$ was originally suggested in [1]. Later more common Gellerstedt equation $(sgny)\,y^m\,u_{xx} + u_{yy} = f(x,y)$ was suggested in [2].

We consider the Tricomi problem for the Gellerstedt equation for the case in which the elliptic part of the boundary is the curve

$$\sigma_\delta = \left\{(x,y):\,\left(x-\frac{1}{2}\right)^2 + \left(\frac{2}{m+2}y^{\frac{m+2}{2}} + \delta\right)^2 = \frac{1}{4} + \delta^2,\, -\infty < \delta < +\infty\right\}.$$

Smooth solutions for this domain have been got in [3]. The notion of an n-regular solution for model Lavrent'ev-Bitsadze equation has been introduced in [4]. In present work we prove a criterion for the existence of n-regular solution of Tricomi problem for Gellerstedt equation.

Keywords: Tricomi problem, Gellerstedt equation, non continuous solutions

2010 Mathematics Subject Classification: 35M10, 35M12

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On I_λ-statistical convergence of weight g

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Abstract: In this paper, following a very recent and new approach of [1], we further generalize the recently introduced summability method of [2] and introduce the new notion namely, $I_\lambda-$statistical convergence of weight g,where $g:\mathbb{N}\to[0,\infty)$ is a function satisfying $g(n)\to\infty$ and $n/g(n)\nrightarrow 0$. We mainly investigate certain properties of this convergence.

Keywords: Ideal, filter, $\mathcal{I}$-statistical convergence of order g, $\mathcal{I}_\lambda-$statistical convergence

2010 Mathematics Subject Classification: 40A05, 40D25

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Asymptotically $\mathcal{I}_\lambda$–Statistical Equivalent Sequences of weight g

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Abstract: This paper presents the following definition which is a natural combination of the definition for asymptotically equivalent of weight g, $\mathcal{I}$ -statistically limit, and λ- statistical convergence, where $g : \mathbb{N} \to [0, \infty)$ is a function satisfying $g(n) \to \infty$ and $n/g(n) \nrightarrow 0$. The two nonnegative sequences $x = (x_k)$ and $y = (y_k)$ are said to be asymptotically $\mathcal{I}^g$- statistical equivalent of weight g to multiple L provided that for every $\epsilon > 0$, and $\delta > 0$,

$$\{n \in \mathbb{N} : \frac{1}{g(\lambda_n)}|\{k \in I_n : |\frac{x_k}{y_k} - L| \geq \epsilon\}| \geq \delta\} \in \mathcal{I},$$

(denoted by $x \overset{S_\lambda^L(I)^g}{\sim} y$) and simply asymptotically $\mathcal{I}^g$- statistical equivalent of weight g if $L = 1$. In addition, we shall also present some inclusion theorems.

Keywords: Asymptotical equivalent, ideal convergence, $\mathcal{I}$-statistical convergence, λ- statistical convergence, statistical convergence of weight g

2010 Mathematics Subject Classification: 40A05, 40D25

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Weak solutions for nonlinear fractional differential equations with fractional separated boundary conditions in Banach spaces

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In this paper, we consider the following boundary value problem of fractional differential equation with fractional separated Boundary Conditions of the form (1)

$$\begin{cases} {}^cD^r x(t) = f(x, x(t)), \quad t \in J = [0,\,1],\ 1 < r \le 2. \\ \\ \alpha_1 x(0) + \beta_1({}^cD^p x(0)) = \gamma_1, \quad \alpha_2 x(1) + \beta_2({}^cD^q x(1)) = \gamma_2,\ 0 < p < 1 \end{cases}$$

where ${}^cD^r$ is the Caputo fractional derivative of order r, $f : J \times E \to E$ is a given function satisfying some assumptions that will be specified later, α_i, β_i, γ_i $(i = 1,\,2)$ are suitably chosen constants in E, with $\alpha_1 \neq 0$. and E is a Banach space with norm $\|.\|$.

The topic of fractional differential equations has been of great interest for many researchers in view of its theoretical development and widespread applications in various fields of science and engineering such as physics, biophysics, chemistry, statistics,economics, blood flow, phenomena, control theory, porous media, electromagnetic, and other fields. Boundary value problems with integral boundary conditions constitute an important class of problems and arise in the mathematical modeling of various phenomena such as heat conduction, wave propagation, gravitation, chemical engineering, underground water flow, thermoelasticity, and plasma physics. They include two-point, three-point, multipoint boundary value problems.

Our investigation relies upon the method associated with the technique of measures of noncompactness and the fixed point theorem of Mönch type.

Keywords and phrases: Caputo fractional derivative, Riemann Liouville integral, measure of noncompactness, fixed point, Banach space.

2010 Mathematics Subject Classification: 26A33

Hardy-type inequality for a fractional integral operator in q-analysis

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Abstract: We consider the operator $I_{q,n}$ the following form

$$I_{q,n}f(x) = \frac{1}{\Gamma_q(n)} \int\limits_0^{\infty} \mathcal{X}_{(0,x]}(s) K_{n-1}(x,s) f(s) d_q s,$$

which is defined for all $x > 0$ [1]. where $K_{n-1}(x,s) = (x - qs)_q^{n-1}$.

Then the q-analog of the two-weighted inequality for the operator $I_{q,n}$ of the form

$$\left(\int\limits_0^{\infty} u^r(x) \left(I_{q,n}f(x)\right)^r d_q x \right)^{\frac{1}{r}} \leq C \left(\int\limits_0^{\infty} v^p(x) f^p(x) d_q x \right)^{\frac{1}{p}}. \tag{1}$$

which has several applications in various fields of science. Where C a positive constants independent of f and $u(\cdot), v(\cdot)$ are positive real valued functions on $(0,\infty)$, i.e. weight functions.

Theorem. Let $1 < r < p < \infty$. Then the inequality (1) holds if and only if $Q_{n-1} < \infty$ holds, where

$$\begin{aligned} Q_m^{n-1} &= \left\{ \int\limits_0^{\infty} \left(\int\limits_0^{\infty} \mathcal{X}_{(0,z]}(s) K_m^{p'}(z,s) v^{-p'}(s) d_q s \right)^{\frac{p(r-1)}{p-r}} \right. \\ &\times \left(\int\limits_0^{\infty} \mathcal{X}_{[z,\infty)}(x) K_{n-m-1}^r(x,z) u^r(x) d_q x \right)^{\frac{p}{p-r}} \\ &\times \left. D_q \left(\int\limits_0^{\infty} \mathcal{X}_{(0,z]}(s) K_m^{p'}(z,s) v^{-p'}(s) d_q s \right) \right\}^{\frac{p-r}{pr}}. \end{aligned}$$

Moreover, $Q_{n-1} \approx C$, C is the best constant in (1).

Keywords: inequalities; Hardy-type inequalities, integral operator; q-calculus; q-integral

2010 Mathematics Subject Classification: 26D10, 26D15, 39A13

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On the applicability of Picard's successive approximations to the numerical solving the problem of heat transfer in domain, degenerate at the initial time

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Abstract: The following problem for the heat equation is considered:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \alpha_0 t, \quad 0 < t < T, \tag{1}$$

$$u(0,t) = \varphi(t), \quad (2) \qquad u(\alpha(t),t) = \psi(t), \quad (3) \qquad \varphi(0) = \psi(0) = 0, \quad (4)$$

where known boundary functions $\varphi(t)$ and $\psi(t)$ are Hölder continuous:

$$|\varphi(t') - \varphi(t'')| < A|t' - t''|^\sigma, \quad |\psi(t') - \psi(t'')| < A|t' - t''|^\sigma, \quad \sigma > \frac{1}{2}. \tag{5}$$

We represent the solution of problem (1)–(5) as $u(x,t) = u_1(x,t) + W(x,t)$, where $u_1(x,t)$ and $W(x,t)$ – are double layer potentials with density $\varphi(t)$ and unknown density $\theta(t)$, respectively. Function $u(x,t)$ satisfies equation (1) and condition (2), and substituting it into equation (3) we obtain the following integral equation:

$$\theta(t) = g(t) + \int_0^t K(t,\tau)\theta(\tau)d\tau, \tag{6}$$

where $W(\alpha_0 t, t) = \int_0^t K(t,\tau)\theta(\tau)d\tau$ is the valeur principale of the potential $W(x,t)$ at $x = \alpha_0 t$, and $g(t) = u_1(\alpha_0 t, t) - \psi(t)$. It was shown in [1] that the equation (6), in general, may not have a solution when $g(t)$ is just the bounded continuous functions. Moreover, the integral operator (6) can have eigenfunction, [3]. Properties of equation (6) was studied Properties of equation (6) were studied in papers [2], [4], [5], etc.

We establish in this work the additional conditions on the boundary functions in (2), (3) for which the solution of equation (6) exist, is unique, and can be found by the Picard's method of successive approximations. For this we define the functional class $M_\beta(0,T)$which contains continuous functions $f(t)$ having estimation $|f(t)| \le M_f t^\varepsilon$, where $\varepsilon > \beta$. The following results are proved:

Theorem 1. If the function $g(t) \in M_{\frac{1}{2}}(0,T)$, then the Picard's iterative process converges uniformly to the unique solution of integral equation and $\theta(t) \in M_0(0,T)$.

Theorem 2. The function $g(t) \in M_{\frac{1}{2}}(0,T)$ under the conditions (4),(5).

Keywords: Heat equation in degenerating domain, Volterra Singular integral operator, Picard's method

2010 Mathematics Subject Classification: 35K05, 35K20, 45D05

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The Hardy-Littlewood type theorems in Besov spaces with the Haar basis

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Abstract: In this paper there were established the coefficient necessary and sufficient conditions that the function belongs to the Besov spaces with the Haar basis.

Let $H = \{h_m(t)\}_{m=1}^{+\infty}$, $t \in [0,1]$ be the orthonormal Haar system [1].

We denote by $\|f\|_p$ the norm of function from $L_p[0,1]$, $1 \le p < +\infty$.

Let $f \in L_p[0,1]$, $1 \le p < +\infty$. Let $E_n(f)_{p\theta}$ be the best approximation of function f in the metrics of Lebesgue space $L_p[0,1]$ by the Haar polynomials of order not higher than n.

Definition 1. *Let* $1 \le p \le +\infty$, $1 \le \theta \le +\infty$, $r > 0$. *We say that function* $f \in L_p[0,1]$ *belongs to the Besov space with the Haar basis, if*

$$|f; B^r_{p\theta}([0,1];H)| = \|f\|_p + \left\{ \sum_{k=0}^{+\infty} 2^{kr\theta} E^{\theta}_{2^k}(f)_p \right\}^{\frac{1}{\theta}}, \quad 1 \le \theta < +\infty;$$

$$|f; B^r_{p\infty}([0,1];H)| = \|f\|_p + \sup\left\{ 2^{kr} E_{2^k}(f)_p \mid k \in \mathbb{Z}^+ \right\}.$$

The following two theorems contain GM-monotone condition on [2] the Fourier-Haar coefficients of function $f \in B^r_{p\theta}([0,1],H)$.

Theorem 1. Let $f \in L[0,1]$. Let $f(x) \sim \sum\limits_{k=1}^{+\infty} a_k h_k(x)$ be its the Fourier-Haar series. If the positive sequence of the Fourier-Haar coefficients $\{a_k\} \in GM$, then for $f \in B^r_{s\theta}([0,1];H)$, where $r > 0$, $1 < s < +\infty$, $1 \le \theta \le +\infty$, it is sufficient that for some p: $1 \le p < s$

$$\left\{ \sum_{k=1}^{+\infty} k^{r\theta+\theta\left(\frac{1}{2}+\frac{1}{p}-\frac{1}{s}\right)-1} a_k^{\theta} \right\}^{\frac{1}{\theta}} < +\infty. \tag{2}$$

Moreover, the following inequality

$$|f; B^r_{s\theta}([0,1];H)| \le A_{psr\theta} \left\{ \sum_{k=1}^{+\infty} k^{r\theta+\theta\left(\frac{1}{2}+\frac{1}{p}-\frac{1}{s}\right)-1} a_k^{\theta} \right\}^{\frac{1}{\theta}}.$$

holds. Here $A_{psr\theta} > 0$ depends only on p, s and r.

Theorem 2. Let $r > 0$, $1 < s < \lambda < +\infty$, $1 \le \theta \le +\infty$. Let $f \in B^r_{s\theta}([0,1]; H)$. If its Fourier-Haar coefficients are positive and GM-monotone, then

$$|f; B^r_{s\theta}([0,1]; H)| \ge D_{\lambda sr\theta} \left\{ \sum_{k=1}^{+\infty} k^{r\theta+\theta\left(\frac{1}{2}+\frac{1}{\lambda}-\frac{1}{s}\right)-1} a_k^\theta \right\}^{\frac{1}{\theta}}.$$

Here $D_{\lambda sr\theta} > 0$ depends only on λ, s, r and θ.

Keywords: Fourier series, Haar system, Fourier-Haar coefficients, GM-monotone sequences, Hardy-Littlewood theorem, Besov space

2010 Mathematics Subject Classification: 42A16, 46E30

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Integrals with homogeneous - difference kernels on the plane and their applications

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Abstract: Two-dimensional singular integrals and one- dimensional generalized integrals of Cauchy type are considered. The applications to the Riemann -Hilbert problem for a general first order elliptic system on the plane are given. More exactly it is shown how to reduce the problem to a system of Fredholm integral equation on the domain and a singular integral equation on its boundary with the help of homogeneous difference integrals.

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Confluent hyper geometric function sand two variables Laguerre polynomials as a solutions of Wilczynski type system

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Abstract: In this work it is shown that in the study of systems, solutions of which are Whittaker functions and two variables Laguerre polynomials, the main role is played by Humbert confluent hypergeometric function $\Psi_2(\lambda, \gamma, \gamma'; x, y)$.

It is defined from Appell functions $F_2(\alpha, \beta, \beta', \gamma, \gamma'; x, y)$ using the passage to the limit [1, p.121], and it is the solution of the following system

$$(1) \qquad \begin{cases} x \cdot Z_{xx} + (\gamma - x) \cdot Z_x - y \cdot Z_y - \lambda \cdot Z = 0, \\ y \cdot Z_{yy} + (\gamma' - y) \cdot Z_y - x \cdot Z_x - \lambda \cdot Z = 0, \end{cases}$$

where $\gamma \neq 0, -1, -2, ...$; $\gamma' \neq 0, -1, -2, ...$; $Z = Z(x, y)$ – general unknown.

The first particular solution of Wilczynski type system (1) is confluent hypergeometric function $Z_1(x, y) = \Psi_2(\lambda; \gamma, \gamma'; x, y)$. In [2] it is shown that Humbert confluent hypergeometric function $\Psi_2(\lambda; \gamma, \gamma'; x, y)$ is the main tool of investigating the Whittaker type systems. Here, it is introduced that it plays the same role at investigation of Laguerre type systems and simplify its construction of solution in the form of two variables Laguerre polynomials.

With this purpose, from confluent hypergeometric system (Ψ_2) at $\gamma = \alpha + 1$ $(\alpha > -1)$, $\gamma' = \beta + 1$ $(\beta > -1)$, it is defined the following system

$$(2) \qquad \begin{cases} x \cdot Z_{xx} + (\alpha + 1 - x) \cdot Z_x - y \cdot Z_y - \lambda \cdot Z = 0, \\ y \cdot Z_{yy} + (\beta + 1 - y) \cdot Z_y - x \cdot Z_x - \lambda \cdot Z = 0. \end{cases}$$

Considering the first particular solution $Z_1(x, y)$, it is easy to be assured that the solution in the form of polynomial exists in the case when $\lambda = -n$ (n – integer), is polynomial

$$Z_1(x, y) = \Psi_2(-n; \alpha + 1, \beta + 1; x, y).$$

Theorem. Let $\alpha > -1$, $\beta > -1$, $\alpha \neq 0$, $\beta \neq 0$. The system of second order partial differential equations (2), where λ – parametric value, has as a solution the two variables polynomial, not equal to zero identically, if and only if λ is in the form of $\lambda = -n$ (n – integer).

Let us call this polynomial the generalized two variables Laguerre polynomial and denote it by

$$L_{n,n}^{(\alpha,\beta)}(x, y) = \Psi_2(-n; \alpha + 1, \beta + 1; x, y).$$

It is also established that the system of the following type

$$\begin{cases} x^2 \cdot U_{xx} - xy \cdot U_y + \left(-\frac{x^2}{4} - \frac{xy}{2} + kx + \frac{1}{4} - \alpha^2\right) \cdot U = 0, \\ y^2 \cdot U_{yy} - xy \cdot U_x + \left(-\frac{y^2}{4} - \frac{xy}{2} + ky + \frac{1}{4} - \beta^2\right) \cdot U = 0, \end{cases}$$

where $k = (\alpha + \beta + 2 - 2\lambda)/2$ is also the Laguerre type system with the solution of the following form

$$U(x, y) = x^{\frac{\alpha+1}{2}} \cdot y^{\frac{\beta+1}{2}} \cdot \exp\left(-\frac{x}{2} - \frac{y}{2}\right) \cdot \sum_{n,m=0}^{\infty} \frac{(\frac{\alpha+\beta}{2} + \lambda)_{m+n}}{(2\alpha + 1)_m \cdot (2\beta + 1)_n} \cdot \frac{x^m}{m!} \cdot \frac{y^n}{n!}.$$

This aspect is still remaining little-investigated. A variety of theorems is proved.

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On one algorithm to find a solution to a linear two-point boundary value problem

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Abstract: A two-parameter family of algorithms for finding an approximate solution to a linear two-point boundary value problem for a system of ordinary differential equations is offered. The convergence conditions for the algorithms are obtained. The necessary and sufficient coefficient conditions for the well-posedness of considered problem are established.

Consider a linear two-point boundary value problem

$$\frac{dx}{dt} = A(t)x + f(t), \qquad x \in \mathbb{R}^n, \qquad t \in (0,T), \tag{3}$$

$$Bx(0) + Cx(T) = d, \tag{4}$$

where $A(t)$ and $f(t)$ are continuous on $[0,T]$, B and C are the given $(n \times n)$ matrices, d is a given n vector, $\|x\| = \max\limits_{i=1:n} |x_i|$, and $\|A(t)\| = \max\limits_{i=1:n} \sum\limits_{j=1}^{n} |a_{ij}(t)| \leqslant \alpha$, $\alpha = \text{const}$.

Present article offers an other family of algorithms of parameterizations method [1]. Herein each algorithm's step also consists of two items. The difference of offering algorithms is in item b), where the unknown functions $u_r(t)$, $t \in [(r-1)h, rh)$, $r = 1 : N$, are to be computed by the recurrent formula.

Keywords: two-point boundary value problem, linear ordinary differential equation, algorithm to find a solution, necessary and sufficient conditions for unique solvability

2010 Mathematics Subject Classification: 34B16, 34B40

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On the convergence of some iterative algorithms to approximate fixed points of nonexpansive mappings

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Abstract: In this paper we present a new iterative algorithm to approximate fixed points of nonexpansive and pseudocontractive mappings. We establish convergence results. We also discuss numerical examples to illustrate theoretical results. In addition, we compare the convergence of these iterations with existing iterations using the numerical computation.

Oscillation of second order half-linear difference equation and discrete Hardy inequality

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This is joint work with R. Oinarov, A.A. Kalybay and S. Kh. Shalginbayeva

Abstract: We consider a second order half-linear difference equation of the following form

(1) $$\Delta(\rho_i|\Delta y_i|^{p-2}\Delta y_i) + v_{i+1}|y_{i+1}|^{p-2}y_{i+1} = 0, \quad i = 0, 1, 2, ...,$$

where $1 < p < \infty$, $\Delta y_i = y_{i+1} - y_i$, $\{\rho_i\}$ - a sequence of positive real numbers and $\{v_i\}$ - a sequence of non-negative real numbers.

In this work for researching oscillation properties of equation (1) we use the variational method, which is reduced to the validity or not validity of the discrete Hardy inequality on the set of finitary sequences. On the basis of the criteria of validity of the discrete Hardy inequality in a local term we establish sufficient, necessary conditions of conjugate, disconjugate on the interval $[m, n]$, $0 \leq m < n \leq \infty$ of equation (1) and also oscillation, nonoscillation of this equation.

From these results as a corollary we obtained the different interesting conditions of oscillation of the equation (1). In particular, we obtained, that if $\{t_k\} \subset \mathbb{N}$, $\mathbb{N} - \{t_k\} \neq \oslash$, $t_k \to \infty$, as $k \to \infty$, $v_{t_k} \neq 0$, $v_i = 0$, $i \in \mathbb{N} - \{t_k\}$ and the following condition

$$v_{t_k} > \rho_{t_k - 1} + \left(\sum_{i=t_k+1}^{\infty} \rho_i^{1-p'}\right)^{1-p},$$

holds for all k, then the equation (1) is oscillatory.

Keywords: discrete Hardy inequalities, oscillatory, nonoscillatory, half-linear equations, disconjugate, conjugate, variational principle

2010 Mathematics Subject Classification: 39A10, 39A21, 34C10, 26D15.

Periodic boundary value problem for a system of ordinary differential equations with impulse effects

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Abstract: In this work, we investigated a nonlinear periodic boundary value problem with impulse effects. We have found conditions of existence isolated solution of periodic boundary value problem for system of nonlinear differential equations with impulse effects.

We consider the following periodic boundary value problem for a system of nonlinear ordinary differential equations with impulse effects on $[0, T]$:

$$dx/dt = f(t, x), \quad t \in [0, T] \setminus \{\theta_1, \theta_2, \ldots, \theta_m\}, \quad x \in R^n,$$

$$0 = \theta_0 < \theta_1 < ... < \theta_{m+1} = T, \tag{1}$$

$$x(0) = x(T), \tag{2}$$

$$x(\theta_i + 0) - x(\theta_i - 0) = J_i\left(\lim_{t \to \theta_i - 0} x(t)\right), \qquad i = \overline{1, m}, \tag{3}$$

where $f : [0, T] \times R^n \to R^n$, is a piecewise-continuous vector valued function with points of discontinuity of the first kind at $t = \theta_i \, (i = \overline{1, m})$ and $J_i(x) \; (i = \overline{1, m})$ are continuous vector valued functions of x. Many authors discuss solvability of problem (1) - (3) [1–5]. Sufficient conditions for the existence of isolated solutions periodic boundary value problem of impulsive.

Keywords: Impules effects, boundray value, differential equation

2010 Mathematics Subject Classification: 34B05, 34B37

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About sedately growing solutions of one generalized Cauchy-Riemann system

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Abstract: In this paper the existence of only zeroth sedately growing solution of the generalized Cauchy-Riemann system is proved in the case when the coefficients of the generalized Cauchy-Riemann system do not belong to the class $L_{p,2}(E)$, $p > 2$, namely $A(z) = az$, $B(z) = b\overline{z}$, where a, b are constants.

Consider the following problem: it is required to find regular solutions of the system

(1) $$\partial w/\partial \bar{z} + azw + b\bar{z}\bar{w} = 0$$

(where a, b are complex numbers), that in the neighborhood of the point of infinity satisfy the following condition

(2) $$|w| < K|z|^N,$$

where N is an arbitrary nonnegative number and K is an arbitrary real number.

The problem, when coefficients of the system (1) from class $L_{p,2}(E)$, $p > 2$, (function $f(z)$ at point $z = x + iy$ belongs to the class $L_{p,2}(E)$, if

$$L_{p,2}(E) = \|f\|_{p,2} = \Big\{ \iint\limits_{|z|\leq 1} |f(z)|^p dx\ dy \Big\}^{\frac{1}{p}} + \Big\{ \iint\limits_{|z|\leq 1} \Big| z^{-2} f(\frac{1}{z}) \Big|^p dx\ dy \Big\}^{\frac{1}{p}}$$

have been studied by I.N.Vekua [1], and when coefficients are constants - by V.S.Vinogradov [2]. In our case, coefficients do not belong to the class $L_{p,2}(E)$, $p > 2$. We will try to find the solution of equation (1) in the form of representation [3]: $f(z) = F(z)e^{if(z)} + G(z)e^{ig(z)}$, where $F(z)$, $G(z)$ are arbitrary analytic functions, and $f(z)$, $g(z)$ are continuous functions.

Theorem 2. *If in a system* (1) *the coefficient a is a real number, then the problem* (1) , (2) *has only trivial solution.*

Keywords: Generalized Cauchy-Riemann system, sedately growing solutions

2010 Mathematics Subject Classification: 30E25

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On a class of fractional elliptic problems with an involution perturbation

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Abstract: To describe the problems, let $\Omega = \{(x,y) \in R^2 : 0 < x < 1, -\pi < y < \pi\}$. The paper is concerned with four boundary value problems concerning the fractional analogue of Helmholtz equation with a perturbation term of involution type in the space variable:

(1) $$D_x^{2\alpha}u(x,y) + u_{yy}(x,y) - \varepsilon u_{yy}\left(x,-y\right) - c^2u(x,y) = 0, (x,y) \in \Omega,$$

where c, ε are real numbers, $D_x^{2\alpha}$ means $D_x^{2\alpha} = D_x^\alpha D_x^\alpha$ and the operator D_x^α acts by the variable x.
Here

$$D_x^\alpha u\left(x,y\right) = \frac{1}{\Gamma\left(1-\alpha\right)} \int\limits_0^x \left(x-s\right)^{-\alpha} \frac{\partial u}{\partial s}\left(s,y\right)ds$$

is a Caputo differentiation operator of the $\alpha \in (0,1]$ order [1].

Regular solution of Equation (1) is a function $u \in C(\bar{\Omega})$, such that $D_x^\alpha u, D_x^{2\alpha} u, u_{yy} \in C(\Omega)$. Since for $\alpha = 1 : {}_C D^1 = \frac{d}{dy}$, then $D_x^2 + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \Delta$. Therefore, Equation (1) is a nonlocal generalization of the Helmholtz equation, which at $\varepsilon = 0$ coincides with the Helmholtz equation.

We obtain for them existence and uniqueness results based on the Fourier method.

Keywords: Caputo operator, Helmholtz equation, involution, fractional differential equation, Mittag-Leffer function, boundary value problem

2010 Mathematics Subject Classification: 34A08, 35R11, 74S25

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Four-dimensional generalized difference matrix and some double sequence spaces

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Abstract: In this study, we introduce some new double sequence spaces $B(\mathcal{M}_u)$, $B(\mathcal{C}_p)$, $B(\mathcal{C}_{bp})$, $B(\mathcal{C}_r)$ and $B(\mathcal{L}_q)$ as the domain of four-dimensional generalized difference matrix $B(r,s,t,u)$ in the spaces $\mathcal{M}_u$, $\mathcal{C}_p$, $\mathcal{C}_{bp}$, $\mathcal{C}_r$ and $\mathcal{L}_q$, respectively. We show that the double sequence spaces $B(\mathcal{M}_u)$, $B(\mathcal{C}_{bp})$ and $B(\mathcal{C}_r)$ are the Banach spaces under some certain conditions. We give some inclusion relations with some topological properties. Moreover, we determine the $\alpha-$dual of the spaces $B(M_u)$ and $B(\mathcal{C}_{bp})$, $\beta(\vartheta)-$duals of the spaces $B(M_u)$, $B(\mathcal{C}_p)$, $B(\mathcal{C}_{bp})$, $B(\mathcal{C}_r)$ and $B(\mathcal{L}_q)$, where $\vartheta \in \{p, bp, r\}$ and $\gamma-$dual of the spaces $B(\mathcal{M}_u)$, $B(\mathcal{C}_{bp})$ and $B(\mathcal{L}_q)$. Finally, we characterize the classes of four dimensional matrix mappings defined on the spaces $B(\mathcal{M}_u)$, $B(\mathcal{C}_p)$, $B(\mathcal{C}_{bp})$, $B(\mathcal{C}_r)$ and $B(\mathcal{L}_q)$ of double sequences.

Keywords: four-dimensional generalized difference matrix; matrix domain; double sequence spaces; alpha-dual; beta-dual; gamma-dual; matrix transformations.

2010 Mathematics Subject Classification: 46A45, 40C05.

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Four-dimensional generalized difference matrix and almost convergent double sequence spaces

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Abstract: The concept of almost convergence for single sequence was introduced by Lorentz [1] and have studied by several mathematicians (see [2–4]), and for double sequences was introduced by Moricz and Rhoades [5].

In this study, the domains $B(\mathcal{C}_f)$ and $B(\mathcal{C}_{f_0})$ of four-dimensional generalized difference matrix in the spaces of almost convergent and almost null double sequences are introduced and some inclusion relations are given with some topological properties. The $\alpha-, \beta(\vartheta)-$ and $\gamma-$ duals of those spaces are calculated, where $\vartheta = \{p, bp, r\}$. Finally, some classes of four dimensional matrix transformations on those new double sequence spaces are characterized with some significant consequences.

Keywords: four-dimensional generalized difference matrix, almost convergence, double sequence spaces, α-dual, β-dual and γ-dual, matrix transformations.

2010 Mathematics Subject Classification: 46A45, 40C05

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Multiplier Amenability of Banach Algebras

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Abstract: Amenability is a distinctive property of locally compact groups and Banach algebras related to them. The amenability of a locally compact group can be characterized by many properties such as the Følner condition and Leptin's condition. Furthermore. the amenability of locally compact groups can also be studied by the properties of certain Banach algebras related to them (such as the Fourier and the Fourier Stieltjes algebras). In this talk we will discuss and present some new results on the amenability properties of Banach algebras in terms of their multiplier algebras.

Keywords: amenable, locally compact, measure algebra, Fourier algebra

2010 Mathematics Subject Classification: 43A07; 43A10; 43A20

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q-Sumudu transforms of basic generalized hypergeometric functions

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Abstract: The object of this paper is to evaluate q-Sumudu transforms of the basic analogue of H-function of two variables. Interesting special cases of theorems are also discussed. Further, the results proved in this paper may find certain applications of q-Sumudu transforms to the solutions of the q-integrodifferential equations involving functions.

Keywords: q-analogues of Fox's H-function of two variables, q-Sumudu transform, q-analogues of Meijjer's G-function of two variables

On periodic wave factorization

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Abstract: We consider a special kind of factorization related to certain domains in multidimensional complex space $\mathbb{C}^m$. This approach was introduced by the author [1, 2] for studying solvability of model elliptic pseudo-differential equations in canonical domains. Now we interested in discrete variant of such a theory, and for this purpose we construct a periodic wave factorization to describe solvability properties of some classes of discrete equations.

Let $D \subset \mathbb{R}^m$ be a convex sharp cone, $T(D) \subset \mathbb{C}^m$ is a domain of the following type

$$T(D) = \{z \in \mathbb{C}^m : \ z = x + iy, x \in \mathbb{T}^m, y \in D\}.$$

We say that a function $A(\xi), \xi \in \mathbb{T}^m, A(\xi) \neq 0, \forall \xi \in \mathbb{T}^m$, admits periodic wave factorization with respect to cone D if it can be represented in the form

$$A(\xi) = A_+(\xi)A_-(\xi),$$

where the factors $A_\pm(\xi)$ admit bounded analytical continuation into complex domains $T(\pm D)$ respectively.

If we consider a discrete equation in a cone then an existence of periodic wave factorization with respect to an appropriate cone is a sufficient condition for unique solvability of such equation in Lebesgue spaces. Some preliminary results for special degenerated cones were obtained in [3, 4].

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Keywords: discrete equation, periodic wave factorization, multidimensional Riemann boundary value problem

2010 Mathematics Subject Classification: 35J46, 42B37

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On multipliers of Fourier series in the Lorentz space

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Abstract: We study the multipliers of Fourier series on the Lorentz spaces, in particular, the sufficient conditions for a sequence of complex numbers $\{\lambda_k\}_{k\in Z}$ in order to make it a multiplier of trigonometric Fourier series of space $L_{p,r}[0;1]$ in the $L_{q,r}[0;1]$. In this work there is a new multipliers theorem which is supplement of the well-known theorems, and given a counterexample.

Keywords: Multiplier, Fourier series, Lizorkin theorem, Lorentz space

2010 Mathematics Subject Classification: 42A45

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About the defect indices of fourth-order differential operator with rapidly oscillating coefficients

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Abstract: We study in the space $L_2(0,\infty)$ of a minimum nonsemibounded differential operator L_0 [1], [2], generated by the differential expression

$$ly = y^{(4)} - (h(x)y')' - q(x)y, \quad x \in (0;\infty) \tag{1}$$

$$y^{(4)} - (h(x)y')' - q(x)y = \lambda y, \tag{2}$$

Where $\lambda \in \mathbf{C}$, $Im\,\lambda \neq 0$, the function $q(x)$ satisfies the conditions of regularity Titchmarsh-Levitan [3], and $h(x)$ is rapidly oscillating disturbance, which does not meet the conditions of the Titchmarsh-Levitan.

$$\varphi(\xi,\lambda) = \frac{h(x)}{4q^{\frac{1}{4}}(x)(q(x)+\lambda)^{\frac{1}{4}}}, \qquad \omega(\xi,\lambda) = \frac{q'(x)}{8q^{\frac{1}{4}}(x)(q(x)+\lambda)}, \qquad x = \nu(\xi).$$

$$\left|\int_{\xi}^{\infty} \varphi(s,\lambda)ds\right| < \infty, \tag{3}$$

$$\varphi_1(\xi,\lambda) = \int_{\xi}^{\infty} \varphi(s,\lambda)ds.$$

$$\mu_0\varphi_1(\xi,\lambda) \in L_1(0,\infty), \tag{4}$$

$$\varphi_1(\xi,\lambda)\omega(\xi,\lambda) \in L_1(0,\infty), \tag{5}$$

$$\mu_0\varphi_1^2(\xi,\lambda) \in L_1(0,\infty). \tag{6}$$

Theorem 3. *Let the*

- **a:** $q(x) \geq c|x|^{\frac{4}{3}+\varepsilon}, \quad c>0, \quad \varepsilon>0, |x| \geq R;$
- **b:** $q'(x)$, $q''(x)$ *do not change sign for sufficiently large* $R>0$ *for* $|x| \geq R$*;*
- **c:** $q'(x) = o\left(q^{\zeta}(x)\right)$ *when* $x \in (0;\infty)$*, where* $0 < \zeta < \frac{5}{4}$*,*

and the following conditions (3)*,* (4)*,* (5) *and* (6)*. Then the deficiency indices of the operator are equal (3.3).*

Keywords: Defect indices, differential operator, rapidly oscillating coefficients.

2010 Mathematics Subject Classification: 34K08

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On a problem for wave equation with data on the whole boundary

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Abstract: In this paper we propose a new formulation of boundary value problem for a one-dimensional wave equation in a rectangular domain in which boundary conditions are given on the whole boundary.

Let $\Omega \subset R^2$ be a rectangular domain, bounded by following lines: $AB : 0 \leq x \leq \ell$, $t = 0$, $BC : x = \ell$, $0 \leq t \leq T$, $CD : 0 \leq x \leq \ell, t = T$ and $AD : x = 0$, $0 \leq t \leq T$. Let $E = (T, T), F = (\ell - T, T)$ be points on a boundary CD.

We consider a nonhomogeneous wave equation in Ω

$$u_{tt} - u_{xx} = f(x, t). \tag{1}$$

It is well known that the Dirichlet problem for the wave equation (1) in a rectangular domain is ill-posed.

Problem. *Find a solution of Eq.(1), satisfying the boundary condition*

$$u|_{AB \cup BC \cup AD} = 0,$$

and conditions on the boundary CD:

$$u_t|_{DE} = 0,$$
$$\alpha u_x + \beta u_t|_{EF} = 0,$$
$$u|_{CF} = 0,$$

where α and β are real numbers.

We prove the well-posedness of boundary value problem in the classical and generalized senses, depending on the coefficients α and β. The proof of the well-posedness of the formulated problem is reduced to question of the existence and uniqueness of solutions of the corresponding functional equations.

Keywords: Wave equation, well-posedness of problems, classical solution, strong solution

2010 Mathematics Subject Classification: 35L05, 35L20, 49K40, 35D35

APPLIED MATHEMATICS

High order finite-difference method for simulation isotropic turbulence flow

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Abstract: The work deals with the modeling of turbulent energy using two different methods: finite-difference and spectral methods. To simulate the turbulent process the filtered three-dimensional non-stationary Navier-Stokes equation is used. The problem is solved by using hybrid methods, where the equation of motion is solved by using finite-difference methods in combination with cyclic penta-diagonal matrix, which allowed to reach high order of accuracy and to simulate turbulence decay at Reynolds number Re=300 for comparing with theoretical solution of Taylor and Green. The spectral method is used for solution of Poisson equation, which is makes it possible to gain the time. For validation of the given algorithm we solved the classical problem of Taylor and Green modeling of isotropic turbulence flow. Where at the work of Taylor and Green [1], the problem is solved analytically and defined all the turbulence characteristics. In the results of our simulation we have compared the turbulence characteristics with analytical solution: the change of turbulent kinetic energy, dissipation energy over the time and longitudinal and transverse one-dimensional spectra are defined. In this paper a three-dimensional non-stationary Navier-Stokes equation is solved for simulation of isotropic turbulence flow. For solving Navier-Stokes equation, we use a splitting scheme by physical parameters that consist of three stages. At the first stage, the Navier–Stokes equation is solved, without taking pressure into account. For approximation of the convective and diffusion terms of the intermediate velocity field finite-difference methods in combination with cyclic penta-diagonal matrix is used [2], which allowed to increase the order of accuracy in time and in space $O(t^3, h^4)$ without changing the amount of points. At the second stage the Poisson equation is solved, which is satisfies the continuity equation with considering the velocity field from the first stage. For solving the Poisson equation we use spectral method in combination with Fourier transform. The obtained pressure field with using fast Fourier transform is translated from the phase space to physical and used at the third stage for the recallculate of the final velocity field [3].

Keywords: finite-difference method, spectral method, Poisson equation, cyclic penta-diagonal matrix

2010 Mathematics Subject Classification: 94B05, 94B15

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A new class of orthogonal polynomials (Adeolu polynomials) for derivation of initial value solvers

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Abstract: This paper presents a set of newly constructed polynomials called Adeolu polynomials, valid in interval [-1, 1] with respect to weight function w(x) = x + 1 and employed as trial function to develop a fast, efficient and reliable block algorithm for the numerical solution of ordinary differential equations with application to third order initial value problems. Collocation and interpolation techniques were adopted for the formulation of self-starting continuous hybrid schemes. Findings from the analysis of the basic properties of the method using appropriate existing theorems show that the developed schemes are consistent, zero-stable and hence convergent. These continuous numerical integration schemes have the ability to yield several output of solutions at the off-grid points without requiring additional interpolation and at no extra cost. On implementation, the schemes compared favourably well with the existing methods owing to the fact that they are accurate and efficient. Further investigation of the properties of these polynomials is ongoing and in future, the scope of application shall be extended to boundary value problems.

Keywords: Collocation, Interpolation, Orthogonal polynomials, Block method, hybrid

2010 Mathematics Subject Classification: 65L05, 65L06, 94B05, 94B15

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Spreading of flame front I by the predictor-corrector scheme

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Abstract: In this paper, we present the algorithm for the numerical solution of non-stationary equations describing the process of distribution of a flame in a two-fuel mixture. For the numerical simulations of the process of flame spreading is desired to use moving irregular meshes which are thickened to a region of high gradients of temperature and concentration of the mixture, since the calculations for such grids can have a high precision even with a small amount of nodes [1].

The proposed finite-difference scheme "predictor-corrector" in step "predictor" splitting method is used with the explicit approximation of the convective terms in the first step and an implicit fractional approximation of diffusive members on the second fractional step. This second step involves the fractional member responsible for the burning process. In addition, in step "predictor" of explicit scheme determined by the flow at the edges of cells. We have proposed a method of selecting grid parameters allowing to calculate profiles of numerical solutions without spurious oscillations.

The proposed method of adapting the grid to a moving flame front tested on scalar nonlinear convection-diffusion equation with a nonlinear right-hand side, as well as a model system of two nonlinear equations. For meshing we used equivalence distribution method that focuses on the use of time-dependent problems. We have shown that the use of adaptive grids allows to obtain a numerical solution with the same precision as in a uniform grid, but with the number of nodes on the order of less.

Keywords: Predictor-corrector, computational modeling, flame front, flame spreading, flame spreading, algorithm of calculation

2010 Mathematics Subject Classification: 97N40, 97M10

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The identification problem for hyperbolic Schrodinger equation

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Abstract: We consider the identification problem for hyperbolic “Schrodinger differential equation

$$\begin{cases} u_{tt} + Au(t) = f + p, 0 < t < 1, \\ iu_t + Au(t) = g + p, -1 < t < 0, \\ u(-1) = \varphi, u(1) = \psi \end{cases}$$

In a Hilbert space with self-adjoint positive definite operator A. We are interested in studying the stability of this problem. In applications, the stability estimates for solutions of the two problems for the hyperbolic Schrodinger equations are defined. The stable difference schemes of the approximate solution of this problem are constructed. The convergence estimates for the solutions of these difference schemes are established. In addition, the theoretical statements are supported by the results of numerical examples.

Key words: Difference scheme, Schrodinger equation, Stability, Well–posedness

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Fractional derivatives and fractional integrals linked to the fractional powers of positive operators

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Abstract: Fractional derivatives and fractional integrals are defined. In addition, definition for positive operators is stated. It is given how fractional derivatives and fractional integrals are connected to fractional positive operators. Finally, the difference formulas for fractional derivative and fractional integral are derived.

Key words: Fractional derivative, Fractional integral, Fractional power, Positive operator

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Source identification problem for an elliptic-hyperbolic equation

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Abstract: In the present paper, the boundary value problem

$$\begin{cases} -\frac{d^2u(t)}{dt^2} + Au(t) = tp + f(t), \ \ (-1 \le t \le 0)\,, \\ \\ \frac{d^2u(t)}{dt^2} + Au(t) = tp + g(t), \ \ (0 \le t \le 1)\,, \\ \\ u(0) = \varphi, u(-1) = \psi, u(1) = \xi, \end{cases} \tag{1}$$

for the differential equation with parameter p in a Hilbert space H with self adjoint definite operator A is investigated. The well-posedness of this problem is established. The stability inequalities for the solution of source identification problem for elliptic-hyperbolic equations are obtained.

Keywords: elliptic equation, hyperbolic equation, boundary value problems, stability

2010 Mathematics Subject Classification: 35M32, 65M06, 35J25, 35L04

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Stable difference scheme for the solution of an elliptic equation with the involution

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Abstract: The theory of functional-differential equations with the involution has received less attention than functional-differential equations. Moreover, one of the unstudied areas of differential equations are partial differential and difference equations with the involution (see, e.g., [1]- [3] and the references given therein).

In the paper [4], the boundary value problem of an elliptic equation with the involution and nonlocal boundary conditions in x was investigated. The stability and coercive stability estimates in Hölder norms in t for the solution of this problem were established. In the present paper a stable difference scheme for the approximate solution of an elliptic equation with the involution is constructed. Theorem on stability and almost coercive stability and coercive stability of this difference scheme is established. The theoretical statements for the solution of this difference scheme are supported by the results of the numerical experiment.

Keywords: Difference scheme, Elliptic equation with the involution, Stability estimates

PACS: 02.30.Jr, 02.30.Ks, 02.60.Lj

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A note on the Dirichlet problem for model complex partial differential equations

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Abstract: Complex model partial differential equations of arbitrary order are considered. The uniqueness of the Dirichlet problem is studied. It is proved that the Dirichlet problem for higher order of complex partial differential equations with one complex variable has infinitely many solutions. **Keywords:** Dirichlet problem, homogeneous partial differential equation, inhomogeneous partial differential equation, complex analysis with one variable **2015 Mathematics Subject Classification:** 94B05, 94B15

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Crank-Nicolson difference scheme for the stochastic parabolic equation with the dependent operator coefficient

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Abstract: In the paper the Crank-Nicolson difference scheme for the numerical solution of the stochastic parabolic equation with the dependent operator coefficient is considered. Theorem on convergence estimates for the solution of this difference scheme is established. In applications, convergence estimates for the solution of difference schemes for the numerical solution of three mixed problems for parabolic equations are obtained. The numerical results are given.

Keywords: difference scheme, stochastic parabolic equation, convergence estimate

2010 Mathematics Subject Classification: 87.10.Ed, 43.35.Ei, 78.60.Mq

Differential operator equations with interface conditions in modified direct sum spaces

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Abstract: In this study we consider a Sturm-Liouville problem involving an abstract linear operator in equation and determined by eigendependent boundary conditions and two supplementary interface conditions. We give a characterisation of some spectral properties of the considered problem. Particularly, it is established such properties as isomorphism and coerciveness and find asymptotic formulas for eigenvalues.

Keywords: Differential operator equations, interface conditions, eigenvalues, isomorphism and coerciveness

2010 Mathematics Subject Classification: 34B4, 34L10

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Inverse problem for the Verhulst equation of limited population growth with discrete experiment data

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Abstract: Verhulst limited growth model with unknown parameters of growth is considered. These parameters are defined by discrete experiment data. This inverse problem is solved with using gradient method with interpolation of data and without it [1, 2]. Approximation of the delta-function is used for the latter case. As an example the bacteria population *E.coli* is considered.

We consider the evolution of biological species in limited habitat. The given system is described by Verhulst equation. The coefficients of the equations are unknown. We would like to determine it from an inverse problem by using the results of the experiment. The data are distributed discretely. The inverse problem is transformed to an extremum one. This problem is solved by gradient method. The gradient of the given functional depends on the solution of the adjoint system. The adjoint equation includes delta-functions due to the discreteness of the data. This difficulty can be leave out by the interpolation of the data. This inverse problem is solved by using gradient method with interpolation of data and without it. The delta-function is approximated by Gauss formula [3]. The calculation are realized for the exact values of data and with noised data. The calculation accuracy is high enough for both cases and algorithms. We find also the parameters of growth for bacteria population E.coli by real experiment of Scientific Centre of Anti-infectious Drugs (Almaty).

Keywords: Verhulst equation, inverse problem, delta-function

2010 Mathematics Subject Classification: 34A34

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A note on the nonlocal boundary value problem for a third order partial differential equation

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Abstract: It is known that various problem in fluid mechanics (dynamics, electricity) and other areas of physics lead to third order partial differential equations, we derive such equations as models of physical systems and consider methods for solving boundary value problems. This type of equations with constant coefficients can be solved using classical methods like Fourier transform method, and Laplace transform method. In the present study, the nonlocal boundary-value problem for a third order partial differential equation in Hilbert space with a self-adjoint positive definite operator is investigated. Applying operator approach, the theorem on stability for solution of this nonlocal boundary value problem is established. In applications, the stability estimates for the solution of three nonlocal boundary value problems for third order partial differential equations are obtained.

Keywords: Third order partial differential equation, Boundary value problems, Self-adjoint positive definite operator, Hilbert Space.

2010 Mathematics Subject Classification: 34G10, 47N20

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Solution of the free boundary problem for the parabolic equations with unknown temperature and velocity

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Abstract: Let $\Omega(t)$ be a domain in $\mathbb{R}^n$, $n \geq 2$, $\gamma(t) := \partial\Omega(t)$, $t \in (0, t)$ and at the initial moment $t = 0$ $\Omega(0) := \Omega$, $\gamma(0) := \Gamma$. $\gamma(t)$ is a free (unknown) boundary.

We consider one–phase multidimensional free boundary problem for the parabolic equations in the unknown domain. Let the substance in $\Omega(t)$ be in a liquid state. The unknown functions determined in $\Omega(t)$ are temperature and velocity of liquid and they satisfy the parabolic equations. Such problem is a mathematical model, for instance, of the transportation of oil, when there is also a paraffined (solid) oil outside of $\Omega(t)$, and the boundary $\gamma(t)$ between the liquid and solid phases is unknown one.

The existence, uniqueness and coercive estimate of the solution of the considered problem locally in time is proved in the Hölder space.

Keywords: Hölder space, parabolic equation, free boundary problem, existence, uniqueness, coercive estimates

2010 Mathematics Subject Classification: 35R35, 35K20, 35B65

On construction of solutions of linear fractional differential equations with constant coefficients

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Abstract: One of the effective methods for finding exact solutions of differential equations is the method based on the operator representation of solutions. The essence of this method is to construct a series, whose members are the relevant iteration operators acting to some classes of sufficiently smooth functions. This method is widely used in the papers of Bondarenko [1] for construction of solutions of differential equations of the integer order. In this paper, the operator method is applied to construct solutions of linear differential equations with constant coefficients and generalized Riemann-Liouville fractional derivative of order α and type γ. Then fundamental solutions are used to obtain the unique solution of the Cauchy problem.

Keywords: Linear fractional differential equations with constant coefficients, Riemann-Liouville derivatives, fundamental solutions, Cauchy problem

2010 Mathematics Subject Classification: 26A33, 34A08

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More general common fixed point theorems under a new concept

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Abstract: A general common fixed point theorem for two pairs of weakly subsequentially continuous mappings (recently introduced) satisfying a significant estimated implicit function is proved. An extension of this result is thereby obtained. Our results assert the existence and uniqueness of common fixed points in several cases.

Keywords: subcompatible mappings, R-weakly commuting mappings of type $(\mathcal{A}_f)$ (resp. $(\mathcal{A}_g)$), sequentially continuous mappings of type $(\mathcal{A}_f)$ (resp. $(\mathcal{A}_g)$), weakly reciprocally continuous mappings, weakly subsequentially continuous mappings.

2010 Mathematics Subject Classification: 47H10, 54H25, 47H06, 47H09.

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Analytical treatment of two-center overlap integral with respect to same screening constants over Slater-type orbitals

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Abstract: By the use of Löwdin- α radial function, an alternative method is introduced for two-center overlap integral over Slater-type orbitals (STOs) at same screening constants (SSC). Notice that the overlap integrals with SSC have an important role in accurate evaluation of multicenter multielectron molecular integrals arising in combined Hartree-Fock-Roothaan (CHFR) theory. This approximation provides us simple and efficient expression for the two-center overlap integrals. To show the effectiveness of established formula we have compared our

results with available literature data [1]. The method ensures high performance with a very short CPU time.

Keywords: Löwdin- α radial function, Hartree-Fock-Roothaan (CHFR) theory

PACS No: 31.15.-p, 31.10.+z

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Problems of the theory of the consolidation solved in the special functions

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Abstract: The soil, which deformation modulus is continuously increased with depth is called continuously heterogeneous. In this paper, this heterogeneity is presented in the form of:

$$E = E_m(\alpha + \beta z)^m \quad (\alpha > 0, E_m > 0, \alpha + \beta z > 0),$$

where E_m, α, β, m are experimental parameters.

On the basis of this dependence the consolidation problems of elastic and elastically creeping inhomogeneous soils are solved in relation to the restricted area of the consolidation. These solutions make it possible to calculate the values of the pore pressure, the amount of the main stresses and vertical displacements of upper surface points of the condensed inhomogeneous soil mass.

In these solutions for highly compressed water-saturated clay soils is also taken into account that at the initial time the part of loading, instantly enclosed load q to the soil which is equal in value of the structural strength of the compression p_{str}, is immediately perceived by a matrix. In addition, Darcy's law is broken, i.e. the initial gradient of pressure is considered. The resulting calculation formulas are presented as a combination of Bessel functions of the first and second kinds. Taking into account that currently one can define any values of the Bessel functions, it is possible to calculate the pressure in the pore fluids and predict the speed of sediments of the compacting mass [1-4].

Keywords: soil, consolidation, stress, deformation modulus, pore pressure

2010 Mathematics Subject Classification: 74A10, 74L10

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The near optimality of the stabilizing control in a weakly nonlinear system with state-dependent coefficients

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Abstract: Currently State-Dependent Riccati Equation (SDRE) technique is quite a popular control approach for the nonlinear feedback control algorithm. However, it needs online solving the SDRE at every sample time that may require high performance computational resources for difficult control tasks. The new SDRE-based approach for constructing nonlinear stabilizing regulator was proposed in [1, 2] to overcome that problem. It deals with weakly nonlinear system with coefficients depending on the state and the formal small parameter. The proposed approach leads to computationally effective control because it uses analytical expressions. This paper is dedicated to near optimality of nonlinear stabilizing control constructed using that approach. First investigation of that problem was made in [3]. Now we use another form of optimal control and gain matrix. The theoretical results analogous to [3] are obtained and appropriate illustrative example of optimal control problem where formal small parameter equals to unit is considered. This work was supported by the Russian Science Foundation (Project No. 14-11-00692).

Keywords: weakly nonlinear systems, stabilizing control, small parameter, state-depended Riccati equations, optimal control.

2010 Mathematics Subject Classification: 49K40, 49K15, 49L20, 49M30, 93D15, 93D05

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Numerical solution of the hyperbolic-Schrödinger equation with the multipoint nonlocal boundary condition

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Abstract: In this work, we suggest a numerical method to solve hyperbolic-Schrödinger partial differential equations with multipoint nonlocal boundary condition. The stability estimates for the solution of the given problem are established. The first and second order of accuracy difference schemes are obtained for the solution of the given problem. These difference schemes are solved by using the method of modified Gauss elimination for one-dimensional hyperbolic-Schrödinger partial differential equations. The results of numerical experiments are given for supporting the method.

Keywords: finite difference equation, partial differential equation, stability

2010 Mathematics Subject Classification: 65N06, 65N12

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Convergence of Modified Homotopy Perturbation Method for Fredholm-Volterra Integro-Differential Equation of order "m'

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Abstract:

Homotopy perturbation method (HPM) [1, 2] is the combination of two methods: the homotopy and perturbation method. In recent years, the application of HPM in mathematical problems has been applied by many researchers. This method deforms complicated problem into a simple problem which is easy to solve.

HPM has been used for a wide range of problems; for finding the exact and approximate solutions of linear and nonlinear: ordinary differential equations(ODEs), Partial differential equations, integral equations and integro-differential equations of Volterra-Fredholm type.

In this note Modied homotopy perturbation method (MHPM) has applied to solve the Fredholm-Volterro integrodiffere-tial equation (FVIDE) of the third kind with the derivative of order m. Selective functions and unknown parameters helped us to obtain two step iterations. This proposed method could avoid complex computations. It is found that MHPM is a semianalytical method and easy to apply for FVIDE. Convergence and numerical examples are given to present the efciency and reliability of the method.

Keywords: Integral equation, Homotopy perturbation method, Numerical method

2010 Mathematics Subject Classification: 65R20, 45D05

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Successive approximations to solve higher order fractional differential equations

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Abstract: In this talk, we discuss the existence and uniqueness of solution of an initial value problem for fractional differential equation involving higher order Caputo's fractional derivative. We transform the posed problem to a Volterra integral equation, then under Krasnoselskii-Krein type conditions and by using successive approximations, we discuss the existence and uniqueness questions.

Keywords: Fractional differential equation, Existence of solution.

2010 Mathematics Subject Classification: 34B15, 34B18, 34A12

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Operational matrices for finding numerical solution of stochastic differential equations arisen in financial mathematics

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Abstract: In this paper, one of the new convenient techniques is used to solve problems formulated by stochastic Volterra integral equations. Cox-Ingersoll-Ross (CIR) model, Vasicek model and Heston model in financial mathematics can be transformed to a stochastic Volterra integral equation. Triangular functions, block pulse functions and their operational matrix and stochastic operational matrix of integration are considered. Properties of the operational matrices are utilized to reduce the computation of this equations to some algebraic equations. This method has several benefits; in addition to validity and good degree of accuracy, arithmetic operations are carried out without the need to derivative or integration, also, computation of presented method is very simple and attractive. The method is applied to a few test examples to illustrate the accuracy and implementation of the method.

Keywords: Orthogonal functions; Operational matrix; Stochastic operational matrix; Ito Integral; Brownian motion.

2010 Mathematics Subject Classification: 91G30, 91G60, 60H35, 65C20

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Problem with inhomogeneous boundary values for the equations of magnetic elektrogazodinamik

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Abstract: Magnetic system of EGD equations in the Lagrangian mass coordinates is:

$$\frac{\partial v}{\partial t}-\frac{\partial u}{\partial x}=0,\quad v=\frac{1}{\rho},\quad \frac{\partial}{\partial t}vH=\frac{\partial}{\partial x}\Big(\frac{\mu_H}{v}\frac{\partial H}{\partial x}\Big),\qquad \frac{\partial E}{\partial t}=-bE\frac{\partial E}{\partial x},$$

$$\frac{\partial v}{\partial t}=\frac{\partial}{\partial x}\Big(\frac{\mu}{v}\frac{\partial u}{\partial x}\Big)-\frac{\partial p}{\partial x}-\mu_e H\frac{\partial H}{\partial x}+\varepsilon E\frac{\partial E}{\partial x},\quad p=r\frac{\theta}{v}, \tag{2}$$

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial x}\Big(\frac{\lambda}{v}\frac{\partial \theta}{\partial x}\Big)-p\frac{\partial u}{\partial x}+\frac{\mu}{v}\Big(\frac{\partial u}{\partial x}\Big)^2+\frac{\mu_e\mu_H}{v}\Big(\frac{\partial H}{\partial x}\Big)^2+b\varepsilon vE^2\frac{\partial E}{\partial x}.$$

The unknown functions satisfy the boundary conditions:

(3)

$$\frac{1}{v}\frac{\partial u}{\partial x}-p\Big|_{x=0}=\frac{1}{v}\frac{\partial u}{\partial x}-p\Big|_{x=1}=H\Big|_{x=0}=H\Big|_{x=1}=0, E\Big|_{x=0}=\frac{\partial E}{\partial x}\Big|_{x=0}=0$$

$$\theta\Big|_{x=0}=\chi_0(t),\quad \theta\Big|_{x=1}=\chi_1(t),$$

$$\chi_i(t)\in W_2^1(0,T),\quad \chi_i(t)\geq m_0>0,\quad \chi_i(0)=\theta(i),\quad i=0,1.$$

At the initial time all the characteristics of the medium are known:

(4)

$$u\Big|_{t=0}=u_0(x), v\Big|_{t=0}=v_0(x), \theta\Big|_{t=0}=\theta_0(x), E\Big|_{t=0}=E_0(x), H\Big|_{t=0}=H_0(x),$$

Theorem. *Let*

$(v_0,u_0,\theta_0,H_0,E_0)\in W_2^2(\Omega),\quad 0<m_0\leq(v_0(x),\theta_0(x))\leq M_0<\infty,\quad E_0'(x)\geq 0.$ *Then, in area $Q=\Omega\times(0,T)$ with any finite T there exists a unique generalized solution of problem (1) - (3), and*

$$(v(t),E(t))\in L_\infty(0,T,W_2^1(\Omega)),\quad (v_t,u_t,\theta_t,H_t,E_t)\in L_2(Q),$$

$(u(t),\theta(t),H(t))\in L_\infty(0,T,W_2^1(\Omega))\cap L_2(0,T,W_2^2(\Omega)),\quad Q=\Omega\times(0,T),\Omega=(0,1)$ $v(x,t),\theta(x,t)$ *– strictly positive, limited functions.*

Keywords: speed, density, temperature, magnetic field, electric field, generalized solution, apriori estimates, existence

2010 Mathematics Subject Classification: 70H03, 70H05

Principle of Mass Conservation for the Boltzmann's Moment System of Equations in Fourth Approximation

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Abstract: We study the one-dimensional non-linear non-stationary Boltzmann's moment system of equations in fourth approximation with the tools developed by Sakabekov in [1-2]. For the third approximation Sakabekov proves the mass conservation law (cf. Theorem in [2]). We consider the initial value problem for the system of hyperbolic partial differential equations in the conservative form. We extend the proof of the principle of mass conservation to the fourth approximation. The principle of mass conservation allows to prove the global existence in time of the solution of the system of nine hyperbolic partial differential equations representing the fourth approximation of the one-dimensional non-linear nonstationary Boltzmann's moment system of equations.

Keywords: Boltzmann equation, moment system, mass conservation, initial value problem, hyperbolic partial differential equations

2010 Mathematics Subject Classification: 35Q20, 35L65, 35L45

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On the well-posedness of the Boltzmann's moment system of equations in fourth approximation

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Abstract: We study the one-dimensional non-linear non-stationary Boltzmann's moment system of equations in fourth approximation with the tools developed by Sakabekov in [1–3]. For the third approximation system Sakabekov proves the mass conservation law (cf. Theorem 2.1 in [2]) and discusses the existence and uniqueness of the solution (cf. Theorem in [3]). We extenISSAGALI 1, Auzhan SAKABEKOVd the analysis of the existence and uniqueness of the solution to the fourth approximation system. In particular, for the fourth approximation system we discuss the well-posed initial and boundary value problem and prove the existence and uniqueness of the solution belonging to the space of functions, continuous in time and square summable by spatial variable.

Keywords: Boltzmann equation, moment system, initial and boundary value problem, hyperbolic partial differential equations, a-priori estimate

2010 Mathematics Subject Classification: 35Q20, 35B45, 35L04

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On solvability of some boundary value problems for a biharmonic equation with periodic conditions

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Abstract: Let $\Omega = \{x \in R^n : |x| < 1\}$ be a unit ball, $n \geq 2, \partial\Omega$ be a unit sphere. For any point we compare its "opposite" point $x^* = (-x_1, ..., -x_n) \in \Omega$, and denote $\partial\Omega_+ = \partial\Omega \cap \{x \in R^n : x_n \geq 0\}$, $\partial\Omega_- = \partial\Omega \cap \{x \in R^n : x_n \leq 0\}$, $I = \partial\Omega \cap \{x \in R^n : x_n = 0\}$.

Consider the following problem in the domain Ω:

$$\Delta^2 u(x) = f(x), x \in \Omega \tag{1}$$

$$D_\nu^m u(x) = g(x), x \in \partial\Omega \tag{2}$$

$$D_\nu^{\ell_1} u(x) - (-1)^k D_\nu^{\ell_1} u(x^*) = g_1(x), x \in \partial\Omega_+ \tag{3}$$

$$D_\nu^{\ell_2} u(x) + (-1)^k D_\nu^{\ell_2} u(x^*) = g_2(x), x \in \partial\Omega_+ \tag{4}$$

where $D_\nu^m = \frac{\partial^m}{\partial \nu^m}, m \geq 1$, $\nu-$ is a normal vector, $k = 1, 2,\ 1 \leq m \leq 3, 1 \leq \ell_1 < \ell_2 \leq 3, \ell_j \neq m, j = 1, 2$.

The exact conditions for solvability of the problems are found. Note, that analogous problems for Poisson equations were studied in [**?**] - [**?**].

Keywords: biharmonic equation, periodic boundary value problem, solvability, existence and uniqueness of solution

2010 Mathematics Subject Classification: 35J15, 35J25

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A conditional stability estimate of continuation problem for the Helmholtz equation

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Abstract: In this paper we consider the continuation problem for the Helmholtz equation. The main result is a conditional stability estimate for a solution to the considered problem. The estimate shows that the closer solution to the surface is more stable.

We consider the initial boundary value problem for the Helmholtz equation in the domain $\Omega = (0,l) \times (0,\pi)$ [1]:

$$u_{xx} + u_{yy} + k^2 u = 0, \qquad (x,y) \in \Omega, \tag{1}$$

$$u_x(0,y) = 0, \quad u(0,y) = f(y), \qquad y \in [0,\pi], \tag{2}$$

$$u_y(x,0) = u_y(x,\pi) = 0, \qquad x \in [0,l], \tag{3}$$

where k, l are given constants. It is required to find a function $u(x,y)$ in Ω from $f(y)$. The main theoretical result is the following

Theorem. Assume that for $f \in L_2(0,\pi)$ there exists a solution $u \in L_2(\Omega)$ to the problem (1) – (3), then the following estimate holds [2]:

$$\|u\|^2(x) \le \Big(\|q\|^2 + \big|\|f_y\|^2 - \frac{k^2}{2}\|f\|^2\big|\Big)^{\frac{x}{l}} \Big(\|f\|^2 + \big|\|f_y\|^2 - \frac{k^2}{2}\|f\|^2\big|\Big)^{\frac{l-x}{l}} e^{2x(l-x)} - \big|\|f_y\|^2 - \frac{k^2}{2}\|f\|^2\big|,$$

where $\|u\|^2(x) = \int_0^\pi u^2(x,y)dy$.

Keywords: Helmholtz equation, stability estimate, inverse problem

2010 Mathematics Subject Classification: 35R30

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Recurrence relation for the moments of order statistics from a beta-Pareto distribution

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Abstract: In this paper, a novel cumulative distribution function ($c.d.f.$) for beta-Pareto (BP) distribution, through two distinct practical frames, is developed. However, the presented models are obviously more pragmatic than the ones being demonstrated in previous works, in the case of extending the further relations. Then, using the exhibited c.d.f.s, certain recurrence relations for the single and product moments of the order statistics of a random sample of size n arising from beta-Pareto distribution are derived.

Keywords: Order statistics, single and product moments, recurrence relations, beta-Pareto

2010 Mathematics Subject Classification: 62G30

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The Motion of Fractional Order Jeffrey Fluid Model

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Abstract: This paper presents some new exact solutions corresponding to three unsteady flow problems of a generalized Jeffrey fluid [1-3] produced by a flat plate between two side walls perpendicular to the plate. The fractional calculus approach in the governing equations is used. The exact solutions are established by means of the Fourier sine transform and discrete Laplace transform. The series solution of velocity field and the associated shear stress in terms of Fox H-functions, satisfying all imposed initial and boundary conditions, have been obtained. The similar solutions for ordinary Jeffrey fluid, performing the same motion, appear as limiting case of the solutions are obtained here. Also, the obtained results are analyzed graphically through various pertinent parameter.

Keywords: Generalized Jeffrey fluid, Fractional derivatives, Fox H-functions, Discrete Laplace transform.

2010 Mathematics Subject Classification: 76A05, 76A10

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Computational algorithms for analysis of data from thin-film thermoresistors on a radio-electronic printed circuit board

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Abstract: In the paper, a printed circuit board with radio-electronic elements is considered as a geometrical structure with thin-film thermoresistors coated on to it. The local resistance of these thermoresistors depends on local temperature. The measurement of these resistances is carried out not in a local way but totally along horizontal or vertical lines. Therefore, the number of these total data on resistance is significantly less than the number of these local zones. During operating the printed circuit board, overheating of separate elements may occur that leads to a temperature increase of its corresponding parts and a resistance increase of separate parts of a chain. We set up the task of determining these areas of overheating and also their resistance and temperature being guided by some additional physical conditions.

Keywords: thin-film thermoresistors, radio-electronic printed circuit boards, local temperature, local resistance, numerical algorithms, underdetermined systems of linear algebraic equations

2010 Mathematics Subject Classification: 65F05, 80A23.

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On solvability of boundary value problem of magnetic gas dynamics with cylindric and spherical symmetries

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Abstract: In this paper we investigate the initial-boundary value problems for the equations motion of a viscous heat-conducting gas, taking into account the magnetic field. The main attention is paid to the case of solutions with cylindrical and spherical symmetry. The paper describes in detail the transition from the basic three-dimensional equations to cylindrical and spherical coordinate system and prove a number of theorems on the existence and uniqueness of the initial-boundary value problems.

$$\frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial (x^m u)}{\partial q} = 0, \;\; P = R\rho\theta,$$

$$\frac{\partial u}{\partial t} + \frac{H^2}{x\rho} = x^m \frac{\partial}{\partial q}\left(\rho \frac{\partial (x^m u)}{\partial q}\right) - x^m \frac{\partial P}{\partial q} - x^m H \frac{\partial H}{\partial q},$$

$$\frac{\partial \theta}{\partial t} = æ \frac{\partial}{\partial q}\left(x^{2m}\rho \frac{\partial \theta}{\partial q}\right) + \rho \left[\frac{\partial (x^m u)}{\partial q}\right]^2 +$$

$$(1) \qquad +\rho x^{2(m-1)} \left[\frac{\partial (xH)}{\partial q}\right]^2 - P \frac{\partial (x^m u)}{\partial q} - \frac{3m}{2} \frac{\partial (x^{m-1} u^2)}{\partial q},$$

$$\frac{\partial H}{\partial t} + x\rho H \frac{\partial (x^{m-1} u)}{\partial q} = x\rho \frac{\partial}{\partial q}\left(\rho x^{2(m-1)} \frac{\partial (xH)}{\partial q}\right),$$

$$(2) \qquad (u, \rho, \theta, H)(q, 0) = (u_0, \rho_0, \theta_0, H_0)(q), q \in [0, b],$$

$$(3) \qquad (u, \frac{\partial \theta}{\partial q}, H)(0, t) = (u, \frac{\partial \theta}{\partial q}, H)(b, t) = 0, t \in [0, T].$$

This article explores the initial-boundary value problems (1)-(3) for the equations motion of a viscous heat-conducting gas, taking into account the magnetic field with cylindrical and spherical symmetry. The paper describes in detail the transition from the basic three-dimensional equation in cylindrical and spherical coordinates. Under certain conditions the data (2) prove a number of theorems on the existence and uniqueness of the initial-boundary value problems.

Keywords: gas dynamics equation, boundary value problems, spherical symmetry

2010 Mathematics Subject Classification: 39A10, 35J66, 35J67

Diffusion equation model for the tumors cell density and immune response

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Abstract: Over the last 20 years increasingly complex mathematical models of cancerous growths have been developed, especially on solid tumors, in which growth primarily comes from cellular proliferation. Consider a procedure for cancer therapy which consists of interaction between immune response (immune cells) and tumor cells without any specific drug. The cytotoxic T lymphocyte (CTL) and tumor necrosis factor (TNF) cause of the immune response. This process is modeled as a system of tumor cell density (TCD) and tumor necrosis factor (TNF). The purpose of this paper is to establish a rigorous mathematical analysis of the model and to explore the density/concentration of tumor cell and immune response (TNF). The result suggests that although TCD capable to growth of tumor but the immune response is block to direct tumor growth. The model assumes that only two factors need be considered for such predictions: net growth rate and infiltrative ability. The model has already provided illustrations of theoretical glioblastomas, but also shows the distribution of the diffusely infiltrating cell.

Investigation of Laplace transforms for Erlangen distribution of the first passage time into zero level of the semi-Markov random process with positive tendency and negative jumps

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Abstract: The investigations of the distributions for the processes of semi-Markov random process have an important value in the random process theory. The purpose of the present article is to find the Laplace transforms for Erlangen distribution of the semi-Markov random processes with positive tendency and negative jumps. Let's the sequence $\{\xi_k, \zeta_k\}_{k\geq 1}$ be given on the probability space $\{\Omega, \Im, P(\cdot)\}$, where the random variables ξ_k and ζ_k, $k \geq 1$ independent and identically distributed. Using these random variables we construct the following semi-markov random process:

$X(t) = z + t - \sum_{i=1}^{k-1} \zeta_i$, if $\sum_{i=1}^{k-1} \xi_i \leq t < \sum_{i=1}^{k} \xi_i,\ k = 1, 2, ...,\ \ t, z \geq 0.$

$X(t)$ process is called as the semi-markov random process with positive tendency and negative jumps. Also let us denotethe random variable τ_z^0 the first falling time of the process into zero-level, as $\tau_z^0 = \min\{t : X(t) \leq 0\}$. We need to find the Laplace transform for distribution of random variable τ_z^0. Let's assume that the random variables ξ_1 and ζ_1 have an Erlangen distribution of third construction with the parameters λ and μ, respectively. According to total

probability formula, we can write the following integral equation for the random variable τ_z^0 :

$$L(\theta \mid z)=\frac{\lambda^3 e^{-\mu z}}{(\lambda+\mu+\theta)^3}+\frac{\lambda^3 \mu e^{-\mu z}}{2} \int_{s=0}^{\infty} s^2 e^{-(\lambda+\mu+\theta) s} \int_{\alpha=0}^{z+s} e^{\mu \alpha} L(\theta \mid \alpha) d \alpha d s.$$

In this case, we get the following fourth order differential equation:

$$\begin{aligned} L^{(4)}(\theta \mid z) & - [3(\lambda+\theta)-\mu] L^{\prime \prime \prime}(\theta \mid z)+3(\lambda+\theta)(\lambda+\theta-\mu) L^{\prime \prime}(\theta \mid z) \\ & - (\lambda+\theta)^2(\lambda+\theta-3 \mu) L^{\prime}(\theta \mid z)-\mu[(\lambda+\theta)^2-\lambda^3] L(\theta \mid z)=0 . \end{aligned}$$

Then the general solution of this differential equation will be as follows:

$$L(\theta \mid z)=\frac{\lambda^3}{[\lambda+\theta-k_1(\theta)]^3} e^{k_1(\theta) z} .$$

This expression is the Laplace transform for relative distribution of random variable τ_z^0. Laplace transform for non-relative distribution of τ_z^0 will be

$$L(\theta)=\int_{z=0}^{\infty} L(\theta \mid z) \lambda^3 z^2 e^{-\lambda z} d z=$$

$$\int_{z=0}^{\infty} c_1(\theta) e^{k_1(\theta) z} \lambda^3 z^2 e^{-\lambda z} d z=\frac{\lambda^3}{[\lambda-k_1(\theta)]^3} C_1(\theta) .$$

Therefore, we will get the following characteristics for $\lambda m>\mu$:

$$\begin{aligned} E \tau_z^0 & =-L^{\prime}(0)=\frac{3(\lambda+\mu)}{\lambda(\lambda-3 \mu)}, \\ E(\tau_z^0 \mid z) & =\frac{3(1+z \mu)}{\lambda-3 \mu}, \end{aligned}$$

and

$$D(\tau_z^0 \mid z)=\frac{3}{(\lambda-3 \mu)^2}+\frac{12(3+z \lambda) \mu}{(\lambda-3 \mu)^3} .$$

Keywords: Laplace transform, semi-Markov random process, random variable.

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Statistical inference for the parameter of Burr-type 2 model using vague information as fuzzy data

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Abstract: Conventional statistical analysis of lifetime models is based on precise data. However, in real world situations, some information about an underlying system might be vague and are represented in the form of fuzzy information. Thus, it is necessary to generalize classical statistical estimation methods for real numbers to fuzzy numbers. Pak et al. ([1], [2], [**?**], [3]) conducted a series of studies to develop the inferential procedures for the lifetime models on the basis of fuzzy numbers. In this study, different methods of estimation are discussed for the parameter of Burr-type 2 when the available data are in the form of fuzzy numbers. For this, we use the numerical methods such as "Newton-Raphson method" when the available observations are described by means of fuzzy information. Furthermore, we present a Bayesian approach to estimate the parameter of the model. Finally, Comparisons are made between these estimators using Monte-Carlo simulation study.

Keywords: Maximum Likelihood Estimate, Markov chain Monte Carlo, EM algorithm, Fuzzy data analysis

2010 Mathematics Subject Classification: 94B05, 94B15

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Differentiation of the functional in an optimization problem for diffusion and convective transfer coefficients of elliptic imperfect contact interface problems

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Abstract: Convection-diffusion problems are typical for mathematical models of liquid and gas mechanics, since heat and impurities transfer can occur not only due to diffusion, but also to the motion of the medium (see [1]). Currently, the most profound results in the theory of numerical solution of problem for PDEs and optimal control problems are obtained for processes with self-adjoint operators.

In this work we investigate issues of numerical solving of optimal control problems for second order elliptic equations with non-self-adjoint operators - convection-diffusion problems. Control processes are described by semi-linear convection-diffusion equation with discontinuous data and solutions (states) subject to the boundary interface conditions of imperfect type (i.e., problems with a jump of the coefficients and the solution on the interface; the jump of the solution is proportional to the normal component of the flux). Controls are involved in the coefficients of diffusion and convective transfer.

The subject of this paper is related to [2]. We prove differentiability and Lipshitz continuity of the cost functional, depending on a state of the system and a control. The calculation of the gradients uses the numerical solutions of direct problems for the state and adjoint problems.

Keywords: optimal control, semi-linear elliptic equation, imperfect contact, convective transfer, cost functional, differentiability

2010 Mathematics Subject Classification: 49J20, 35J61, 65N06

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Ranking DNA words Using Nonextensive Statistical Mechanics

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Abstract: Functional DNA subsequences and genome elements are spatially clustered through the genome just as keywords in literary texts. Therefore; some of the methods for ranking words in texts can also be used to compare different DNA subsequences. One of these methods is based on nonextensive statistical mechanics [1]. Here we claim that the distribution of distances between the successive subsequences (words) has q-exponential form which is the Tssalis distribution function in nonextensive statistical mechanics [2].Thus we introduce the q-parameter as a measure for estimating the clustering level of words. In this study, we analyzed the distribution of distances between consecutive occurrences of 16 possible dinucleotides in human chromosomes by comparing their corresponding q-parameters and found that the CGs have the highest clustering level which is a biologically important word concerning its methylation. We extended our study by comparing the genome of some selected organisms and concluded that the clustering level of CG dinucleotides increases in higher evolutionary organisms compared to lower ones

Keywords: Statistical Mechanics, DNA sequences, q-exponential function

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Positiveness of second order differential operators with interior singularity

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Abstract: The main goal of this study is to provide an operator-theoretic framework for the investigation of discontinuous Sturm–Liouville problems with eigenparameter appearing in the boundary conditions. We introduce some self-adjoint compact operators in suitable Sobolev spaces such a way that the considered problem can be reduced to an operator-pencil equation. We define a new concept so-called generalized eigenfunctions and prove positiveness of operator-pencil corresponding to the considered problem.

Keywords: Sturm-Liouville problem, operator-pencil, interior singularity

2010 Mathematics Subject Classification: 34B4, 34L10

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Transmission conditions for biharmonic equations with discontinuous coefficients and numerical solution of boundary value problem

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Abstract: The mathematical model of bending problem for the plate system which is composed of inhomogeneous elastic plates with differing elastic properties is presented by boundary value problems for the biharmonic differential equations of with discontinuous coefficients. Providing continuity at the common borders of the plates, determination of transmission conditions among them and the development of methods for finding suitable numerical aproximate solution for the problem keep up to date for applied mathematics.

The fundamental aim of this study is to obtain the transmission condition of plates system composed of elastic plates with differing elastic properties, their difference equations and then the numerical solution at different boundary conditions. Firstly, we can represent the potantial energy of plates system. Later, finite difference approximations of transmission condition at common boundary of plates is obtained by functional approximation method with the help of numerical approximation of derivative and integral.

In this paper, we obtain numerical solution of biharmonic equation with discontinuous coefficient at different boundary conditions and the analysis of numerical results is given.

Keywords: biharmonic equation,transmission condition

2010 Mathematics Subject Classification: 65N06, 74G15, 35J40

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Some properties of eigenvalues and generalized eigenvectors of one boundary-value problem

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Abstract: We investigate one discontinuous boundary value problem which consist of a Sturm-Liouville equation with piecewise continuous potential together with eigenparameter-dependent boundary conditions and supplementary transmission conditions. We establish some spectral properties of the considered problem. In particular it is shown that the generalized eigenfunctions form a Riesz basis of the adequate Hilbert space.

Keywords: Discontinuous boundary value problem, boundary conditions, Riesz Basis

2010 Mathematics Subject Classification: 34B4, 34L10

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On a method of finding a solution of semi-periodic boundary value problem for hyperbolic equations

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Abstract: In this paper we investigate the floor semi-periodic boundary value problem for a system of hyperbolic equations with mixed derivative. An algorithm for finding an approximate solution of this problem.

In this paper, a semi-periodic boundary value problem for a system of linear hyperbolic equations is considered:

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + C(x,t)u + f(x,t), \quad (x,t) \in \overline{\Omega} = [0,\omega] \times [0,T],$$

$$u(0,t) = \psi(t), \quad u(x,0) = u(x,T), \quad t \in [0,T], \quad x \in [0,\omega],$$

where $A(x,t), C(x,t)$ matrix of $(n \times n)$ order, $f(x,t)$ n-vector-function in continuous $\overline{\Omega}$, n-vector-function $\psi(t)$ continuously-differentiable in $[0,T]$. For her study, the interval $[0,\omega]$ is divided into M parts, and parameterization method is applied for each area. The proposed method for finding approximate solutions of semi-periodic boundary value problem for a system hyperbolic equations is illustrated by an example.

Keywords: Hyperbolic equations, decomposition, algorithm, the boundary value problem

2010 Mathematics Subject Classification: 35K20, 44A10, 45D05, 35L20

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A note on numerical solution of the parabolic-Schrödinger equation

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Abstract: The nonlocal boundary value problem for a parabolic-Schrödinger equation is considered. The stability estimates for the solution of the given problem are established. For aproximately solving this nonlocal boundary problem the first and second order of difference schemes are presented. The theoretical statements for the solution of these difference schemes are supported by the result of numerical examples.

Keywords: finite difference equation, partial differential equation, stability

2010 Mathematics Subject Classification: 65N06, 65N12

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Well-posedness of high order of accuracy difference scheme

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Abstract: The simply nonlocal boundary value problem was investigated for the first time by A.V. Bitsadze and A.A. Samarskii in the paper [1]. In recent years, [3]– [5], the Bitsadze–Samarskii type nonlocal boundary value problem and its generalizations for various differential and difference elliptic equations have been studied. In this work, the Bitsadze-Samarskii type nonlocal boundary-value problem with the integral condition

$$(1) \qquad \begin{cases} -\frac{d^2u(t)}{dt^2} + Au(t) = f(t), \ 0 < t < 1, \\ u(0) = \varphi, \ u(1) = \int\limits_0^1 \rho(\lambda)u(\lambda)d\lambda + \psi \end{cases}$$

for the differential equation of elliptic type in a Hilbert space H with the self-adjoint positive definite operator A with a closed domain $D(A) \subset H$ is considered. The well-posedness of the fourth order of accuracy difference scheme for approximate solution of nonlocal boundary-value problem (1) in Holder spaces with a weight is proved. The theoretical statements for the solution of this difference scheme are supported by the results of a numerical example.

Keywords: elliptic equation, nonlocal boundary value problem, well-posedness

2010 Mathematics Subject Classification: 35J40, 34B10, 49K40

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Rank 0 invariant solutions of dynamics of two-phase medium

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Abstract: There is considered a system of partial differential equations, which describes dynamics of two-phase medium [1]

$$\frac{d\rho_1}{dt_1} + \rho_1 \mathrm{div}\ \vec{u}_1 = 0,$$
$$\frac{d\rho_2}{dt_2} + \rho_2 \mathrm{div}\ \vec{u}_2 = 0,$$
$$\rho_1 \frac{d\vec{u}_1}{dt_1} + m_1 \nabla P(\rho_1, \rho_2) = -\frac{\rho_2}{\tau}(\vec{u}_1 - \vec{u}_2),$$
$$\rho_2 \frac{d\vec{u}_2}{dt_2} + m_2 \nabla P(\rho_1, \rho_2) = \frac{\rho_2}{\tau}(\vec{u}_1 - \vec{u}_2),$$

Algebra Lie of symmetry group of this system was found [2]. Optimal systems of subalgebras all dimensionals for this algebra were found in papers [3,4]. Each subalgebra from these optimal systems provides dissimilar invariant, partially invariant, or differential invariant submodel [5]. Solutions of this submodels are called invariant, partially invariant, or differential invariant solutions of original system.

Four-dimensional subalgebras from optimal systems of subalgebras give partially invariant, or invariant solutions. All rank 0 invariant solutions are found. They are received using some 4-dimensional subalgebras. All other 4-dimensional subalgebras will give only partially invariant submodels of this system.

Keywords: two-phase medium, rank 0 invariant solutions, symmetry group, partially invariant submodels

2010 Mathematics Subject Classification: 35B06, 35Q35

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Numerical solution of the elliptic-Schrödinger equation with the multipoint nonlocal boundary condition

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Abstract: The boundary value problem for an elliptic-Schrödinger equation with the multipoint nonlocal boundary condition is considered. The stability estimates for the solution of the given problem are established. For numerically solving this multipoint nonlocal boundary problem the first and second order of difference schemes are presented. The theoretical statements for the solution of these difference schemes are supported by the result of numerical examples.

Keywords: finite difference equation, partial differential equation, stability

2010 Mathematics Subject Classification: 65N06, 65N12

References:

[1] Ashyralyev A., Fattorini H.O., "On uniform difference scheme for second order singular perturbation problems in Banach spaces", *SIAM J. Math. Anal.*, Vol.23, No.1, pp. 29-54, 1992.

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On parabolic inverse problems with overdetermination data on spatial manifolds

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Abstract: We examine the question on finding a solution to a parabolic system together with a right-hand side and coefficients in parabolic systems of equations. Let G be a domain in $\mathbb{R}^n$ with a boundary Γ of class C^{2m} and $Q = (0,T) \times G$. The parabolic system is of the form

(1) $$u_t + A(t,x,D)u = g = \sum_{i=1}^{r} b_i(t,x)q_i(t,x') + f, \quad (t,x) \in Q,$$

where $x' = (x_1, x_2, \ldots, x_k)$, $x'' = (x_{k+1}, x_{k+2}, \ldots, x_n)$ and A is a matrix elliptic operator of order $2m$ with matrix coefficients of dimension $h \times h$, representable as

$$A(t,x,D) = \sum_{i=r+1}^{sh} q_i(t,x')A_i(t,x,D_x) + A_{hs+1}(t,x,D_x),$$
$$A_i = \sum_{|\alpha|\le 2m} a_{i\alpha}(t,x)D^\alpha,$$

(2) $$u|_{t=0} = u_0, \quad B_j u|_S = \sum_{|\beta|\le m_j} b_{j\beta}(t,x)D^\beta u|_S = g_j(t,x),$$

where $m_j < 2m$, $j = 1,2,\ldots,m$, and $S = (0,T) \times \Gamma$. The unknowns are a solution u and functions $q_i(t,x')$ $(i = 1,2,\ldots,sh)$ occurring into the right-hand side of (1) and in the operator A. The overdetermination conditions are of the form

(3) $$u|_{S_i} = \psi_i(t,x'), \ S_i = (0,T) \times \Gamma_i, \quad i = 1,\ldots,s$$

where $\{\Gamma_i\}$ $(i = 1,2,\ldots,s)$ is a set of smooth $n-1$-dimensional surfaces lying in G. Under certain conditions on the data, we establish the existence and uniqueness theorems for solutions to these inverse problems.

Keywords: inverse problem, parabolic system, existence, uniqueness

2010 Mathematics Subject Classification: 35R30, 35K41, 35K35

Statistical property of earthquakes network: active and passive points

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Abstract: Recent results in complex systems have indicated that network theory, which is known as graph theory in mathematics, is a powerful method to handle unsolved complex problems. Earthquakes manifest spatio-temporal complex behavior that can be studied using complex networks. It is so essential for these studies to construct an appropriate network. Then we use the statistical mechanics to study the property of such a network.

We propose a new combination method of Abe - Suzuki method [1] and Telesca - Lovallo method [2] for constructing earthquakes network. We divide a geographical region into small square cells that these cells cover the entire region without any overlapping. If an earthquake with any value of magnitude occurs in a cell, we identify it as a vertex of a network. Then for connecting links between vertexes we use the visibility condition. Indeed in our method two events are connected to each other if visibility condition holds true between them.

For Iran and California earthquakes we divide the longitudinal and latitudinal ranges into cells (cell sizes changes from $4km$-$220km$). We show that the constructed networks are scale free and theirs degree distribution obey the q-exponential function which is used in non-extensive statistical mechanics [3]. The diagram of q parameter in terms of the cell size has a peak at $31km$ for Iran and $44km$ for California. Due to dependence of network characteristics on each other for cell sizes less than peak size, only one of the cell sizes is enough to describe the earthquakes network. We found that such model network results in both the Gutenberg-Richter and Omori laws.

Also we find the universal behavior of links and nodes number with time for same areas and same resolution of Iran and California earthquakes. This behavior is power law and similar to Omori law but there are differences between them.

Furthermore, analogy to Darooneh- Lotfi [4] by using the concept of PageRank, we find the passive and active points in the geographical region of Iran.

Keywords: Statistical Mechanics, Earthquake Network, PageRank

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Monge-Ampere equation and pluripotential theory

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Abstract: Pluripotential theory based on the Monge-Ampere operator $(dd^c u)^n$, where $d = \partial + \overline{\partial}$, $d^c = \frac{\partial - \overline{\partial}}{4i}$ are well-known notations, is one of the actual areas of modern mathematics. It is a direct continuation and further development of the classical potential theory, based on the Laplace operator.

In 1959 Bremermann proved that the maximal plurisubharmonic (psh) function $\omega(z) = \sup\{u \in psh(D)) : u|_{\partial D} \leq \varphi\}$, where $D \subset \mathbb{C}^n$ is a srictly pseudoconvex domain and $\varphi \in C(\partial D)$, has many properties of harmonic functions, and is connected with the complex Monge-Ampere equation $(dd^c u)^n = 0$. Later Kerzman (1977) proved the opposite, that if $(dd^c u)^n = 0$ then u is maximal. Since maximal psh functions are the basis of the pluripotential theory, then during the 1977-82 years was carried out intensive research aimed at building the pluripotential theory in the complex space $\mathbb{C}^n$: have been introduced and studied the basic objects of the theory, such as extremal Green function $V(z, K)$, $\mathcal{P}-$ measure $\omega(z, K, D)$, pluripolar sets, capacity values $\mathcal{P}(K, D), C(K, D)$, etc.

The most importantly, these studies have been used successfully in solving various problems, that have accumulated in multidimensional complex analysis and in the theory of plurisubharmonic functions. By 1982 the foundation pluripotentil theory was practically built. All the basic fundamental theory of the objects have been identified, the method of their application has been developed.

In this paper we consider a generalized Monge-Ampère equation (equation in Hessian)

$$(dd^c u)^m \wedge \beta^{n-m} = 0, \quad 1 \leq m \leq n, .. \tag{2}$$

where $\beta = dd^c|z|^2$ is the volume form in $\mathbb{C}^n$. Equation (1) for $m = 1$ gives the Poisson equation, and for $m = n-$ the Monge-Ampère equation. Equation (1) and properties of their solutions have been intensively studied in recent years. So, the real equations in the Hessian dedicated by authors such as A. Caffarelli, L.Nirenberg, J.Spruck, N. Trudinger, N. Ivochkina, X. Wang, etc. Complex Hessian equation (1) was first considered by S. Li, Z. Błocki, S. Dinev, S. Kolodziej (2005-2012). The remarkable fact in their research is to determine the class of $m-$subharmonic $(m - sh)$ functions in which it is possible to solve the equation (1). Remember, a function $u \in C^2(D)$ is called $m - sh$, if

$$(dd^c u)^k \wedge \beta^{n-k} \geq 0 \; for \; all \; k = 1, 2, ..., m.$$

The class of $m-$subharmonic functions plays the same role in the solution of equation (1), as the class of plurisubharmonic functions for the Monge-Ampère equation $(dd^c u)^n = 0$. Therefore, it is important to further in-depth study of the class $m-sh$ functions, in particular its potential-capacity properties. In this way, we formulate a few following fundamental results (see [4-5])

Theorem 1. *Locally $m-$polar set is globally $m-$polar in $\mathbb{C}^n$, i.e. if a set $E \subset \mathbb{C}^n$ s locally $m-$polar, then there exists a function $u \in m-sh(\mathbb{C}^n)$ such that $u \not\equiv \infty$, $u|_E = -\infty$.*

Recall that the set $E \subset D \subset \mathbb{C}^n$ is called $m-$ polar, if there exists a function $u \in m-sh(D)$ such that $u \not\equiv \infty$, $u|_E = -\infty$.

Theorem 2. *If the domain $D \subset \mathbb{C}^n$ is $m-$ regular, then in it the Dirichlet problem*

$$(dd^c u)^m \wedge \beta^{n-m} = 0, \;\; u|_{\partial D} \equiv \varphi(\xi) \tag{3}$$

has a unique solution $u \in m-sh(D)) \bigcap C(\overline{D})$ for any given boundary function $\varphi(\xi) \in C(\partial D)$.

Theorem 3 (Quasicontinuity, analogue of Luzin's theorem)**.** *Arbitrary $m-$ subharmonic function is continuous almost everywhere with respect to the capacity, i.e. if $u \in m-sh(D)$, then for any $\varepsilon > 0$ there exists open set $U \subset D$ such that its capacity $C(U) < \varepsilon$ and the function u is continuous in $D \setminus U$.*

Keywords: Monge-Ampere equation, plurisubharmonic functions, Dirichlet problem

2010 Mathematics Subject Classification: 32U05, 32U15

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Maxwell-Auzhan boundary conditions for Boltzmann's moment system equations in third approximation

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Abstract: In this article we prove existence and uniqueness of the solution of the problem with initial and Maxwell-Auzhan boundary conditions [2] for nonstationary nonlinear one-dimensional Boltzmann's six-moment system equations [1] in space of functions continuous in time and summable in square by a spatial variable. In order to obtain a priori estimation of the initial and boundary value problem for the nonstationary nonlinear one-dimensional Boltzmann's six-moment system equations we get the integral equality then use the spherical representation of vector. Then we obtain the initial value problem for Riccati equation. We have managed to obtain a particular solution of this equation in an explicit form [3-4].

Keywords: Maxwell-Auzhan Boundary conditions, Boltzmann's moment system equations

2010 Mathematics Subject Classification: 35Q20, 35B45, 35L04

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Numerical solution of the relaxational filtration model in relaxationaly-compressed porous environment realized by the linear Darcy law by Monte Carlo and probability difference methods

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Abstract: The model of linear relaxational filtration is considered. Initial and boundary conditions are set for them (Dirichlet, Neumann and mixed). Obtained problem solved by Monte Carlo methods – "random walk on spheres", "random walk on balls" and "random walk on lattices" of Monte Carlo methods and by probability difference methods. Filtration model in relaxationaly-compressed porous environment realized by the linear Darcy law.

$$\chi\partial_x^2\Big(p(t,x)+\lambda_m\partial_t p(t,x)\Big)=\partial_t\Big(p(t,x)+\lambda_m'\partial_t p(t,x)\Big), \tag{4}$$

$$\mathbf{W}(t,x)=-\frac{\kappa}{\mu}\mathrm{grad}_x p(t,x). \tag{5}$$

Here $p(t,x)$ is pressure, $\mathbf{W}(t,x)$ is velocity of filtration, χ, λ_m, $\lambda_m'=\lambda_m\frac{\beta_*}{\beta}$, β_*, β is parameters, and all parameters are nonnegative given numbers. The model in bounded region of filtration $\Omega\in\mathbb{R}^3$ with boundary $\partial\Omega$, and for $t\in[0,T]$, and for pressure $p(t,x)$ we consider equation (4). Then we set initial conditions for them. We consider three boundary conditions: Dirichlet, Neumann and mixed for equation (4). Obtained problems solved by Monte Carlo methods – "random walk on spheres", "random walk on balls" and "random walk on lattices" of Monte Carlo methods and by probability difference methods. [1].

Keywords: Numerical modeling, model, relaxation, filtration, random walk, Monte Carlo, probability, difference, method, approximation

2010 Mathematics Subject Classification: 65C05

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New Economy Form (NEF) as a Plan-Market Economy

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Abstract: We propose to research a theory of growing income with a Plan-Market Economy in the New Economy Form. Humanity tends to get an increasing income to make a prosperous life. The method that leads us to the rising income has been a dream of economists. However, they fail to find it. The mentioned problem belongs to the class of economic growth and development. We started the research with the model of economic growth and development of income equal to the sum of costs.

Keywords: New Economy Form (NEF), improved Harrod-Domar mode, old style, abstract model

2010 Mathematics Subject Classification: 330.564:64.066.2

Consider the abstract model of Harrod-Domar model as follows

(1) $$y'(t) = \lambda u(t), t \in [t_0, +\infty)$$

(2) $$y(t) = u(t) + c(t), t \in [t_0, +\infty)$$

Initial income $y(t_0) = y_0$ (3)

New Economy Form (NEF) as the new style of economic work. We split the time interval $[t_0, +\infty)$ into n parts as follows. Take $n+1$ points $t_0, a_1, a_2, ..., a_n \in [t_0, +\infty)$. Then we get a sequence of time intervals

$$[t_0, a_1] \cup [a_1, a_2] \cup ... \cup [a_{n-1}, a_n] \subset [t_0, +\infty)$$

Then we look at the improved Harrod-Domar model from (1) - (3) on this sequence of time intervals.

Obviously, this is the only way to fulfill the plan in a plan-market economy:

$$y' = \lambda y(t) + \lambda c(t), t \in [t_0, a_1] \cup ... \cup [a_{n-1}, a_n] \subset [t_0, +\infty)$$

with the following conditions $y(t_0) = y_0, y(a_1) = y_1, ..., y(a_n) = y_n$.

With the help of the given model we could find the economic unknown quantities.

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Initial–Boundary–Problems for Vlasov–Poisson Equations with Homogeneous Magnetic Field

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Abstract: We consider the Vlasov-Poisson equations in infinite cylinder with the Dirichlet boundary condition for electric potential [1]. The Vlasov-Poisson system describes evolution of electric potential and distribution functions for densities of charged particles in high-temperature rarefied plasma. The cylindrical shape of domain corresponds to thermonuclear reactor that is called "mirror trap". We study classical solutions with supports of distribution functions strictly inside domain. Such solutions are modelling plasma confinement. We obtain explicite relations providing existence and uniqueness of such solutions in Hölder spaces.

This work was supported by Ministry for Education and Science of Russia, project No.02.a03.21.0008.

Keywords: Vlasov–Poisson equations, classical solutions, plasma confinement

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Numerical solution of the multiphase one dimensional model of gas lift process

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Abstract: Study of the processes of field development is of great practical importance in the oil and gas industry due to the increasing need to improve production methods. A very large number of works is dedicated to the mathematical modeling of these processes [1–3]. We consider and conduct a numerical simulation of a one-dimensional model of gas-lift process, where the movement in gas-lift well is described by partial differential equations of hyperbolic type. Difference schemes for the gas-lift model of the process on a non-uniform grid condensing near the boundaries of subdomains with gas, liquid and a gas-liquid mixture are developed.

In this paper, the mathematical model of gas lift wells is developed. A one-dimensional model of gas-lift wells is examined in which it is assumed that the flow is two-phase and isothermic in the annular portion and in the well. The system, which describes the process under study, consists of the equations of motion and continuity equations of thermodynamic state, concentration, and hydraulic resistance. At the interface, matching conditions for pressure, velocity and concentration are set that allows to obtain a formula for determining the density of the liquid phase in an explicit form. Finite-difference scheme on adaptive non-uniform grid condensing on the boundaries of the gas, liquid and liquid-gas phase, is developed. When building a grid, cubic spline function is used. The results of the proposed algorithm is illustrated by the example of a real well.

Keywords: gas lift process, multiphase flow, finite difference method, numerical solution

2010 Mathematics Subject Classification: 47.85.Dh

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Numerical implementation of the method of fictitious domains for elliptic equations

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Abstract: In the paper we study the elliptical type equation with strongly changing coefficients. We are interested in studying such equation because the given type equations are yielded when we use the fictitious domain method. In this paper we suggest a special method for numerical solution of the elliptic equation with strongly changing coefficients. We have proved the theorem for the assessment of developed iteration process convergence rate. We have developed computational algorithm and numerical calculations have been done to illustrate the suggested method effectiveness.

Keywords: elliptical equation, Dirichlet problem, the equation with rapidly changing coefficients, computational algorithm, iterative process, fictitious domain method, boundary conditions

2010 Mathematics Subject Classification: 97N40

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The gyroscope movement with variable moments of inertia

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Abstract: It took more than two hundred years since the publication of the equations of rigid body dynamics with a fixed point, but research is still ongoing. Great interest in this problem is caused by the fact that in the motion of rigid body with a fixed point are observed gyroscopic effects, widely used in modern technology. The topicality of considering the problem about motion of rigid body with a fixed point is also due to the need to allow for perturbing gravitational, electrical, magnetic and other forces, variability of inertia moment of the object and a wide application in practice.

This kind of problems can be reduced to the study of systems of nonlinear and with variable coefficients differential equations, generating analytical solutions of which offers great mathematical difficulty, and it is possible in a relatively small number of cases. Therefore, the construction of analytical solutions for a wide class of such problems is topical.

This paper discusses motion equations of an axisymmetric rigid body with a fixed point with variable moment of inertia, which is described by a system of nonlinear dynamic Euler equations

$$(1) \qquad \begin{cases} \dfrac{d[A(t)]p}{dt} + [C(t) - B(t)]qr = M_x, \\ \dfrac{d[B(t)]q}{dt} + [A(t) - C(t)]rp = M_y, \\ \dfrac{d[C(t)]r}{dt} + [B(t) - A(t)]pq = M_z, \end{cases}$$

where $A(t)$, $B(t)$, $C(t)$ are body's inertia moments relative to x, y, z axes connected with the body; p, q, r are projections of the body's angular velocity vector on these axes; M_x, M_y, M_z are moments of external resistance forces.

The system of differential equations (1) is considered jointly with initial conditions

$$(2) \qquad \begin{gathered} t = 0 : p(0) = p_0, \quad q(0) = q_0, \quad r(0) = r_0, \\ \dot{p}(0) = 0, \quad \dot{q}(0) = 0, \quad \dot{r}(0) = 0. \end{gathered}$$

The solution of this problem is obtained by the technique of partial discretization of non-linear differential equations, built by A.N. Tyurekhodzhayev on the base of the generalized functions theory.

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On the stability of the telegraph equation with time delay

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Abstract: Telegraph equation is mostly interested in physical systems. Many physicists and engineers use telegraph equation without time delay [1]-[**?**]. However, for several reasons, in problems encountered in real life, time delay should be considered in modeling. In this study, the initial value problem for telegraph equations with delay

$$\left\{\begin{array}{l} \frac{d^2v}{dt^2} + \alpha \frac{dv}{dt} + Av(t) = aAv([t]), t > 0, \\ \\ v(0) = \varphi, v'(0) = \psi \end{array}\right. \tag{1}$$

in a Hilbert space H with a self-adjoint positive definite operator A, is considered. Here $A \geq \delta I$, $\delta \geq \frac{\alpha^2}{4}$ and $0 \leq a < 1$. Theorem on stability estimates for the solution of this problem is established. As a test problem one-dimensional delay telegraph equation with Dirichlet boundary conditions is considered. Numerical solutions of this problem are obtained by first and second order of accuracy difference schemes.

Keywords: initial and boundary value problems, difference schemes, delay telegraph equations.

2010 Mathematics Subject Classification: 35L20, 65N06

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Determination of a control parameter of the r-modified Crank-Nicholson difference scheme for the Schrödinger equation

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Abstract: The differential equations with parameters have been studied extensively by many researchers (see, [1]- [9] and the references therein). However, such problems were not well-investigated in general. In the present paper, the second order of accuracy r-modified Crank-Nicholson difference schemes are presented for the numerical solution of the boundary value problem for the Schrödinger differential equation with parameter p

$$\begin{cases} i\frac{du(t)}{dt} + Au(t) + iu(t) = f(t) + p, \ 0 < t < T, \\ u(0) = \varphi, \ u(T) = \psi \end{cases}$$

in a Hilbert space H with self-adjoint positive definite operator A. The well-posedness of this difference schemes are established. The stability inequalities for the solution of difference schemes for three determination of a control parameter problems for the Schrödinger equation are obtained.

Keywords: determination of a control parameter problem, difference scheme, Schrödinger equation, stability, well-posedness

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On the analytical solution methods for sine-Gordon equation

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Abstract: In this work some solution methods are considered to obtain the analytical solution of nonlinear sine-Gordon equation. Theoretical aspects and some basic concepts on solution of this equation are given. Several applications of sine-Gordon equation in biology and biophysics are introduced.

Keywords: soliton solutions, nonlinear waves, traveling wave solutions

2010 Mathematics Subject Classification: 35C08, 74J30, 35C07

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On the numerical solution of sine-Gordon equation

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Abstract: In this work the approximate solution of well-known nonlinear equation is studied. Finite difference method and the fixed point iteration are used to obtain numerical solution of the nonlinear equation. Error analysis is presented using MATLAB implementation.

Keywords: nonlinear boundary value problems, difference equations, numerical analysis

2010 Mathematics Subject Classification: 34B15, 39A10, 97N40

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Dynamic analogues of Green and Gauss's formulas for unsteady dynamics of anisotropic elastic medium at antiplane deformation

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Abstract: The paper is devoted to the solution of non-stationary boundary value problems (BVP) for an anisotropic elastic medium at antiplane deformation. Wave propagation in continuous medium is described, as a rule, by hyperbolic equations and systems [1, 2]. In this case we have hyperbolic equation of second order with constant coefficients

$$(a_{ij}\partial_i\partial_j - \rho\partial_t^2)u(x,t) = G(x,t), \;\; i,j = \overline{1,N}$$

where $a_{ij} \geq \lambda\|\xi\|$, $\lambda = const > 0$, $\rho = const$, G is regular function. For the solution of the non-stationary BVP for this equation the Method of Generalized Functions (MGF) [3] is used. This method allows the original BVP to lead to the solution of the equations in the space of generalized functions with a certain right-hand side of the class of singular generalized functions such as simple and double layers and use the properties of fundamental solutions of equations for the construction of their generalized solutions.

Here MGF is used for investigation of unsteady dynamics of anisotropic elastic medium at antiplane deformation. The statement of two initial boundary problems are given. Dynamic analogues of Green and Gauss's formulas constructed [4] and their integral representations obtained.

Keywords: hyperbolic equation, boundary value problem, generalized solutions, elastic medium

2010 Mathematics Subject Classification: 35L15, 74J05

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Optimization of turbulent flow in a radial reactor with fixed bed

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Abstract: The data of the computation of turbulent flow in the CF-π and CP-π configurations of the radial reactor with a fixed bed are presented.

Turbulent flow in the annular duct and in the central pipe is described by the Reynolds equations [1]. The macroscopic k-ϵ turbulence model [2, 3] in the variables averaged over the local volume of the catalyst bed and is used for determining eddy viscosity in a porous media.

The macroscopic variables determined over a local volume of the porous medium are used in a fixed bed for flow description [4]. The motion equations with the Ergan drag law in macroscopic variables are used for flow description in the catalyst bed.

The computational data are obtained for the averaged and turbulent characteristics, and it is shown that the flow in the fixed bed causes the generation of the turbulence kinetic energy and its dissipation rate; the flow in the CF-π configuration is distributed more uniformly as compared to the CP-π configuration of the radial reactor. Computed data are compared with the experimental ones.

Keywords: radial reactor, fixed bed, k-ϵ turbulence model.

2010 Mathematics Subject Classification: 02.60.Cb, 44.15.+a

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Instability of control system in the neighborhood of program manifold

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Abstract: We will consider the problem of construction of the steady systems of automatic control on the program manifold. Given program $\omega(t,x)=0$ is performed only subject to condition, if initial values of the state vector of the system satisfies to equality $\omega(t_0,x_0)=0$. But these equalities can be not always performed. Therefore at the construction of the systems of differential equations it is necessary to have some requirements on quality properties of the program manifold as stability, convergence, dissipatedness relatively to some function. To the construction of the systems of differential equations on the program manifold, possessing properties of stability, to the optimality and establishment to estimations for quality of transient's indexes in the neighborhood of program manifold, were devoted great number of works. The review of these works is conducted in [1 - 3].

In this work the control system with respect to vector-function ω is investigated with nonlinearity $\varphi(\sigma)$, satisfying to the local quadratic conditions. For this system was built Lyapunov function of the form

$$V(\omega,\ \xi)=\omega^T L\omega+\int_0^\sigma \varphi^T\beta d\sigma,$$

where $L=L^T,\ \beta=diag\,(\beta_1,\ldots,\beta_r)$ are matrices not necessarily sign definiteness. The sufficient conditions of instability are obtained with respect to vector-function ω by means of this function.

Keywords: program manifold, automatic control system, absolute stability, Lyapunov's function

2010 Mathematics Subject Classification: 34K20, 93C15, 34K29

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Role of non-commutative geometry in gravitational collapse scenarios

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Abstract: The collapse process of a homogeneous scalar field has been studied by many authors and it has been shown that such a process would end in a curvature singularity which can be either hidden behind a horizon or be visible to external observers. In the present work, we revisit the collapse process of a spherically symmetric homogeneous scalar field (in FRW background) minimally coupled to gravity, when the phase-space deformations are taken into account. Such a deformation is mathematically introduced as a particular type of noncommutativity between the canonical momenta of the scale factor and of the scalar field. In the absence of such deformation, the collapse culminates in a spacetime singularity. However, when the phase-space is deformed, we find that the singularity is removed by a non-singular bounce, beyond which the collapsing cloud re-expands to infinity. More precisely, for negative values of the deformation parameter, we identify the emergence of a negative pressure term, which slows down the collapse to finally avoid the singularity formation. Depending on the model parameters, one can find a minimum value for the boundary of the collapsing cloud or correspondingly a threshold value for the mass content below which no horizon would form. Such a setting predicts a threshold mass for black hole formation in stellar collapse and manifests the role of non-commutative geometry in physics and especially in stellar collapse and supernova explosion. [1–3].

Keywords: gravitational collapse, non-commutative geometry, non-linear differential equations, numerical method

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MATHEMATICS EDUCATION

Geometry study using the interactive whiteboard

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Abstract: Information technology can be used in different learning modes, and most importantly - in the mode of the graphic illustration of the studied material.

The effect of information technology on the secondary mathematics classroom has been studied by several authors ([1–4] and references therein).

The computer as a tracer has a number of advantages compared with compass and straightedge. So much for the image of bodies of rotation required to build the image of the circle is an ellipse. However, ruler and compass can construct an approximate image of the ellipse, it has not always been good quality. Using SMART Board interactive whiteboard and SMART Notebook software and other computer softwares, it can be created a wide variety of models of geometric figures, which is difficult in the case of material models, both technically and in material terms.

In geometry lessons students work a lot with graphic representation of spatial geometrical figures, which are not always clearly reflect their properties. Therefore, of particular interest are image editors allow teacher and students to create and modify computer models of geometric objects.

The potential of information technology in conducting computer simulation in order to independently obtain new knowledge about the geometric objects based on the study of computer models makes these technologies in learning an instrument of knowledge.

In this paper, methodocial aspects of using information technologies in the teaching geometry. The prospects of development of the learning environment in geometry study by using interactive whiteboard are revealed.

Keywords: information technologies, secondary school, teacher professional development, interactive whiteboard

2010 Mathematics Subject Classification: 97U30, 97U70

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Usage of recognizing algorithm handwriting in mathematical education

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Abstract: Improving quality of modern mathematical education can be achieved by using modern information technologies ([1–4]).

It is well known that recognizing algorithms handwriting can be divided to various types by objective of application.

In this work, we use Neural networks method. We propose software which based on recognizing algorithm. Proposed programm permit us to organize lessons, save education materials and view of created early lessons.

The applied programm is an addition tool for teachers and allows them to show students in class all handwritten, painted and tagged items.

The program permit us to save the classified lessons, drafts and help fast viewing.

Keywords: mathematics education, Neural networks method, recognating algorithm

2010 Mathematics Subject Classification: 97D40, 97U30

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Fundamentalization of mathematical knowledge while teaching students inverse and incorrect problem

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Abstract: Fundamentalization of mathematical knowledge of students of physical and mathematical areas of training in higher educational institutions is considered in the article [1, 2].

Training inverse and incorrect problems, the solution of which requires the development of a number of mathematical theories has a significant opportunity to improve the fundamental mathematical knowledge. To confirm this, the article presents several productions of inverse problems and methods of their study, included in the content of training inverse and incorrect problems of the future specialist of physical and mathematical profile [3–5].

Keywords: training of inverse and incorrect problems, mathematical education, competence in the field of applied mathematics, student.

2010 Mathematics Subject Classification: 65L09, 65M32

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Geometric heritage of al-Farabi in education

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Abstract: On the base of studying geometrical treatise of al-Farabi and A.Kubesov's works devoted to their research the expediency of introduction geometric heritage of al-Farabi to modern mathematics education due to its uniqueness, which is reflected in use of algorithmic approach in solving geometrical problems and its application-oriented direction is proved in the article.

Keywords: mathematical heritage, geometric construction, construction with help of compass and ruler, algorithm, algorithmic approach, application-oriented direction, mathematics education.

2010 Mathematics Subject Classification: 373+51.7+004

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Features and advantages of training teachers to use means of informatization to profile teaching mathematics

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Abstract: The usage advantages of modern means of informatization of mathematics education are discussed in the article. Aspects of training teachers to effective use of such means within the framework of profile teaching mathematics pupils are considered.

Keywords: profile education, mathematics, informatization of education, training teachers

2010 Mathematics Subject Classification: 373+51.7+004

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Value of new technologies in improving the branch of learning

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Abstract: The creation of electronic mathematical textbooks with multimedia devices and realization of school lessons using a computer is studied. A practical program that allows us to automate the use of new technologies in practical departments is being created. Practical program enables upgrading of schoolwork. The PHP web-programming language and MySQL information management system of practical program has been prepared.

The practical program offers new additional tools for school technique using modern technologies.

The practical program is an additional tool for teachers, it makes possible to show all the things shown, drawn and marked by teacher directly on students' computers. Besides, the program gives students the opportunity to carry out home works through this program. And it allows the teacher to identify academic student performance on a particular lesson or a specific topic. Because the teacher is able to not only see the result of the performed work, but also to monitor the performance of tasks from the beginning of the process to the end. Practical program gives some information about the user by using the handwriting recognition algorithms. Interpretation of lessons and works into the electronic form is made in the form of vector graphics. The difference between vector graphics and video inveterate method is that it takes 20-30 times less space and preserves the quality of the recording regardless of the demonstration screen. Therefore, it can increase the number of participants in electronic form during the lesson and spreading of these lessons in the program with ease.

Keywords: mathematics education, PHP, MySQL

2010 Mathematics Subject Classification: 97U30

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Formation of project and research skills of students in calculation of limits

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Abstract: Today different goals and objectives are qualitatively assigned to the educational system, which are defined by the dictated social order, by a new model of expert and professional. Consequently, it is necessary to form a personality, which is able to organize his professional career in the ever-changing socio-cultural conditions. Therefore, current priorities of higher education are aimed at development of specialist personality, the leading component of which is the possession of an independent creative and research activities [1].

According to the heads of project and research works, the most of students are not able to independently propose and substantiate a hypothesis, plan an activity, formulate a goal, search and analyze necessary information, represent results of research, realize reflection, competently write a report. This is due to the fact that not only the students but also their teachers are not trained to project and research activities [2].

The method of rationalization [3], applied to rationalize various irrational and trigonometric expressions, and hence to solve irrational and trigonometric equations and inequalities, to calculate limits, etc., is a generalization of the well-known idea about the use of rationalize integration substitutions [4], where these substitutions are widely used to rationalize the various classes of differential expressions by I. Newton, L. Euler, P. Chebyshew and other mathematicians.

Keywords: project-research activity, radical expression, rationalization, indeterminacy, rational

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Methods of professionally-oriented training of complex function theory

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Abstract: This article describes the interactive methods of professionally-oriented education of theory analytic functions for the purpose of formation of professional competence of students; The paper defined the concept of "professional competence" and "activity approach to professional-focused training to the mathematician" and the classification of interactive forms and methods of teaching.

Keywords:

2010 Mathematics Subject Classification: 97I80, 97I20, 97I30

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Student-centered model for teaching numerical methods course

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Abstract: The quality of modern education is strongly influenced by students' personal motivation. Long scientific and educational experience confirms that the success of students depends on various factors mainly related to their personal ambitions and preferences. The purpose of this work is to prove applicability of student-centered model for teaching "Numerical methods" course that weakens students' personal and psychological barriers to conscious material learning and contributes to their computational thinking development.

Keywords: student-centered model, computational thinking, "Numerical methods" course

2010 Mathematics Subject Classification: 97Q30, 97N99

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Didactic bases of system formation of learners' methodological knowledge

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Abstract: The article deals with the role and significance of methodological knowledge in education. Studied the basic sources, factors of educational content and the expected results of the inclusion of methodological knowledge in school education. Didactic conditions of formation and learning of methodological knowledge in teaching the basics of sciences of a school course. Analyzes the principles and criteria for acquiring methodological knowledge of learners.There are some demands representation of methodological knowledge in educational subject. The selection procedure of the complex methodological knowledge and its didactic and methodological foundation. [1]. On the basis of analysis of the main sources, shows that methodological knowledge are reflected in all elements of the content of education, at all levels in the form of the whole complex of knowledge. Methodological knowledge included in the content of education in an implicit form of knowledge: about the knowledge of scientific methods (universal, general scientific and private scientific or specific); logic of scientific knowledge (formal and dialectical); regularities of the process of scientific cognition; the logic of scientific activity. Methodological knowledge present in the content of education and in dissected forms. In studying academic subject is recommended to show the nature of scientific cognition and its effectiveness, specific techniques of every field of knowledge, and use those and other methods that have the general scientific and educational character. [2].

Keywords: methodological knowledge, school, teacher training, academic subject

2010 Mathematics Subject Classification: 00A35, 97C70

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Analysis of theoretical and methodological bases of teaching object-oriented programming languages in higher school

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Abstract. In this paper, we affect one of the most urgent calls of our time - problem of educating at higher school.

One of the most pressing challenges of our time is the problem of teaching in higher school. Training of young people in the sphere of the latest computer technologies is the teachers' task. When teaching teachers should change the methodology of training specialists in higher school in accordance with the adequately changing development pace. Software changes due to the appearance on the market of new information technologies. The subject area of computer science changes in an extremely dynamic way. Teachers of higher school that teach subjects associated with the technology of software creation are constantly faced with the problems of changing the content of curricula, working programs, methodical support (lectures, labs, tests, etc.), the development of new teaching and learning aids.

In connection with the application in the educational process of the various versions of educational software, teaching guides on the use of this software have to be accordingly changed. A certain bias towards the creation of software for complex systems has been recently observed; the tasks under consideration become much more complicated, so the old techniques and methods previously used to create simple or formalized programs are not enough. At the modern stage of social development, the needs of educational, industrial, commercial entities have significantly increased, and simple programs no longer meet them. Education of students of information specialties has not been provided yet with the necessary theoretical basis for the creation of complex software systems, the necessary subjects are often presented in the curricula fragmentary, in the list of elective courses.

Keywords: analysis, development, complex systems, information, object-oriented approach, model, education, methods, pedagogics, algorithmization

2010 Mathematics Subject Classification: 68N15

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History of mathematics and its teaching in higher school

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Abstract: Currently teaching hours for basic mathematics are reduced because of the education reform. This is typical for the specialty "*Mathematics*", and for the natural sciences and technical fields where mathematics is not critical discipline. This adversely affects the quality of trained specialists. We cannot change this situation generally. However, we would like somehow influence it.

Modern educational programs usually provide elective courses. Then we could offer a special non-standard mathematical course for the students. It will be able to interest them, is aimed at improving the general mathematical culture of pupils and, thus, to fill the numerous gaps in their mathematical education. It will be able to interest them, will aimed at improving the general mathematical culture of pupils and, thus, to fill the numerous gaps in their mathematical education. This is "*History of Mathematics*". We are teaching this course for many years at the Mechanics and Mathematics Faculty of al-Farabi Kazakh National University as an elective course. We recommend [1] as an appropriate textbook for it.

Keywords: history of mathematics, mathematical education

2010 Mathematics Subject Classification: 01A05

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Methodological questions of filling the virtual university program modules

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Abstract: Methodological questions of filling the virtual university program modules. Consider the questions filling Virtual University program modules with teaching tools. Here are the main teaching tools used in the content and also the types of computer training programs:

Work module tutorial (unit). Units are designed by each module, the average during one academic period. Each student receives 25 -27 unit and methodical recommendations for work with workers modular textbook.

Training didactic or methodical tools logically complement each other and enhance didactic functions of the discipline. General view of the virtual university materials must be more than 2000 items of educational products. Among them - the simulated movies, music videos, video lectures (review and modular), satellite TV lectures, author's lecture courses, slide lectures. Computer training programs . The university (virtual) should have more than 300 computer-based training programs for all academic disciplines. Work with such programs aimed at enhancing the student's mental activities, the formation and consolidation of skills.

Training programs represent one of the forms of independent work of students on the stage when a certain knowledge is already formed.

Collective training is a classroom group session aimed at updating and processing of professional skills and allows students unleash your creativity in an interactive way. Student Appraisal System. Continuous monitoring and assessment of assimilation quality of each module in accordance with standard criteria is also the original difference of this technology. Certification system is divided into a learning module, current and complete[1].

We note the characteristic features of virtual education: flexibility, modularity, parallelism, large audience, economical, workability, social equality, internationalization, the new role of the teacher, positive impact on student (listener), quality.

Creating information systems focuses on the complex means of interaction with the end user who performances a "smart interface" role on modern level and enables interactive solution of information problems on a computer. Note that for this purpose capabilities of the Web environment are widely used; this capabilities ensure the provision of a user interface for working with one or more databases. Thus, it can be seen that the known three-tier client-server architecture of the database is displayed naturally on the Web environment, where the Web - browser plays a role of a "thin" client, a Web-server - the role of the application server.

Keywords: supertyutor, proftyutor, training, certification, module, unit, integration, telecommunications, infrastructure, configuration, portal, environment.

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OTHER TOPICS

Furstenberg type theorem for a multiplicative product of Markov random matrices

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Abstract: In the work, considering the certain conditions of "general situation", Furstenberg type theorem is proved for multiplicative product of a special type of Markov random matrices.

The classical theorem of Furstenberg of strict positivity of the largest Lyapunov exponent for the product of a large number of independent matrices and its generalization to the case of the dependent (for example, forming a Markov chain) random matrices play a key role in the whole range of issues related to the spectral properties of disordered structures.

Currently, there are several different approaches to proof theorems of Furstenberg [1], [2], [3]. In this paper we prove the the Furstenberg type theorem for multiplicative product (in terms of [4] multiplicative integral) Markov random matrices of a special form.

Note that, the necessity of studying of such multiplicative products of random matrices arise in the problems of the evolution of the magnetic field in so-called Markov linear incompressible flows [5].

Keywords: random matrix, a multiplicative product, transition function, transition density, the formula of Kac-Feynman, the Lyapunov exponent

2010 Mathematics Subject Classification: 15B52, 37H15

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On asymptotic normality of solutions of the Cauchy problem of a parabolic equation with random right side

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Abstract: Consider the Cauchy problem for the heat equation

$$\frac{\partial u(t,x)}{\partial t} = \frac{1}{2}\frac{\partial^2 u(t,x)}{\partial x^2} + C(t,x), \quad u(0,x) = f(x), \tag{1}$$

where $t \geq 0,\ x \in \mathbb{R}, f(x),\ C(t,x)$are continuous bounded (generally speaking, random) functions. Then to solve the problem (1) we have

$$u(t,x) = M_x\left[f(W_t) + \int_0^t C(t-s, W_s)ds\right],$$

where W_s is Wiener process and sign M_x means taking the conditional expectation on all facing at the initial time t = 0 of point x, trajectories of W_t process. Next we consider the special case of the equation (1), namely, consider the equation

$$\frac{\partial u(t,x)}{\partial t} = \frac{1}{2}\frac{\partial^2 u(t,x)}{\partial x^2} + g(x)\dot{W}_t, u(0,x) = f(x), \tag{2}$$

where $t \geq 0,\ x \in \mathbb{R}, f(x), g(x)$ are limited continuous functions, $\dot{W}_t$ is "white noise" $[1,2]$.

Theorem 1. Let $f(x), g(x)$ be continuous and bounded functions. Then, if the function $g(x)$ at $|x| \to \infty$ satisfies relation $|g(x) - \sigma(x)| \to 0$, where $\sigma(x) = \sigma_1\ , x > 0;;\ \sigma(x) = \sigma_2\ ,\ x < 0$; then distribution of the random function $\frac{u(t,x)}{\sqrt{t}}$, where $u(t,x)$ is a solution of Cauchy problem given by equation (2), which at $t \to \infty$ converges to distribution of the normal random variable with parameters $\left(0, \frac{(\sigma_1+\sigma_2)^2}{4}\right)$.

Keywords: parabolic equation, Wiener process, white noise, asymptotic normality, infinitesimal operator

2010 Mathematics Subject Classification: 35R60, 60H15

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Modeling of thorium transport by the water flow in the river Kechi-Kemin

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Abstract: Radioactivity of the Kechi-Kemin river is mainly due to transported thorium. The problem of the spread of radioactive pollution of the river Kechi-Kemin and their impact on the life is an important task that requires permanent monitoring and research. An important part of such work is mathematical modeling.

The current talk presents the modeling of thorium transfer in the section of the river Kechi-Kemin. To solve this problem, we used the numerical method [1] for the Saint-Venant equation to describe behaviour of water flow. This method has used together with thorium advection-diffusion transfer model along the river watercourse. By the use of river regime information and thorium source data we have simulated the possible behavior of thorium propagation in water flow. This report contains a description of the methods and the main results of the calculations.

Keywords: Kechi-Kemin, radioactive, river modeling

2010 Mathematics Subject Classification: 35Q35, 35Q86

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One-types in ordered theories

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Abstract: Let A be a subset of a saturated enough model N of T. The analysis of one-types is based on using the properties of A-definable family of 1-formulas (**Points A, B**) [1], [2], [3], of 2-formulas (**Point C**) and properties of interactions of one-types (**Point D**) [4]. Taking into consideration the points **A, B, C** it is possible to show the existence of 18 kinds of one-types in ordered theories. The properties of of 1-formulas permits to give an example complete ordered theory with two non-orthogonal 1-types one is definable, second is non-definable.

Let $p, q \in S_1(A)$. It said to be p is *weakly orthogonal to* q ($p \perp^w q$) if if $p(x) \cup q(y)$ is complete two-type over A [5].

Definition 1. *Type p is definable if and only if $\forall\varphi(x, \bar{y})$ $\exists\psi_\varphi(\bar{y}, \bar{a})$, such that*

$$A \models \psi_\varphi(\bar{b}, \bar{a}) \Leftrightarrow \varphi(x, \bar{b}) \in p$$

Theorem 2. *There exist linear ordered structurel $M = \langle M; =, <, E^2, S^2\rangle$, infinite set $A \subset M$, two one-types over A, $p, q \in S_1(A)$ such that $p \not\perp^w q$, p is definable and q is non-definable.*

Keywords: ordered structures, 1-formula, 1-type.

2010 Mathematics Subject Classification: 03C64.

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On the Tietze extension theorem in soft topological spaces

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Abstract: The theory of soft set was introduced by Molodtsov in 1999 as a new mathematical tool for deal with uncertainties. The idea of soft topological space was first given by Shabir and Naz. The purpose of this paper is to prove Tietze Extension Theorem using Uryshon's Lemma in the so-called "soft topological space". After giving the necessary separation axioms based on the soft point definition in [3] such as soft completely regular space and soft normal space, we prove important theorem for soft separation axioms. Later we give Uryshon's Lemma. Finally we prove Tietze Extension Theorem using Uryshon's Lemma.

Keywords: Soft set, soft completely space, soft normal space, Urysohn Lemma and Tietze Extension Theorem.

2010 Mathematics Subject Classification: 54D15, 54D35

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A study on intuitionistic fuzzy supra soft topological spaces

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Abstract: The aim of this paper is to introduce some necessary definitions on the so called "intuitionistic fuzzy supra soft topological spaces". Furthermore, we give definitions of fuzzy strongly supra soft connected and fuzzy supra soft continuous mapping. We present characterization theorems of their.Finally, we define intuitionistic fuzzy supra soft compactness and invesitigate some results of intuitionistic fuzzy soft compactness.

Keywords: intuitionistic fuzzy supra topology, fuzzy strongly supra soft connected

2010 Mathematics Subject Classification: 03E75, 54A40

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The probability distribution function for the sum of squares of independent random variables

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Abstract: In the paper, the probability distribution function is derived for the sum of squares of random variables for nonzero expectations. This distribution function enables one to develop an efficient one-step algorithm for phase ambiguity resolution when determining the spatial orientation from signals of satellite radio-navigation systems. Threshold values for rejecting false solutions and statistical properties of the algorithm are obtained.

Keywords: probability distribution functions normal distribution, variance, squared standard deviation

2010 Mathematics Subject Classification: 60A05, 60C05

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Hyperimmunity and A-computable universal numberings

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Abstract: Whether there exists a computable universal numbering for a computable family is the key question in theory of numberings. In a very general setting, this problem was explored in [1]. For sets A that are Turing jumps of the empty set, the problem was treated in [2] and other papers.

It is well known that, in the classical case, every finite family of c.e. sets has a universal (principal) computable numbering.

S.A. Badaev and S.S. Goncharov have showed that for every set A such that $\emptyset' \leq_T A$, a finite family S of A-c.e. sets has an A-computable universal numbering if and only if S contains the least set under inclusion. But also they have showed that for an infinite family of A-c.e. sets if $\emptyset' \leq_T A$ then the presence of the least set under inclusion is neither necessary nor sufficient to have an A-computable universal numbering, [3].

We considered a family F of total functions, computable relative to oracle A. Taking into the account an infinite set A is hyperimmune iff no recursive function majorizes A, [4], we have showed that if A is a hyperimmune set and if A-computable (finite or infinite) family F of total functions contains at least two elements, then F has no universal A-computable numbering. However, if Turing degree of a set A is hyperimmune-free then every A-computable finite family of total functions has A-computable universal numbering.

Keywords: universal numbering, hyperimmune set, hyperimmune-free degree

2010 Mathematics Subject Classification: 03D45, 68Q15

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A system dynamics model to forecasting of the United States gross savings

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Abstract: This work presents a macroeconomic model for long-term forecasts describing compounding process of the gross saving on based the feedback loops of the theory of system dynamics and their empirical evidence on the statistical data of the U.S. economy [1] with the help of econometric analysis under different assumptions on the function of adjusting time. The system dynamics model for the forecasting indicators of economy, business and finance are widely used and they was studied detail in [2–9].

Indeed, to build the model compounding gross saving used of variables: $y_t > 0$, $t \in Z$ - a initial state, $y_{t+T} > 0$, $T = 1, 2, \ldots$ - a stock; $m \gg 1$ - a S-shaped limit, $r_t \geq 0$ - a growth rate; $y_{t+j}^{4/5} > 0$, $j = 0, 1, \ldots, T-1, \ldots$ - a reinforcing feedback, $1 - (y_{t+j}/m)^{1/5} > 0$ - a balancing feedback, $\Delta(t+j)$ - a function of adjusting time, $\Delta y_{t+j} = y_{t+j+1} - y_{t+j}$ - a endogenous flow, $r_t y_{t+j}^{4/5}(1 - (y_{t+j}/m)^{1/5})\Delta(t+j)$ - a exogenous flow and for stock-flow relation chosen the Von Bertalanffy logistic growth equation [10], then the forecasting problems of the gross savings was reduced to following econometric model:

$$\log_a(1-(y_{t+t}/m)^{1/5}) = \log_a(1-(y_t/m)^{1/5}) - r_t \sum_{j=0}^{T-1} \Delta(t+j)/(5m^{1/5}\ln a) + \varepsilon_{t+T-1},$$

$$E(\varepsilon_{t+T-1} \mid \mathrm{X}) = 0,$$
$$Var(\varepsilon_{t+T-1} \mid \mathrm{X}) = \sigma^2 \mathrm{I},$$
$$Cov(\varepsilon_{t+T-1}, \varepsilon_{t+S-1} \mid \mathrm{X}) = 0,$$
$$\varepsilon_{t+T-1} \mid \mathrm{X} \approx N(0, \sigma^2 \mathrm{I}),$$

where ε_{t+T-1} - random errors, X - a matrix of the observations, $E(\cdot)$ - a expectation, $Cov(\cdot)$ - a covariance, $Var(\cdot)$ - a variance, I - a identity matrix, $N(\cdot, \cdot)$ - a Normal distribution and $a > 0$, $a \neq 1$ - parameter for adjusting time and this parameter determined by Otelbaev average conditions [11]: $T_t = \inf_{T=1,2,\ldots} \left\{ T \mid r_t \sum_{j=0}^{T-1} \Delta(t+j)/(5m^{1/5}\ln a) \geq 1 \right\}$.

Keywords: model of system dynamics, von Bertalanffy growth model, econometric measures

2010 Mathematics Subject Classification: 91B62, 91B82, 93A30

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Selections, games and Karamata theory

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Abstract: We present various results which show how ideas from selection principles theory (developed in 1920s and 1930s by K. Menger, W. Hurewicz, F. Rothberger, W. Sierpiński) and corresponding game theory can be applied to the classical Karamata theory (1930s) for sequences of real numbers. These results are related to regularly and rapidly varying sequences and a number of their variations, as well as to rates of divergence of such sequences. Statistical versions of the mentioned notions will be also discussed.

Keywords: Selection principles, game theory, regular variation, rapid variation, rate of divergence

2010 Mathematics Subject Classification: 26A12, 40A05, 91A05

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Application of the theory of games for selection of effective defense

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Abstract: This article aims to prove that the use of games theory in decision making can aid in choosing the best methods of defense against attack weighed against the possible costs of such methods. Decision making in games theory assumes interaction between the seller or buyer with the environment as the environment fluctuates and the outcome is the expected payoff for such decisions. The decision maker makes decisions based on the possibility of risk and the expected payoff and whether or not the risk will be advantageous. The Wald Test is justified in situations where the possibility of environmental health is unknown, the solution occurs only once, and there is no risk taken. The Hurwitz pessimism-optimism theory is used when environmental information is missing or unreliable, recalculations must be made for each state of the environment, the number of implemented solutions is small, and some risk is allowed. Integrating these methodologies into the possible attacks and protections, we can evaluate the methods of protection in theory and in practice. Given the possible methods of defense being none, firewall, intrusion detection tools, and link redundancy, we can evaluate each method individually and in combination. Although the most effective method in theory would be to use firewall, intrusion detection tools, and link redundancy together, in practice the cost is prohibitive. If cost is factored into the equation, then the most effective method in theory is a firewall, however this fails to take into account the possible loss if an attack is successful. Taking into account this factor as well, a firewall works in a limited number of attacks, link redundancy for higher, and the most effective is to use all three methods in combination (firewall, intrusion detection tools, and link redundancy).

Keywords: game theory, Wald test, Hurwitz pessimism-optimism theory, security threat, firewall, link redundancy, intrusion detection tools, effective defense

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Binarity for almost ω-categorical quite o-minimal theories

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Abstract: The present lecture deals with the notion of *weak o-minimality*, which initially deeply studied in [1]. A subset A of a linearly ordered structure M is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \ldots \rangle$ such that any definable (with parameters) subset of the structure M is a finite union of convex sets in M.

In the following definitions (introduced in [2] and [3] respectively) we assume that M is a weakly o-minimal structure, $A \subseteq M$, M is $|A|^+$-saturated, and $p, q \in S_1(A)$ are non-algebraic types. We say that p is not *weakly orthogonal* to q ($p \not\perp^w q$) if there are an A-definable formula $H(x, y)$, $\alpha \in p(M)$, and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$. We say that p is not *quite orthogonal* to q ($p \not\perp^q q$) if there is an A-definable bijection $f : p(M) \to q(M)$. We say that a weakly o-minimal theory is *quite o-minimal* if the relations of weak and quite orthogonality for 1-types coincide.

Almost ω-categoricity has been introduced in [4] and studied in [5].

Theorem. Any almost ω-categorical quite o-minimal theory is binary.

Keywords: almost ω-categoricity, weak o-minimality, quite o-minimality, binary theory

2010 Mathematics Subject Classification: 03C64

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Prospective study on the potential of big data in railways

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Abstract: Now new information technologies are actively implemented and used for development of different areas of industry, transportation, including the railway industry. In our work solutions to the problems of railway industry based on Big Data are presented. Key terms, basic characteristics and stages of big data are presented. Global achievements in railway industry are analysed, provided examples of foreign authors, functional block diagram of big volumes of data is developed.

Keywords: big data, data analysis, railway engineering.

2010 Mathematics Subject Classification: 68U35, 68P01

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An Efficient Method for the Determination of Fourth Virial Coefficient with Lennard-Jones (12-6) Potential and Its Application

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Abstract: In this work, a new theoretical approach is proposed for calculating fourth virial coefficient with Leonard-Jones potential(12-6). The established algorithm can be used to evaluate the thermodynamics properties and the intermolecular interaction potentials of liquids and gases with an improved accuracy. Note that the evaluation of the high-order virial coefficients is very valuable for accurate calculation of thermodynamic parameters. By using the suggested method, the fourth virial coefficient of CH_4, Ar, C_2H_6 and SF_6 molecules are evaluated. The calculation results are useful for accurate interpretation of the experimental data and of the determination of related physical properties [1].

Keywords: Virial equation of state, Fourth virial coefficient, Lennard-Jones (12-6) potential

PACS No: 51.30.+i, 05.10.-a

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Analyzing High-Dimensional Survival Data Using Random Forests

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Abstract: Recently developments in genetics create cases that contain numerous covariates and few observations. These kinds of data are called High-dimensional data. In these cases, use of usual statistical methods is not appropriate. Especially in survival analysis, modeling of high-dimensional survival data is of utmost importance. One of the usual methods for analyzing survival data is the use of logistic regression. In this paper will be shown that the performance of logistic regression is not acceptable for analyzing high-dimensional survival data [1]. Generalized Breiman's Random Forests method for analyzing high-dimensional survival data, namely Random Survival Forests (RSF), has a strong efficiency for modeling survival data [2-3]. The main advantage of this method is that there is no distributional assumption on the regression model. Then the efficiency of RSF and usual methods for analyzing survival data such as logistic regression model are compared for modeling a real high-dimensional survival dataset and results are evaluated.

Finally using random forests and logistic regression models, a real dataset has been modeled and the accuracy of results are compared by Akaike Information Criterion [4]. It is shown that the random forests method and logistic regression model may presented different significant covariates. So, based on the data conditions the researchers should precisely choose the models.

Keywords: Random Survival Forests, Logistic regression, High-dimensional data, Survival analysis.

2010 Mathematics Subject Classification: 62F01, 62F02, 62F99

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Features in simulation of crystal growth using the hyperbolic PFC equation and the dependence of the numerical solution on the parameters of the computational grid

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Abstract: We investigate the three-dimensional mathematical model of crystal growth called PFC (Phase Field Crystal) in a hyperbolic modification. This model is also called the modified model PFC (originally PFC model is formulated in parabolic form) and allows to describe both slow and rapid crystallization processes on atomic length scales and on diffusive time scales [1, 2]. The modified phase field crystal model describes a continuous field of atomic density $\phi(x,t)$ and it can be expressed by the following equation:

$$\tau\frac{\partial^2\phi}{\partial t^2}+\frac{\partial\phi}{\partial t}=\nabla^2\mu, \tag{1}$$

where t is the time, τ is the relaxation time of the atomic flux to its stationary state, and μ represents the chemical potential, obtained from the free-energy functional

$$\mathcal{F}[\phi,\nabla\phi,\nabla^2\phi]=\int_\Omega\left[f(\phi)-|\nabla\phi|^2+\frac{1}{2}(\nabla^2\phi)^2\right]\mathrm{d}\Omega, \tag{2}$$

associated to the domain Ω. The chemical potential can be obtained as the variational derivative of the free-energy functional $\mathcal{F}$, namely

$$\mu(\phi)=\frac{\delta\mathcal{F}}{\delta\phi}=f'(\phi)+2\nabla^2\phi+\nabla^4\phi. \tag{3}$$

The function f represents the homogeneous part of the free energy density. It takes on the form

$$f(\phi)=\frac{1-\epsilon}{2}\phi^2+\frac{\alpha}{3}\phi^3+\frac{1}{4}\phi^4. \tag{4}$$

Here, $\epsilon=(T_c-T)/T_c$ is the undercooling, where T and T_c are the temperature and critical temperature of transition, respectively. α is a coefficient which means a measure of metastability.
The solution of this equation is possible only by numerical methods. Previously, authors created the software package for the solution of the Phase Field Crystal problem, based on the method of isogeometric analysis (IGA) [4] and PetIGA program library [5].
Using the modified PFC model for the simulation allows description of fast and slow dynamics of structure rearrangement from homogeneous unstable phase to the emerged periodic structure [3]. We consider that if the element size is larger than lattice parameter of the emerged structure on the interface, then the type of finite elements and its shape functions predetermine the behavior of the interface front movement, because of approximating of the atomic density inside the element using inappropriate functions. In this report, we show solutions for

both stationary and non-stationary problems which confirm our assumption of a necessary condition for the computational cell size: to be at least less then the lattice parameter.

This work was supported by the Russian Science Foundation [grant number 16-11-10095]. We also thank Immers Ltd. (Michurinskiy Ave, 19, bld. 3, 119192, Moscow, Russia, www.immers.ru) for technical support and ability to run numerous calculations on the newest computing cluster of their production.

Keywords: crystal growth, phase field crystal, isogeometric analysis, crystallization

2010 Mathematics Subject Classification: 74E15, 74S05

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The Moutard transformation of two-dimensional Dirac operators and the Mobius geometry

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Abstract: We discuss the Moutard type transformation for two-dimensional Dirac operators of the form

$$\mathcal{D} = \begin{pmatrix} 0 & \partial \\ -\bar{\partial} & 0 \end{pmatrix} + \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix}$$

with U a real-valued potential. In particular, we show that such a transformation has a very simple geometrical meaning. Given a solution of the equation $\mathcal{D}\psi = 0$ an a point $x_0 \in \mathbb{R}^3$, it maps the Dirac operator $\mathcal{D}$ into an operator $\tilde{\mathcal{D}}$ of the same form with another potential $\tilde{U}$ and gives an explicit procedure how to describe all solutions of the equation $\tilde{\mathcal{D}}\psi = 0$ in terms of solutions of the equation $\mathcal{D}\psi = 0$. However such a function ψ and a point x_0 determine a surface Σ via the Weierstrass representation such that U is the potential of the surface. The inversion of $\mathbb{R}^3$ maps Σ into a surface $\tilde{\Sigma}$ such that $\tilde{U}$ is the potential of the Weierstrass representation of $\tilde{\Sigma}$.

We use this transform for constructing blow-up solutions of the modified Novikov–Veselov equation which is a two-dimensional generalization of the modified Korteweg–de Vries equation.

Keywords: two-dimensional Dirac operator, Moutard transformation, inversion, modified Novikov–Veselov equation, blow-up.

2010 Mathematics Subject Classification: 35A30, 35Q41, 35Q53, 37K10, 53A30

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On stochastic stability of the integral manifold under permanently acting random perturbations

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Abstract: In this paper the stability in probability under an action of permanently acting random perturbations of the analytically given integral manifold is investigated. On the one hand these results generalize the theorems of stability under an action of permanently acting perturbations of the invariant set [1] in the class of ordinary differential equations. On the other hand they extend the theorems of stability under permanently acting perturbations of the unperturbed motion [2] to the case of invariant sets. To proof these theorems on stochastic stability under an action of permanently acting random perturbations of the analytically given invariant set we used the theorems in [3].

Keywords: integral manifold, stochastic stability

2010 Mathematics Subject Classification: 34H05,60H10

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Automorphisms of differential polynomial algebras

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Abstract: It is well known that all automorphisms of polynomial algebras and free associative algebras in two variables are tame. It is also known that polynomial algebras and free associative algebras in three variables over fields of characteristic zero have wild automorphisms.

The basic concepts of differential algebras can be found in [1]. Let $\Delta = \{\delta_1, \ldots, \delta_m\}$ be a basic set of derivations and let $A = \mathbf{k}\{x_1, x_2\}$ be the differential polynomial algebra in two free differential variables x_1, x_2 over an arbitrary field $\mathbf{k}$ of characteristic zero. We assume that $m \geq 2$, i.e., the number of basic derivations is at least two. Let θ be the automorphism of A defined by

$$\theta(x_1) = x_1 + \delta_2(w), \quad \theta(x_2) = x_2 + \delta_1(w), \quad w = \delta_1(x_1) - \delta_2(x_2).$$

This automorphism represents an analogue of the Anick automorphism for polynomial and free associative algebras in four variables. The Anick automorphism of free associative algebras in three variables is wild [2].

We prove that θ is a wild automorphism of A. The question on the tameness of automorphisms of ordinary (with only one basic derivation) differential polynomial algebras in two variables remains open.

Keywords: Automorphisms, polynomial algebras, differential polynomial algebras, derivations.

2010 Mathematics Subject Classification: 12H05, 14H37, 13N15

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Combining goodness of fit tests for multivariate normality

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Abstract: Multivariate statistical analyses often require the data to come from multivariate normal distribution. Vector-valued tests for multivariate normality were introduced in [1]. The authors proposed two-dimensional procedures by combining several scalar tests such as non-parametric, Wald's type chi-squared tests [2] and Henze-Zirkler test. Two types of rejection regions, intersection and union, were considered there. Their preliminary simulation study to compare powers of the tests was limited to the multivariate t distribution with 10 degrees of freedom as an alternative and rejection regions with equal weights for the marginal significance levels. In this work we investigate the power of the vector-valued tests as a function of the weight w for the rejection regions of intersection type. Scalar tests of different origins are combined to form two-dimensional vector-valued tests. Moreover, vector-valued tests based on Neyman-Pearson intervals are developed and their performance is investigated through simulation study. This study assesses power of the proposed testing procedures implemented in a contributed package `mvnTest` [3] for the statistic software R [4].

Keywords: vector-valued goodness of fit tests, multivariate normality, modified chi-square type tests

2010 Mathematics Subject Classification: 62H15, 62H10

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The stability of forcing companion for center of Jonsson set's fragment

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Abstract: Let T is Jonsson theory [1] complete for the existential sentences in the language L and C is its semantic model. Let X is Jonsson subset [**?**] of C and M is existentially closed submodel of semantic model C of considered Jonsson theory T, where $dcl(X) = M$. $Th_{\forall\exists}(M) = T_M$, where T_M is the Jonsson fragment of Jonsson set X.

Let $A \subseteq C$, $\sigma_\Gamma(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma$, where $\Gamma = \{P\} \cup \{c\}$. Consider the following theory $T_\Gamma(A) = Th_{\forall\exists}(C, a)_{a\in A} \cup \{P(c)\} \cup \{"P \subseteq"\}$, where $\{"P \subseteq"\}$ is an infinite set of sentences, which says that a interpretation of symbol P is positively existentially closed submodel in the signature σ. This theory is not necessarily complete. Through S_Γ denoted the set of all Σ-completions of theory $T_\Gamma(A)$. Theory T is said to be $P-\lambda$-stable, if $|S_\Gamma| \leq \lambda$ for any A, such that $|A| \leq \lambda$.

Consider all completions of the center T^* of the theory T in a new signature σ_Γ, where $\Gamma = \{c\}$. By virtue that T^* is Jonsson theory, there exist its center and we denote it so T^C. If restrictions T^C to the signature σ, that the theory T^C becomes a complete type. This type we call a central type of theory T.

Theorem 1. Let λ be an arbitrary infinite cardinal, T_M is perfect Jonsson theory, complete for positive existential sentences. Then the following conditions are equivalent:

(1) T_M^* is $P-\lambda$-stable;
(2) $\left(T_M^*\right)^F$ is λ-stable in the classical sense, where $\left(T_M^*\right)^F$ is forcing companion of theory T_M^* in enriched signature;
(3) T_M^C is λ-stable in the classical sense.

Keywords: Jonsson theory, Jonsson set, forcing companion, stable theory

2010 Mathematics Subject Classification: 03C05, 03C25, 03C45

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Categorical Jonsson fragments in existentially prime convex Jonsson theory

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Abstract: The theory Ò is called convex if for any its model $\mathfrak{A}$ and for any family $\{\mathfrak{B}_\mathfrak{i} \mid \mathfrak{i} \in \mathfrak{I}\}$ of substructures of $\mathfrak{A}$, which are models of the theory T, the intersection $\bigcap_{i \in I} \mathfrak{B}_\mathfrak{i}$ is a model of T. If it is assumed that this intersection is not empty. If this intersection is never empty, then the theory is called strongly convex.

The inductive theory Ò is called the existentially prime if

1. It has a prime algebraic (AP) model, (the class of its AP denote by T_{AP})
2. Class (E_T) non trivial intersects with class AP, i.e. $T_{AP} \bigcap E_T \neq 0$.

Let Jonsson theory T [1] is complete for the existential sentences in the language L and C is its semantic model.

Let X is Jonsson subset [**?**] in T, and M is existentially closed submodel of semantic model C of considered Jonsson theory T, where $dcl(X) = M$.

$Th_{\forall\exists}(M) = T_M$, T_M is the Jonsson fragment of Jonsson set X [**?**].

Theorem 1. Let theory T will be existential prime strongly convex Jonsson existentially complete theory. Then the following conditions are equivalent:

(1) T_M^* is ω- categorical;

(2) T_M is ω - categorical.

Theorem 2. Let T be an existential prime strongly convex Jonsson existentially complete theory. Then the following conditions are equivalent:

(1) T_M^* is ω_1 - categorical;

(2) any countable model E_{T_M} has a prime algebraic extension in E_{T_M}.

Keywords: Jonsson theory, Jonsson set, Jonsson fragment

2010 Mathematics Subject Classification: 03C05, 03C35

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Strongly minimal fragments in existentially prime convex Jonsson theory

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Abstract: Let Jonsson theory T [1] is complete for the existential sentences in the language L and C is its semantic model.

Let X is Jonsson subset [**?**] of T, and M is existentially closed submodel of semantic model C of considered Jonsson theory T, where $dcl(X) = M$.

$Th_{\forall\exists}(M) = T_M$, T_M is the Jonsson fragment of Jonsson set X [**?**] .

Let $I(E_T, \aleph_0)$ denotes the number of countable existentially closed models of Jonsson theory T. On strongly minimality for Jonsson sets in the frame of the Jonsson theories one can find in [**?**].

Theorem 1. If theory T is the strongly minimal existentially prime convex Jonsson theory complete for existential sentences, then T_M is categorical for $k \geq \aleph_1$ and $I(E_T, \aleph_0) \leq \aleph_0$.

Theorem 2. If theory T is Jonsson theory complete for the existential sentences is uncountable categorical and it has Jonsson strong minimal $\mathfrak{L}$-formula, then either T_M is $\aleph_0$- categorically or $I(E_{T_M}, \aleph_0) = \aleph_0$.

Theorem 3. If theory T Jonsson existentially prime convex theory complete for the existential sentence is uncountable categorical, but not $\aleph_0$-categorical, then $I(E_{T_M}, \aleph_0) \geq \aleph_0$.

Keywords: Jonsson theory, Jonsson set, forcing companion, stable theory

2010 Mathematics Subject Classification: 03C05, 03C25, 03C45

References:

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MS1

MATHEMATICAL MODELLING

Numerical solution to inverse source problems for hyperbolic equation

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Abstract: In this work, we consider two inverse source problems with respect to processes described by boundary value problems of hyperbolic type. In particular, these classes of problems arise in the study of boundary value problems with nonlocal (integral) boundary conditions. The specific character of the considered classes of inverse problems is that the identifiable coefficients belong to the right-hand side, and they depend only on the time or only on the space. This specific character allows reducing the problem to specially built Cauchy problems with respect to a system of ordinary differential equations using the method of lines [1–3]. Thus, we do not use any iterative procedures in the approach proposed in the work. Some of the results of the carried out numerical experiments are given. The obtained results show the efficiency of the proposed approach.

Keywords: inverse problem, method of lines, hyperbolic equation, space and time dependent

2010 Mathematics Subject Classification: 35R30, 35K20, 65N40, 65N21

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Implementation of scalar filtered density function for large eddy simulation of turbulent reacting flow using a high-order discontinuous Galerkin method

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Abstract: Turbulence is interesting and useful flow phenomena of nature. In case of reacting flows, turbulence promotes effective mixing of reactants. Due to fine mixing a chemical reaction is going to be highly productive. Hence, turbulent reactive flows are especially needed in industry. Nowadays engineering problems require highly accurate simulation of turbulent reacting flow. Therefore mathematical models and numerical methods must be sufficiently accurate.

In present work, a Filtered Density Function methodology (FDF) coupled with high-order Discontinuous Galerkin (DG) method is applied for Large Eddy Simulation (LES) of turbulent reacting flow. The FDF method has proven to be very effective for LES of turbulent reactive flows [1, 2] from the other side DG method is highly accurate and useful method, and at the same time is compact and relative easy [3]. In order to compare results there are provided results of computations of discontinuous Galerkin LES/FDF and finite difference direct numerical and large eddy simulations. In addition a DG-LES/FDF numerical code is parallelized with CUDA technology, which accelerated computations more that 10 times.

Keywords: filtered density function, high-order method

2010 Mathematics Subject Classification: 76F25, 76F65

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Dynamics of the massif in the vicinity of the tunnel any profile of section under action of transport loadings

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Abstract: The mathematical model has been considered of dynamics of not supported tunnels of any cross section of a deep pawning under the action of the transport loadings of the established profile moving with a constant subsonic speed (a speed is smaller then the speeds of elastic waves in the massif). For the circular tunnels such problems were considered in [1]. Here with use General Functions Method [2,3] the computer programs of calculation of the motion of the surrounding pedigree massif are developed and some calculations of dynamics of the surrounding massif are given in the vicinity of tunnels of circular and vaulted profiles of cross section.

The considered model and a computer programs allow to research the deflected mode of the massif in vicinities of transport tunnels taking into account a look and the speed of transport loading, mechanical parameters of the pedigree massif and design features of the construction. Especially it is important at design of lines of subways in an urban environment for an assessment of their seismic impact on land constructions and buildings.

Keywords: elastodynamics, boundary value problem, transport loading

2010 Mathematics Subject Classification: 74B05,45E99

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An asymptotical method for determining hydraulic resistance coefficient of gas-lift process

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Abstract: A mathematical model in oil production is formulated to determine the coefficient of hydraulic resistance [1], during the motion of the gas-liquid mixture in the lift. As it is known, the motions of the gas and gas-liquid mixture in the tubes are described by the system of partial differential equations of hyperbolic type in the asymptotic case [2]:

$$\left\{ \begin{array}{l} \frac{\partial P}{\partial t} = -\frac{c^2}{F}\frac{\partial Q}{\partial z}\varepsilon, \\ \frac{\partial Q}{\partial t} = -F\frac{\partial P}{\partial z}\varepsilon - 2aQ, \end{array} \right. \tag{1}$$

where the parameters in (1) defined as [3]. Applying the lines method, we obtain from (1)

$$\dot{x} = (A_0(\lambda_c) + A_1\varepsilon)x + B\varepsilon u + V\varepsilon,$$

with initial condition $x_0 = \left[P_1^0, Q_1^0,\ P_2^0,\ Q_2^0\right]'$. It is required to minimize the functional

$$f(\lambda_c) = \sum_{i=1}^{N}\left[\tilde{Q}_2^i - Q_2^i\right]^2.$$

On a concrete example the comparison of the values of the obtained hydraulic resistance coefficient with the statistical value of hydraulic resistance is given. It is shown that they differ from each other to the order 10^{-3}.

Keywords: gas-lift, the coefficient of hydraulic resistance, lines method

2010 Mathematics Subject Classification: 49J15, 49J35

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Modeling and calculation of flotation process in one-dimensional formulation

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Abstract: In the framework of the assumptions of the mechanics of the multiphase media is constructed a mathematical model of the flotation process in the dispersed mixture of liquid, solid and gas phases, taking into account the degree of mineralization of the surface of the bubbles. Application of the constructed model is demonstrated on the example of one-dimensional stationary flotation and it is shown that the equations describing the process of ascent of the bubbles are singularly perturbed ("hard"). Analyzed the effect of size and concentration of bubbles and the volumetric content of dispersed particles on the flotation process.

Nowadays the flotation method is widely used for division of disperse mixes in various industries [1-7] that stipulates an urgency of a considered problem. Modeling of the flotation process in the three-phase environment is a difficult task. Theoretical study of the process of flotation is based, in the main, on the kinetic model [1-5] where many important hydrodynamic factors are not considered. Consequently, it is necessary to construct the mathematical model of the flotation process with considering the influence of the hydrodynamic factors [8].

Keywords: Flotation process, multiphase media, bubble

2010 Mathematics Subject Classification: 76T10, 35Q35

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The Fujita type a critical exponent for a double nonlinear parabolic equation and system

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Abstract: Consider in $Q = \{(\mathrm{t}, \mathrm{x}) : \mathrm{t} > 0,\ x \in R^N\}$the following problem Cauchy to a degenerate parabolic system with double nonlinearity

$$(1)\qquad \begin{cases} \frac{\partial u}{\partial t} = \nabla\left(v^{m_1-1}\left|\nabla u\right|^{p-2}\nabla u\right) + div(c(t)u) + u^{\beta_1}, \\ \frac{\partial v}{\partial t} = \nabla\left(u^{m_2-1}\left|\nabla v\right|^{p-2}\nabla v\right) + div(c(t)v) + v^{\beta_2} \end{cases}$$

$$(2)\qquad u(0,x) = u_0(x) \geq 0,\ v(0,x) = v_0(x) \geq 0,\ x \in R^N$$

This problem describes the different nonlinear processes [1-4]. The solution of the problem (1), (2) depending on value of numerical parameters m_i, β_i , $i = 1, 2, p$and functions $c(t)$, $\gamma(t)$ have a property of a finite speed perturbation, space localization, blow-up and so on [1,2]. Now these properties of the solution to the problem Cauchy for a different nonlinear equation in the case single equation and system intensively studied by many authors (see [1-4] and the literature therein). Many works (see [1-3] and literature therein) are devoted to the Fujita type critical exponents for semi-linear system (1) ($m_i = 1,\ \beta_i\ ,\ i = 1, 2, p = 2$). In this case in [4] the following condition $(\beta_i + 1)/\beta_1\beta_2 - 1 = N/2,\ i = 1, 2$ of the critical exponents were established.

In this work an algorithm for establishing value of a critical exponent for the system (1) and for some other degenerate system based on approximately self similar approach is suggested. Using comparison principle the condition of a global solvability, property a finite speed of perturbation, space localization of the solution, blow-up is established. The following condition of a global solvability
$\tau(t)\bar{u}^{\beta_1-(p-2)}\bar{v}^{-(m_1-1)} < (\beta_1 - 1)N/p$, $\tau(t)\bar{u}^{-(m_2-1)}\bar{v}^{\beta_2-(p-2)} < (\beta_2 - 1)N/p$,
where $\bar{u}(t) = (T + \int\gamma(t)dt)^{-1/(\beta_1-1)}$, $\bar{v}(t) = (T + \int\gamma(t)dt)^{-1/(\beta_2-1)}$, $T \geq 0$, $\tau(t) = \int[\bar{u}(t)]^{(p-2)}[\bar{v}(t)]^{(m_1-1)}dt$
which consists in particular the result of th work [4] is established.

Keywords: double nonlinear, parabolic system, property, critical exponent

2010 Mathematics Subject Classification: 35B20, 35B33

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The features of a non-stationary state of stress in the elastic multisupport construction

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Abstract: The paper deals with the problem of propagation of unsteady elastic waves in an elastic multisupport construction, which is a rectangular strip. The mixed problem is formulated in terms of the stress and velocity and is numerically modeled using an explicit difference scheme through computation based on the method of spatial characteristics. The main objective of this study is to analyze the impact of the gap in the boundary conditions on the propagation of wave processes in the internal points of the studied elastic medium. The concentration of dynamic stresses was investigated in the vicinity of the gap of the boundary conditions. The results of the study were brought to the numerical solution.[1-8]

Keywords: Speed, voltage, load, plane strain, stress concentration, numerical solution, wave propagation

2010 Mathematics Subject Classification: 65L12, 74G70

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Modeling and solving the two-dimensional non-stationary problem in an elastic body with a rectangular hole

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Abstract: In this work we consider the problem of the propagation of non stationary stress waves in an elastic body with a rectangular hole in the linear formulation. The wave process is caused by applying an external dynamic load on the front boundary of the rectangular region and the lateral boundaries are free of the stress. The lower boundary of the rectangular region is rigidly fixed, and the contour of the rectangular hole is free from the stress. The problem is solved by using the difference method of the spatial characteristics. On the basis of the developed numerical methods it is obtained the computational finite - difference relations of the dynamic problems at the corner points of the rectangular hole, where the first and second derivatives of the unknown functions have a discontinuity of the first kind. We analyze the dynamic stress fields in an elastic body with a rectangular hole and we studied the concentration of dynamic stresses in the vicinity of the corner points of the rectangular opening.[1-8]

Keywords: speed, stress, load, plane strain, stress concentration, numerical solution, wave propagation

2010 Mathematics Subject Classification: 65L12, 74G70

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On the stability estimation of differential-difference analogue of the integral geometry problem with a weight function

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Abstract: A great deal of the work has been devoted to problems of integral geometry, that is, the problems of determining a function from its integral along a given family of curves. One of the stimuli for studying such problems is their connection with multidimensional inverse problems for differential equations [1]. In some inverse problems for hyperbolic equations were shown to reduce to integral geometry problems and, in particular, a problem of integral geometry was considered in the case of shift-invariant curves. Mukhometov [2] showed the uniqueness and estimated the stability of the solution of a two-dimensional integral geometry on the whole. His results were mainly based on the reduction of the two-dimensional integral geometry problem

$$V(\gamma, z) = \int\limits_{K(\gamma,z)} U(x,y)\rho(x,y,z)ds, \quad \gamma \in [0,l], \quad z \in [0,l] \tag{1}$$

where $U \in C^2(\overline{D}), \rho(x,y,z)$ is a known function to the boundary value problem

$$\frac{\partial}{\partial z}\left(\frac{\partial W}{\partial x}\frac{\cos\theta}{\rho} + \frac{\partial W}{\partial y}\frac{\sin\theta}{\rho}\right) = 0, \quad (x,y,z) \in \Omega_1, \tag{2}$$

$$W(\xi(\gamma), \eta(\gamma), z) = V(\gamma, z), \quad V(z,z) = 0, \quad \gamma, z \in [0,l]. \tag{3}$$

Differential - difference analogue of the problem (2)-(3) are studied. The stability estimate for the considered problem (2)-(3) are obtained.

Keywords: integral geometry problem, differential-difference problem, stability, uniqueness.

2010 Mathematics Subject Classification: 34K29

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Soft matrices on soft multisets in an optimal decision process

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Abstract: In this paper, we introduce a concept of a soft matrix on a soft multiset, and investigate how to use soft matrices to solve decision making problems. An algorithm for a multiple choose selection problem is also provided. Finally, we demonstrate an illustrative example to show the decision making steps.

Keywords: Soft matrix, soft multiset, soft max-min decision making

2010 Mathematics Subject Classification: 03E75, 03E99

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On a problem of reconstruction of a discontinuous function by its Radon transform

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Abstract: A problem of reconstruction of a discontinuous function by its Radon transform is considered. Almost all of mathematical models of tomography include objects with discontinuous properties, but the vast majority of the algorithms applying for reconstruction of a searched-for function require its smoothness or at least continuity. Thus an employment of such algorithms for reconstruction of discontinuous functions leads to errors that increase the errors arising usually at the reconstruction of smooth functions in 3–10 times. Hence a development of the algorithms and numerical tools for reconstruction of discontinuous objects by tomographic data is an important task. An expression "breaks recovery" includes few problems [1–3]. First of them is the problem of visualization of a set of breaking points. This problem consists in obtaining the image, on which the set of breaking points of a function can be recognize easily. A result of the second problem (the identification of discontinuities) should be mathematical description of the points or lines of discontinuities. The third problem consists in determination of jump values.

The problem of reconstruction of a set of breaking points is connected closely with the problem of reconstruction of discontinuous functions. One of approaches to the numerical solution for the problem of reconstruction of discontinuous functions reconstruction consists in the next sequential steps: a) a visualization of a set of breaking points; b) an identification of this set; c) a determination of jump values; d) an eliminations of discontinuities. We consider three of listed problems except for the problem of jump values determining. The problems are investigated by mathematical modeling with usage of numerical experiments. The descriptions of simulations and their results are represented. The results of simulation are satisfactory and allow us to hope for the further development of the approach.

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Keywords: tomography, Radon transform, discontinuous function, visualization of breaking points, identification, elimination of breaks, simulation

2010 Mathematics Subject Classification: 44A12, 65R32, 94A08

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A new combined DG-MC solver for large eddy simulation of reacting turbulent flows

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Abstract: In the presented article, a new computational methodology is developed and implemented to simulate turbulent reacting mixing layers of two opposite parallel flows. The large eddy simulation (LES) approach and the methodology named "filtered density function" (FDF) are used for turbulence modelling. In the FDF methodology, the effects of the unresolved scalar fluctuations are taken into account by considering the probability density function of subgrid scale scalar quantities. A big advantage of the methodology is that in the FDF transport equation the effect of chemical reactions appears in a closed form. The base filtered transport equations are solved numerically by a Discontinuous Galerkin (DG) method. The FDF transport equation is solved by a particle based lagrangian Monte-Carlo (MC) method. The power of DG methods is that it can provide high order accuracy with fewer degrees of freedom. It is shown that the developed DG-MC solver is a powerful tool for large eddy simulation of reacting turbulent flows.

Throughout this note we use techniques from the works [1], [2].

Keywords: : turbulence, filtered density function, large-eddy simulation, Monte Carlo method, discontinuous Galerkin methods

2010 Mathematics Subject Classification: 76F10, 76F65, 76D06, 65C05, 65L60

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On a method of mathematical modeling of the pyrolysis process to produce nanomaterials from natural gas

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Abstract: In this paper we developed a method for implementing a mathematical model of the methane catalytic pyrolysis to produce nanomaterials in periodic and continuous mode. It is shown that the Laplace transform method is mostly appropriate for this case, which is applicable both for the differential equations for the solid-phase components and for calculating the periodic mode. The adequacy of the resulting solution with known experimental data has been checked. Mathematic formulation is of the following mode :

$$\frac{\int \partial C_i}{\partial t} + v_1 \frac{\partial C_i}{\partial x} = D_i \frac{\partial^2 C_i}{\partial x^2} + D_i \frac{\partial^2 C_i}{\partial r^2} + \frac{D_i}{r}\frac{\partial C_i}{\partial r} + \alpha_{i-1} C_{i-1} + \beta_{i+1} C_{i+1} + J_i,$$

$$C_i(t=0,x,r) = C_i^0(x,r),$$

$$C_i(t=0,x,r) = C_i^L(r),$$

$$\frac{\partial C_i}{\partial x} \mid_{x=1} = 0,$$

$$D_i \frac{\partial C_i}{\partial r} \mid_{r=0} = \sum_{j=1}^{M_2} v_j^i W_j, if x = I_{ap}/2,$$

$$D_i \frac{\partial C_i}{\partial r} \mid_{r=0} = 0, if x \neq I_{ap}/2,$$

$$D_i \frac{\partial C_i}{\partial r} \mid_{r=D_{ap}/2} = 0,$$

where $i = 1, 2, ..., n; I_{ap}, D_{ap}$- device length and diameter, $m; \tilde{N}_i^L$ - gas constituent original concentration at the reactor inlet, mole/ $m^3; v_i^j$ gas phase i-component stoichiometric coefficient in the j-surface reaction; W_j- j-surface reaction velocity mole/(m^3, s) (1) - modified equation system for the gas constituent concentrations with the optional term, allowing for the interim radicals concentrations impact; $\alpha_0; \alpha_{n+1} = 0$; (2) - initial conditions. In this Article we prove the solvability of the model (1) - (7), solubility and the iterative method stability in solving the system differential equations (1).

Keywords: catalytic pyrolyze, differential equation, Laplace transform. **2010 Mathematics Subject Classification:** 94B05, 94B15

Mathematical modeling of the thermal effect on the aquatic environment from thermal power plant by using two water discharged pipes

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Abstract: In this work presents a mathematical modeling of the thermal load on the aquatic environment by using two water discharged pipes of TPP. It is solved by the Navier-Stokes and temperature transport equations for an incompressible fluid in a stratified medium. [1], [3], [4]. Numerical method is based on the projection method [2] which was approximated by the finite volume method. The numerical solution of the equation system is divided into four stages. At the first step it is assumed that the momentum transfer was carried out only by convection and diffusion. Intermediate velocity field is solved by 5-step Runge-Kutta method. At the second stage, the pressure field is solved by found the intermediate velocity field. Poisson equation for the pressure field is solved by Jacobi method. The third step is assumed that the transfer is carried out only by pressure gradient. The fourth step of the temperature transport equation is also solved as momentum equations. The obtained numerical results of stratified flow for two water discharged pipes were compared with experimental data and with numerical results for one water discharged pipe. General thermal load in the reservoir-cooler decreases comparing with one water discharged pipe and revealed qualitatively and quantitatively approximately the basic laws of hydrothermal processes.

Keywords: Stratified medium, Navier-Stokes equation, Thermal discharge, Operational capacities of thermal power plant

2010 Mathematics Subject Classification: 76D05, 76D50, 65N08

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Mathematical modeling of non-equilibrium sorption

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Abstract: We consider the system of equations modeling the process of non-equilibrium sorption. Difference approximation of differential problem by the implicit scheme is formulated. The solution of the difference problem is constructed using the sweep method. Based on the numerical results we can conclude the following: when the relaxation time decreases to 0, then the solution of non-equilibrium problem tends with increasing time to solution of the equilibrium problem.

Keywords: The system of equations of non-equilibrium sorption, difference approximation, the implicit scheme, sweep method, numerical experiments.

2010 Mathematics Subject Classification: 35Q35, 65M06, 76S05

Exact solution of two phase spherical Stefan problem with two free boundaries

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Abstract: Solution of the heat equation in a spherical domain with two free boundaries (two-phase Stefan problem) when one of the subdomains degenerates at the initial time is considered. The use of conventional finite-difference methods in these cases is not expedient because of the degenerate domain. The solution is found in the form of combination of Integral Error functions series, [1] and then recurrent solvability of nonlinear algebraic equations for determining the coefficients of the series is proved.

The mathematical model includes the spherical heat distribution with constant coefficients for each of the phases

$$\frac{\partial\theta_1}{\partial t} = a_1^2\left(\frac{\partial^2\theta_1}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_1}{\partial r}\right), \qquad \alpha(t) < r < \beta(t), \tag{1}$$

$$\frac{\partial\theta_2}{\partial t} = a_2^2\left(\frac{\partial^2\theta_2}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_2}{\partial r}\right), \qquad \beta(t) < r < \infty, \tag{2}$$

with initial condition

$$\theta_1(r,0) = 0, \tag{3}$$

$$\theta_2(r,0) = f(r), \tag{4}$$

subjected to free boundary conditions

$$\theta_1(\alpha(t),t) = \theta_m, \tag{5}$$

$$-\lambda_1\frac{\partial\theta_1\big(\alpha(t),t\big)}{\partial r} = Q, \tag{6}$$

where $Q = \frac{P(t)}{2\pi\alpha^2(t)}$

$$\theta_1(\beta(t),t) = \theta_2(\beta(t),t) = \theta_m, \tag{7}$$

the Stefan's condition

$$-\lambda_1\frac{\partial\theta_1(\beta(t),t)}{\partial r} = -\lambda_2\frac{\partial\theta_2(\beta(t),t)}{\partial r} + L\gamma\frac{d\beta}{dt}, \tag{8}$$

as well as the condition at infinity

$$\theta_2(\infty,0) = 0. \tag{9}$$

To solve this problem we firstly reduce it to the linear problem. Then suggest exact solution in the form of Heat polynomials and series combination of integral error functions whose coefficients have to be determined.

Keywords: Stefan problem, degenerate domain, Integral Error functions.
Mathematics subject classification: 80A22

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Mathematical modeling of the temperature dependence of molten viscosity of metal according to the concept of chaotized particles

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Abstract: Employees of the Chemical and Metallurgical Institute J.Abisheva proposed a new approach based on the Boltzmann distribution [1]. According to this approach, all three aggregate states can be viewed from the perspective of subordination Boltzmann distribution and to connect virtually every state with important characteristics of matter on the basis of excess or excess energy barriers melting and boiling. As in all cases considered particles, differing only in the amount of energy of chaotic motion, then their union and differential display can be described as the concept of chaotized particles. According to the concept of chaotized particles, crystal sliding and steam sliding, liquid sliding particles are present in each of the states of aggregation of matter.

The temperature dependence of viscosity fusion is defined on the basis of the concept of randomized particles in the full range of the liquid state on the uniform model considering degree of association of the elementary clusters of dynamically existing crystal mobility particles. The received form of temperature dependence of viscosity can be used for calculation of energy of activation of a viscous current fusion in a combination with Frenkel's equation. A new semi-empirical model of viscosity tested on 28 common metals for which there are reference data on the temperature dependence of viscosity. Obtained high values of correlation coefficients for the proposed model points to its functional characteristics. On this basis there were recommended calculated dependence for each metal.

Keywords: viscosity, chaotic particles, degree of associative clusters, the temperature dependences of the viscosity, liquid metals, fixed point

2010 Mathematics Subject Classification: 97M10

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The mathematical models of electromagnetic field dynamics and heat transfer in closed electrical contacts including Thomson effect

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Abstract: We represent mathematical models of electromagnetic field dynamics and heat transfer in closed symmetric and asymmetric electrical contacts including Thomson effect, which are essentially nonlinear due to the dependence of thermal and electrical conductivities on temperature [1]. Suggested solutions are based on the assumption of identity of equipotential and isothermal surfaces, which agree with experimental data [2] and valid for both linear and nonlinear cases. Well known Kohlrausch temperature-potential relation is analytically justified and agrees with [3-4].

Keywords: Thomson effect, symmetric and asymmetric electrical contacts

Mathematics subject classification: 74Nxx

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Solvability of a stationary problem of magnetohydrodynamics

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Abstract: This work is devoted to study the following stationary problem of magnetohydrodynamics consisting in finding the functions $\vec{v}(x)$, $p(x)$, $\vec{H}(x)$ and $\vec{E}(x)$ in $\Omega \subset R^3$:

$$-\nu\Delta\vec{v} + (\vec{v}\cdot\nabla)\vec{v} - \frac{\mu}{\rho}(\vec{H}\cdot\nabla)\vec{H} + \frac{1}{\rho}\nabla(p(x) + \frac{\mu}{2}|\vec{H}|^2) = \vec{f}(x), \quad x \in \Omega, \tag{1}$$

$$div\vec{v}(x) = 0, \quad x \in \Omega, \tag{2}$$

$$rot\vec{H}(x) - \sigma(\vec{E}(x) + \mu[\vec{v} \times \vec{H}]) = \vec{j}(x), \quad x \in \Omega, \tag{3}$$

$$div\mu\vec{H}(x) = 0, \quad x \in \Omega, \tag{4}$$

$$rot\vec{E}(x) = 0, \quad x \in \Omega, \tag{5}$$

$$\vec{v}(x)|_S = 0, \vec{E}_\tau(x)|_S = 0, \vec{H}\cdot\vec{n}|_S = 0. \tag{6}$$

Here $\vec{n}$ is the unit outward normal to S, and $\vec{E}_\tau = \vec{E} - \vec{n}\left(\vec{n}\cdot\vec{E}\right)$. $\Omega \subset R^3$ is the bounded domain with smooth boundary S.

Using the results in [1]- [3], we prove unique solvability of (1)-(6) in Sobolev and Hölder spaces.

Keywords: magnetohydrodynamics, generalized solution, stationary problem

2010 Mathematics Subject Classification: 76W99, 35D30, 60G10

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Mathematical modelling and control theory analysis of artificial satellite

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Abstract: In this paper, an analysis of Mathematical Control Theory is carried out which concludes controllability of artificial satellites. Control Theory is application-oriented mathematics that deals with the basic principles underlying the analysis and design (control) systems. To control means that one has to influence the behaviour of the system in a desirable way. A preliminary discussion is provided which acts as stairs for reaching final conclusions and includes an analytical approach to model the physical system into dynamical equations of motions which is termed as Mathematical Modelling. Since most of the real life systems are nonlinear in nature therefore an approach for linearization is described. By doing so, we will able to apply numerous linear analysis methods to study further behaviour of the system. Then a mathematical approach used to compute Transition Matrix for getting solution of linear system by using variation of parameter method. Throughout the analysis we are looking for minimum norm (optimal) steering controls,we will also introduce linear oprator technique for calculating the steering controls of the system, after then we made some interesting conclusions by using Kalman's Controllability test. Finally, a graphical simulation is provided using MATLAB which includes computations of controllability Grammian and controller with related MATLAB codes.

Keywords: Control Theory, Transition Matrix, Optimal Control, Kalman's Controllability test.

New effects for high speed collision of blunt bodies with atmospheric inhomogeneities

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Abstract: The results of numerical investigation of collision of blunt bodies moving in the atmosphere with supersonic speeds with local inhomogeneities – low density and high density gas bubbles are presented. The effect of appearing of anomalous pressure and density jumps in a critical point of a body essentially exceeding values calculated for 1D Riemann problem for the interaction of a bow shock wave with plain layer is studied. In case of the interaction of blunt bodies with low density bubbles the effect was previously observed but was not enough clearly explained in [1]. Similar results were obtained both experimentally and theoretically for the interaction of blunt bodies with high temperature plasma formations induced by pulsed laser energy deposition in supersonic upstream flow.

Now it is shown that the main reason for appearing of anomalous pressure and density jumps in a critical point of a body is the preceding fast process of focusing of transversal shock wave in small zone on the symmetry axes ahead of a body which is accompanied in some cases by formation of thin axial high speed cumulative jets. The mechanism of focusing processes was in general agreement with well investigated shock focusing scenarios for the interaction of plane shock with low density and high density gas bubbles [2] providing of course taking into account some specific of this particular case due to the curvature of a bow shock wave and the existence of a body closely near the focusing region.

For low density gas bubbles according to the "divergent" scenario of the interaction the "weak focusing" regime was realized. The secondary weak convergent-type shock wave propagating inside a shock layer was focused along the symmetry axes. Shortly after that the pressure and density jumps in a critical point of a body were induced by a retransmitted shock wave reflected from the rear interface of a gas bubble which was amplified by defocusing shock wave propagating from the shock focusing region.

For high density gas bubbles the "strong focusing" regime was realized. According to "convergent" scenario of the interaction the bow shock wave was diffracted around the bubble. Then the strong converging diffracted bow shock wave and the transmitted shock wave were collapsed in a very small zone (almost a point) at the rear pole of the bubble on the symmetry axes. The collapsing process was accompanied by formation of thin supersonic upstream and downstream cumulative jets. The interaction of downstream jet with a body surface was resulted in dramatic pressure and density jumps.

Parametric investigations for ellipsoidal gas bubbles of different density ratio, size and elongation were carried out. It was concluded that to realize the cumulation effect the density ratio should be more than some critical value, the gas bubble shape should be closed to spherical and the longitudinal size – about a bow shock wave distance. There were no focusing or cumulation effects for the interaction with a flat layer or ellipsoidal gas bubbles of too large cross size.

The investigations were carried out in 2016 in the Institute of Mechanics of Moscow State University (project AAAA-A16-116021110195-4) and were financially supported partially by the Council on Grants of the President of Russian Federation (project NSh-8425.2016.1), Russian Science Foundation (project 14-11-00773) and the Russian Foundation for Basic Research (projects 14-01-00891-a, 16-29-01092-ofi).

Keywords: supersonic flow, numerical simulation, blunt body, gas bubble, shock focusing, cumulation

2010 Mathematics Subject Classification: 76M20

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Initiation of 3D Detonation in Supersonic Flows in Channels of Special Geometry

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Abstract: The problem of the formation of detonation in 3D channel with square variable cross-section have been investigated. Detailed stationary and unsteady flow patterns with and without detonation have been found. The critical values of gasdynamic parameters separating different flow regimes were obtained. Hypersonic analogy of flat and 3D flows, allowing the use of 2D solutions to estimate the parameters of three-dimensional supersonic flows, have been confirmed. The limitations of the analogy concerned with chocking effect of the burning flow have been determined. A numerical study of detonation of the combustible mixture in 3D cylindrical channels with helical obstacles of various shapes have been conducted: 1) with a twisted lateral surface of elliptic cross-section; 2) with a screw body of elliptic cross-section, located at the axis of the channel; 3) with the stellate body composed of a number of twisted thin blades located at the axis of the channel; 4) with a number of twisted thin blades attached to the lateral surface of the channel, inducing an analogy with a rifled barrel in the artillery. All of these forms of static 3D channels with the obstacles can be obtained from the 2D rotating regions through their simultaneous movement along the axis of the channel and rotation. According to the hypersonic analogy, the distribution of parameters, which arise in the solution of plane problems with rotation, should be close to the distributions of the parameters in cross-sections of the 3D channel with helical elements upon injection of supersonic flow of combustible mixture in the channel. As a result of numerical calculations of unsteady flows in the specified channels, detailed stationary flow patterns with detonation waves near the obstacles have been obtained. These waves had complicated structure and interacted with each other and with the channel walls. The comparison with 2D calculation was made and the above-mentioned analogy was confirmed. The study is carried out in a framework of one-step combustion kinetics by numerical method based on the S.K. Godunov scheme. The original parallelized software package with graphical interface was developed and used to perform numerical investigation. The calculations have been performed on the MSU "Lomonosov" supercomputer.

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Keywords: detonation of the combustible mixture, combustion kinetics

2010 Mathematics Subject Classification: 76M20

Supersonic multi-species flow with particle dispersion on a mixing layer

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Abstract: Particle-laden flows play an important role in high-speed technologies such as solid rocket propulsion systems and high-speed fuel combustors. The flow physics in such devices is very complex due to shock dynamics, turbulence and particle dispersion in mixing layers. Direct numerical simulation (DNS) has been successfully used for compressibility effects in the turbulent shear layer [1], single-phase and two-phase flows [2]. The particle dispersions in the mixing layers for the different Stokes numbers have been obtained in [3]. In spite of that there are a few investigations of a compressible multispecies shear layer flow with the dispersion of particles.

This work is our first stage of the simulation in a comprehensive study of the flow-particle interactions in a three-dimensional multispecies turbulent medium. The direct numerical simulations of the flowfield structures and the properties of the particle dispersion in the quasi-2D turbulent mixing layer (hydrogen-air) are performed by solving the time-dependent, compressible Euler equations. The 3D numerical code using the high-order essentially non-oscillatory (ENO) scheme is developed. The dispersion of the particles is studied by following their trajectories in the mixing layer with the Lagrangian method. In detail, the effect of the initial mass fraction of hydrogen and the number of particles on the growth of vortices and their thickness is studied. The simulation reveals that the capturing of the particles by the vortices essentially depends on the density of particles.

Keywords: supersonic shear flow, mixing layer, particle dispersion, multispecies flow, ENO-scheme.

2010 Mathematics Subject Classification: 02.60.Cb, 02.70.Bf

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Modeling and solving an optimization problem of supply of chemical raw materials

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Abstract: In this article the optimization problem of supply of the chemical raw materials is solved in the following formulation. The trading company is specialized in the supply of the chemical raw materials to the industrial enterprises. The delivery is carried out by self - the transport of buyers. It is given the nomenclature data set of the chemical raw materials, the cost of its purchase, delivery and warehousing. The profit from the sale of each type of the chemical raw materials is fixed. The financial resources for the planning period are limited. In addition, on the basis of existing and potential customer's orders for the planning period, the limits on the number of purchased raw materials of every kind were defined.How many of each type of the chemical raw materials is necessary to buy for the company to get the best profit under the condition that all the purchased raw materials will be sold in the planning period? The problem is solved by the method of the linear programming. At the same time a mathematical formulation of the problem is carried out. This means that the controlled variables and the target function of the problem are identified. Then, by means of these variables the objective function and constraints are described in the form of linear relations. [1-3].

Keywords: Quantitative methods in management, linear programming, numerical optimization, objective function, simplex method, Mathcad, Qbasic

2010 Mathematics Subject Classification: 65K05, 90C05, 90B50

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Fragmenting of algorithm for solving the problem of clustering miRNA families

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Abstract: The division of miRNAs into families does not adequately reflect the degree of nucleotide sequence similarity, and thus the categorisation of miRNAs into families requires quantitative criteria defining the differences between miRNA families. The genomes of different organisms have orthologous miRNAs that should be distributed into families. Than it is necessary to establish the degree of similarity of orthologous miRNAs and their belonging to different families. The present research is aimed at solving the problem of clustering miRNAs based on their nucleotide sequence similarity using the fragmented programming method.

Using this developed method, the miRNA nucleotide sequences of Arabidopsis thaliana were clustered. The results identified a conserved polynucleotide, which serves as the Ath-miR156a-j binding site in paralogous SPL mRNAs. These nucleotides encode the conserved ALSLLS motif and the miR156 and miR157 subfamilies belong to the same family. The human miR-1273 family includes miRNAs with very different nucleotide sequences; therefore, based on our program they belong to different miRNAs families.

Keywords: miRNA family, algorithm, fragmented programming method.

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Iterative method of finding hydraulic conductivity characteristics of soil moisture

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Abstract: The work considers the initial boundary value problem for a nonlinear equation of hydraulic conductivity. A method of finding a nonlinear diffusion coefficient was developed and hydraulic conductivity of soil moisture was found. Numerical calculations were conducted.

Keywords: moisture conduction, inverse problem, conjugate problem, difference scheme, diffusion coefficient

2010 Mathematics Subject Classification: 80A20, 35K55, 65M06, 65M32

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Spectral properties of a Laplace operator with Samarskii–Ionkin type boundary conditions in a disk

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Abstract: In this paper we consider a spectral problem for a model differential operator of the second order with involution. The operator is given by a differential expression

(1) $$Lu = -u''(-x), -1 < x < 1$$

and boundary conditions of the general form:

(2) $$\alpha_j u'(-1) + \beta_j u'(1) + \alpha_{j1} u(-1) + \beta_{j1} u(1) = 0, \;\; j = 1, 2.$$

Coefficients of the boundary condition are numbers, which can be complex.

A criterion of the basis property of systems of eigenfunctions of the operator is set in the sense of coefficients of the boundary conditions. The interest to studying the basis properties of operators of the form (1), (2) has arose relatively recently. In works of T.Sh. Kal'menov and his disciples the *self-adjointness* property of the Cauchy operator:

(3) $$L_K u = -u''(-x), \; -1 < x < 1, \quad u'(-1) = 0, \;\; u(-1) = 0$$

is successfully applied in solving initial-boundary value problems for a heat equation with inverse time direction, in finding a criterion of well-posedness of the Cauchy problem for the Poisson operator, in formulating an asymptotic solution to problems with a small parameter.

Keywords: generalized spectral problem, involution, eigenfunctions, eigenvalues, basis property

2010 Mathematics Subject Classification: 34L10, 34L15

Spectral properties of a Laplace operator with Samarskii–Ionkin type boundary conditions in a disk

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Abstract: One of the most important problems in the theory of harmonic functions are Dirichlet and Neumann boundary value problems. In the one-dimensional case, or when considering the problem in a multidimensional parallelepiped, the main problems include also the periodic boundary value problems.

In this paper we consider spectral properties of a Laplace operator with boundary conditions of Samarskii-Ionkin type in a disk and prove the completeness of eigenfunctions.

Let $\Omega = \{z = (x, y) = x + iy \in C : |z| < 1\}$ be a unit disk, $r = |z|, \varphi = \arctan(y/x)$, $\Omega^+ = \Omega \cap \{y > 0\}$, and $\Omega^- = \Omega \cap \{y < 0\}$. We consider the spectral problem corresponding to the Laplace operator

$$-\Delta u(z) = \lambda u(z), |z| < 1$$

with local boundary conditions

$$u(1, \varphi) = 0,\ \ 0 \leq \varphi \leq \pi,$$

and with one of the nonlocal boundary conditions

$$\frac{\partial u}{\partial r}(1, \varphi) - \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0,\ \ 0 \leq \varphi \leq \pi,$$

or

$$\frac{\partial u}{\partial r}(1, \varphi) + \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0,\ \ 0 \leq \varphi \leq \pi.$$

We note that unlike the one-dimensional case the system of root functions of the problems consists only of eigenfunctions.

Keywords: Poisson equation, Samarskii–Ionkin type problem, eigenfunctions, eigenvalues

2010 Mathematics Subject Classification: 35J05, 35J25, 35P10

Analytical solution of two-phase spherical Stefan problem by heat polynomials and integral error functions

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Abstract: On the base of the Holm model [1], we represent two phase spherical Stefan problem and its analytical solution, which can serve as a mathematical model for diverse thermo-physical phenomena in electrical contacts [2]. Spherical Stefan problem is reduced to linear problem with degenerate domain. Suggested solution is obtained from Integral Error Function and its properties which are represented in the form of series whose coefficients have to be determined:

$$u_1(x,t) = \sum_{n=0}^{\infty} A_{2n}(2a_1\sqrt{t})^{2n}\left[i^{2n}erfc\frac{-x}{2a_1\sqrt{t}} + i^{2n}erfc\frac{x}{2a_1\sqrt{t}}\right]$$

$$+\sum_{n=0}^{\infty} A_{2n+1}(2a_1\sqrt{t})^{2n+1}\left[i^{2n+1}erfc\frac{-x}{2a_1\sqrt{t}} - i^{2n+1}erfc\frac{x}{2a_1\sqrt{t}}\right], \qquad (1)$$

$$u_2(x,t) = \sum_{n=0}^{\infty}(2a_2\sqrt{t})^{n}\left[B_n i^{n}erfc\frac{-x}{2a_2\sqrt{t}} + C_n i^{n}erfc\frac{x}{2a_2\sqrt{t}}\right]. \qquad (2)$$

We find free interphase boundary, unknown coefficients A_{2n+1}, A_{2n}, B_n and C_n from boundary conditions utilizing Faa Di Bruno formula, Bell polynomials and Lebniz differentiation formula. To simplify calculations, we use Hermite polynomials to represent series (1) in the following form:

$$u_1(x,t) = \sum_{n=0}^{\infty} A_{2n}\sum_{m=0}^{n} x^{2n-2m}t^m B_{2n,m} + \sum_{n=0}^{\infty} A_{2n+1}\sum_{m=0}^{n} x^{2n-2m+1}t^m B_{2n+1,m}.$$

Convergence of solution series is proved.

Keywords: Stefan problem, integral error function, heat polynomials

Mathematics subject classification: 80A22

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The mathematical modeling of grouping the dipole water clusters

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Abstract: In the paper, a physical-mathematical model and a computational algorithm implementing the model are proposed to study the behavior of particles having an electric dipole moment in an external electric field. Computational experiments demonstrate the orientation dynamics of water clusters with the increase of the generated field. The dipole properties of some water clusters were previously determined using Hyperchem program.

Keywords: electric dipole, water clusters, physical-mathematical model, computational experiment, Runge-Kutta methods

2010 Mathematics Subject Classification: 93A30, 70-08

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Simulation of shock wave boundary layer interaction in flat channel with jet injection

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Abstract: The flow around jets has been comprehensively investigated experimentally [1] and simulated numerically. Nowadays the interactions between boundary layer and shock wave in the channel flow field with perpendicular jet injection aren't well understood. Due to complexity of these interactions, usually they are studied by parts. One part of these studies focuses on the properties of the shock wave structures in the region of the jet injection [2], other considers the zone of the shock wave boundary layer interaction [3]. In the present we simulate and study the shock wave structures in the supersonic channel flow with jet injection where both processes will be present. For describing the process of the multispecies supersonic gas flow in the flat channel with perpendicular jet injection, Favre-averaged Navier-Stokes equations coupled with $k-\omega$ turbulence model are applying. High order Weighted Essentially Non-Oscillatory (WENO) scheme is used to approximate convective terms. During the investigation of flow physics in detail, the three shock wave structures are observed: in the region of the jet (barrel, bow, oblique and closing shocks), on the upper boundary layer (reflection, transmitted and reattachment shocks), and new structures behind the jet on the lower boundary layer, which are analogous to the structures on the upper boundary layer.

Keywords: supersonic flow, SWBLI, flow control, separation, Navier-Stokes equations, WENO-scheme, shock wave

2010 Mathematics Subject Classification: 02.60.Cb, 02.70.Bf

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Application of of fuzzy logic for matching modes use of wind power plant with an electrical load schedule

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Abstract: This article proposed method, based on the theory of fuzzy sets, taking into account the uncertainty of wind power, as an energy carrier, it is a random uncontrollable natural process, for for matching modes use of wind power plant with an electrical load schedule. - *in Absheron, Typical for Caspian coast.* The average annual of wind speed of this zone is more than $4m/sec$.

Wind power unit runs on unmanaged schedule, in connection with the many uncertainties and unpredictable parameters. The consumer often does not allow for interruptions in the supply of energy.

Bases of the analysis mode of operation the wind turbine in fuzzy-defined conditions based on the principle of constructing a fuzzy model based on binary and conventional fuzzy relationship. The first of these fuzzy relations based on two basic sets X and Y, and the second – on two basic sets Y and Z. Here X – it describes a number of characteristics of wind power, taken on the Beaufort scale, Y - the set of characteristics of power output, and Z - set of daily intervals load. Fuzzy relation L - describes development of capacity at a certain wind turbine, and N - a covering daily load. For the construction of fuzzy relation N of a covering daily load function used concept of a conditional fuzzy subset:

$$\mu(y_i, z_j) = max(min(\mu(y_i, z_j), \mu(y_i))). \tag{1}$$

Using the method of construction of fuzzy relations, provided possibility of comparing produced cabacity to the load schedule, based on which it can be concluded about the use of electric energy.

Keywords: alternative energy resources, wind power plant, solar baterries, wind power, wind speed, wind energy, wind linguistic change, construction method of uncertain relation, the theory of fuzzy sets.

2010 Mathematics Subject Classification: 94D05, 00A71

Numerical solution of $2D$-vector tomography problem using the method of approximate inverse

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Abstract: The problem of vector tomography is considered in this work in the following formulation. Let a certain vector field $\mathbf{v}$ be given in a bounded domain of $\mathbb{R}^2$ filled by a medium in which the probing radiation propagates along straight lines. We have to reconstruct this field from its known longitudinal and (or) transverse ray transforms.

We propose an algorithm for solution of the vector tomography problem. The algorithm based on so called method of approximate inverse developed by A.K. Louis and his pupils [1–3]. The idea of the method of approximate inverse is as follows. Let $A : H \to K$ be a linear bounded operator. It is required to find an approximate solution (a function f) of the operator equation $Af = g$ for given $g \in K$. Mollifiers e^y_γ are used for solving. This functions have a properties $\langle e^y_\gamma, e^y_\gamma \rangle_H = 1$ and $\langle f, e^y_\gamma \rangle_H \approx f(y)$. Let A^* be an adjoint operator for A. Hence, equation $A^* \psi^y_\gamma = e^y_\gamma$ has a solution $\psi^y_\gamma \in K$ and

$$f(y) \approx \langle f, e^y_\gamma \rangle_H = \langle f, A^* \psi^y_\gamma \rangle_H = \langle Af, \psi^y_\gamma \rangle_K = \langle g, \psi^y_\gamma \rangle_K.$$

The authors were supported partially by the Russian Foundation for Basic Research (project 14-01-31491-mol_a)

Keywords: approximate inverse, vector tomography, ray transform, solenoidal vector field, potential vector field

2010 Mathematics Subject Classification: 44A12, 65R10, 65R32, 94A08

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Duality property of the noncommutative ℓ_∞ and ℓ_1 valued symmetric Hardy spaces

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Abstract: In this paper we consider the noncommutative $H_E(\mathcal{A};\ell_\infty)$ and $H_E(\mathcal{A};\ell_1)$ spaces and obtain some result on duality for these spaces, i.e we obtain the following result:

Theorem. Let E be an r-convex symmetric Banach function space on $[0;1]$ for some $0 < r < \infty$ and E do not contain c_0 or separable. Then

$$(i)\ \ (H_E(\mathcal{A};\ell_1))^* = L_{E^\times}(\mathcal{M};\ell_\infty)/J(H^0_{E^\times}(\mathcal{A};\ell_\infty))$$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_E(\mathcal{A};\ell_1)$ and $y \in H_{E^\times}(\mathcal{A};\ell_\infty)$, where $J(H^0_{E^\times}(\mathcal{A};\ell_\infty)) = \{x^* : x \in H^0_{E^\times}(\mathcal{A};\ell_\infty)\}$.

$$(ii)\ \ (L_E(\mathcal{M};\ell_1)/J(H^0_p(\mathcal{A};\ell_1)))^* = H_{E^\times}(\mathcal{A};\ell_\infty)$$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_E(\mathcal{A};\ell_1)$ and $y \in H_{E^\times}(\mathcal{A};\ell_\infty)$, where $J(H^0_{E^\times}(\mathcal{A};\ell_1)) = \{x^* : x \in H^0_{E^\times}(\mathcal{A};\ell_1)\}$.

This theorem is analogue of Theorem 5 in [1].

Keywords: von Neumann algebra, subdiagonal algebras, noncommutative vector valued symmetric Hardy spaces, duality.

2010 Mathematics Subject Classification: 46L51, 46L52.

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Lateral boundary conditions for the Klein-Gordon-Fock equation

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Abstract: In this paper we consider an initial-boundary value problem for the Klein-Gordon-Fock equation. In [**?**], Authors considered one dimentional potential

$$u(x,t) = \int_{\Omega} (-\frac{1}{2}|x-y|) f(y) dy \tag{1}$$

in $\Omega = (0,1)$ and the equation

$$-u^{''}(x) = f(x), \;\; x \in \Omega, \tag{2}$$

and they showed that if they are solved the equation (2) with the boundary conditions

$$u^{'}(0) + u^{'}(1) = 0, \;\; -u^{'}(1) + u(0) + u(1) = 0. \tag{3}$$

then they were finded an unique solution of that boundary value problem in the form (1).

This simple method finds equivalent boundary value problems of ODE for one dimensional potential integrals. However this task becomes tedious for PDE, and in works [1, 2] are obtained boundary conditions of the volume potentials for elliptic and hyperbolic equations and showed some their applications. In particular, in [2], by using a new non-local boundary value problem, which is equivalent to the Newton potential, Authors found explicitly all eigenvalues and eigenfunctions of the Newton potential in the 2-disk and the 3-ball. The aim of this work is to find lateral boundary conditions for the Klein-Gordon-Fock equation as shown in [**?**].

Keywords: Potential, fundamental solution, Klein-Gordon-Fock equation

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25, 35P05

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A modelling the dynamics of in-situ leaching process at the microscale

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Abstract: Some physical and chemical processes take place at interfaces between the solid and the liquid. One of them is in-situ leaching, an effective and promising method of extraction of valuable components from ores by their selective dissolution. To explore the main features of the movement of fluids media and dissolution mechanism of rocks by acids, it is considered a mathematical models at the microscopic level [1], using equations of continuum mechanics [2] and well-known chemical laws. The important point here is a derivation of new conditions at the free boundary and the dynamics of this boundary.

The efficient and accurate numerical simulation at the microscale plays an important role in understanding of the basic physical properties of rocks leaching processes. The application of complex numerical algorithms on a huge grids leads to the high demands on computer resources. Therefore, the numerical simulation of exact mathematical models is based on parallel computing.

Keywords: solid-liquid interface, microscopic dynamics, parallel computing

2010 Mathematics Subject Classification: 76D45, 76S05

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MS2
SPECTRAL THEORY
OF
DIFFERENTIAL
EQUATIONS

The first regularized trace of integro-differential Sturm-Liouville operator on the segment with punctured points at integral perturbation of transmission conditions

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Abstract: The report is devoted to calculating a first regularized trace of one integro-differential operator with the main part of the Sturm-Liouville type on a segment with punctured points at integral perturbation of "transmission" conditions. The Sturm-Liouville operator

$$-y''(x) + q(x)y(x) + \gamma \int_0^\pi y(t)dt = \lambda y(x)$$

given on the segments $\frac{\pi}{n}(k-1) < x < \frac{\pi}{n}k$, $k = \overline{1,n}$; $n \geq 2$ is considered. Boundary conditions of the Dirichlet type:

$$y(0) = 0, \quad y(\pi) = 0$$

are given on the left-hand and right-hand ends of the segment $[0,\pi]$. The functions continuous on $[0,\pi]$, the first derivatives of which have jumps at the points $x = \frac{\pi}{n}k$, are solutions. The value of jumps is expressed by the formula

$$y'\left(\frac{\pi k}{n} - 0\right) = y'\left(\frac{\pi k}{n} + 0\right) - \beta_k \int_0^\pi y(t)dt, \quad k = \overline{1, n-1}.$$

The basic result of the report is the exact formula of the first regularized trace of the considered differential operator.

Keywords: spectral problem, first regularized trace, integro-differential operator, inner-boundary condition

2010 Mathematics Subject Classification: 34L10, 34L15, 34L20

On the solvability of differential equations in logarithmic scale of Hilbert spaces

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Abstract: Consider a Hilbert space H and selfadjoint semibounded operator Λ with lower bound $\mu > 1$. This operator generates the scale of Hilbert spaces $H^\alpha = D(\Lambda^\alpha)$ with the norm $\|u\|^{(\alpha)} = \|\Lambda^\alpha u\|$.

With H^α we consider the logarithmic scale of Hilbert spaces L^s with norms $\|u\|_{(s)} = \|(\ln \Lambda)^s u\|$.

Consider the Cauchy problem

(1) $u''(t) - Au(t) = f(t), \quad u(0) = u_0, \quad u'(0) = u'_0,$

where $A : H \to H$ is an unbounded linear operator. We assume that A is a continuous operator from L^s to L^{s+2}.

Let $C([0,T] \to H^\alpha)$ denote the space of all continuous functions $u : [0,T] \to H^\alpha$.

Theorem 1. *Let $\alpha > 0$ and $T > 0$. For any $f \in C([0,T] \to H^\alpha)$ and $u_0, u'_0 \in H^\alpha$ there exists unique solution of the Cauchy problem (1) from $C([0,T] \to H^\alpha)$.*

As application to the hypersingular integro-differential equations consider $H = L_2(\mathbb{R}^n)$ and $\Lambda f(x) = F^{-1}(1+|\xi|^2)^{1/2} F f(x)$, where F is Fourier transform. In this case H^α coincides with Sobolev space $W_2^\alpha(\mathbb{R}^n)$.

Let A be the singular integral operator with kernel (see [1], page 50)

(2) $K(x,y) = [(x-y) \otimes (x-y)]|x-y|^{-n-2}, \quad x, y \in \mathbb{R}^n, n \geq 3.$

The integral operator with kernel (2) maps L^s to L^{s+2} continuously (see [2], Proposition 3.5). Moreover, the convolution $K * K$ has the same property. Hence, it follows from theorem 1 that the corresponding equation has solution in $W_2^\alpha(\mathbb{R}^n)$ for any $\alpha > 0$.

Keywords: integro-differential equation, singular integral, peridynamics.

2010 Mathematics Subject Classification: 45J05, 46N20.

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On spectral problems for loaded two-dimension Laplace operator

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Abstract: In domain $Q = \{x, y : -\pi < x < \pi, \ -\pi < y < \pi\}$ we consider following two spectral problems:

$$(1) \quad \begin{cases} -\triangle\varphi(x,y) + \alpha\varphi(0,y) = \lambda\varphi(x,y), \ \ \{x,y\} \in Q, \\ \dfrac{\partial^j\varphi(-\pi,y)}{\partial x^j} = \dfrac{\partial^j\varphi(\pi,y)}{\partial x^j}, \ \ \dfrac{\partial^j\varphi(x,-\pi)}{\partial y^j} = \dfrac{\partial^j\varphi(x,\pi)}{\partial y^j}, \ \ j = 0, \, 1; \end{cases}$$

$$(2) \quad \begin{cases} -\triangle\varphi(x,y) + \alpha\varphi(0,y) + \beta\varphi(x,0) = \lambda\varphi(x,y), \ \ \{x,y\} \in Q, \\ \dfrac{\partial^j\varphi(-\pi,y)}{\partial x^j} = \dfrac{\partial^j\varphi(\pi,y)}{\partial x^j}, \ \ \dfrac{\partial^j\varphi(x,-\pi)}{\partial y^j} = \dfrac{\partial^j\varphi(x,\pi)}{\partial y^j}, \ \ j = 0, \, 1; \end{cases}$$

where Δ is Laplace operator, α, $\beta \in \mathbb{C}$ are given complex numbers, $\lambda \in \mathbb{C}$ is a spectral parameter.

Note that the need to investigate of spectral problems (1) and (2) arise in solving of the stabilization problem for a loaded heat equation with help of the boundary control actions.

Depending on the values of α and β, we give a complete description of the eigenvalues, eigenfunctions and associated functions of the set of spectral problems (1) and (2).

Keywords: Loaded Laplace operator, spectrum, eigenfunction

2010 Mathematics Subject Classification: 35J05, 35P05, 35R10

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Spectrum of Volterra integral operator of the second kind

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Abstract: In this paper we consider the singular Volterra integral equation with spectral parameter $\lambda \in \mathbb{C}$ of form

$$\varphi(t) - \lambda \int_0^t K(t,\,\tau)\,\varphi(\tau)\,d\tau = f(t), \quad t > 0, \tag{1}$$

where

$$K(t,\,\tau) = K^{(1)}(t,\,\tau) + K^{(2)}(t,\,\tau), \tag{2}$$

$$K^{(1)}(t,\,\tau) = \frac{1}{2a\sqrt{\pi}} \cdot \frac{t^\omega + \tau^\omega}{(t-\tau)^{\frac{3}{2}}} \exp\left(-\frac{(t^\omega + \tau^\omega)^2}{4a^2(t-\tau)}\right), \tag{3}$$

$$K^{(2)}(t,\,\tau) = \frac{1}{2a\sqrt{\pi}} \cdot \frac{t^\omega - \tau^\omega}{(t-\tau)^{\frac{3}{2}}} \exp\left(-\frac{(t^\omega - \tau^\omega)^2}{4a^2(t-\tau)}\right), \ \omega\, 1/2. \tag{4}$$

We call such equations as the Volterra integral equations with 'incompressible' kernel [1]. It is shown that the corresponding homogeneous equation on $|\lambda| \geq \exp\{|\arg\lambda|\}, \quad \arg\lambda \in [-\pi, \pi]$ has a continuous spectrum, and the multiplicity of the characteristic numbers grows with increasing $|\lambda|$. We use the Carleman-Vekua regularization method. We introduce the characteristic integral equation. We prove that the initial integral equation has eigenfunctions, the multiplicity of which depends on the value of the spectral parameter λ. We prove the solvability theorem of the nonhomogeneous equation (1)–(4) in a case when the right-hand side of the equation belongs to a certain class.

Keywords: Volterra integral equation, spectrum, eigenfunction

2010 Mathematics Subject Classification: 45C05, 45D05, 45E10

References:

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On a regular problem for an elliptic-parabolic equation with a potential boundary condition

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Abstract: In this paper we construct a lateral boundary condition for an elliptic-parabolic equation in a finite domain. Theorem on existence and uniqueness of the solution of the problem considered are proved by method of theory potential. For construction of problem we use the Bitsadze-Samarskii problem with boundary conditions.

Bitsadze-Samarskii conditions binds internal traces $u(x,0)$ and $\frac{\partial u}{\partial \tau}(x,0)$ with boundary conditions $u(x,t)$ and $\frac{\partial u}{\partial \tau}(x,t)$ on the boundary of the domain.

Theorem *For each*

$$f(x,t) \in C^{\alpha}(\overline{D^-}) \bigcap C^{\alpha}(\overline{D^+})$$

Bitsadze-Samarskii problem has a unique solution $u(x,t)$ and

$$u(x,t) \in C^{\alpha}(D) \bigcap C^{2+\alpha}(\overline{D^-}) \bigcap C^{2+\alpha,1+\alpha}_{x,t}(\overline{D^+}).$$

Throughout this note we mainly use techniques from our works [1-3].

Keywords: an elliptic-parabolic equation, Bitsadze-Samarskii problem, lateral boundary condition

2010 Mathematics Subject Classification: 35Q79, 35K05, 35K20

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Structure of fractional spaces generated by the two dimensional neutron transport operator

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Abstract: In this study, the structure of fractional spaces generated by the two-dimensional neutron transport operator A defined by formula $Au = \omega_1 \frac{\partial u}{\partial x} + \omega_2 \frac{\partial u}{\partial y}$ is investigated.The positivity of A in $C\left(R^2\right)$ and $L_p\left(R^2\right),\ 1 \leq p < \infty$ is established. The structure of fractional spaces generated by the operator A is studied. It is established that for any $0 < \alpha < 1$ and $1 \leq p < \infty$ the norms in the spaces $E_{\alpha,p}\left(L_p\left(R^2\right),\ A\right)$ and $E_\alpha\left(C\left(R^2\right),\ A\right),\ \ W_p^\alpha\left(R^2\right)$ and $C^\alpha\left(R^2\right)$ are equivalent, respectively. The positivity of the neutron transport operator in Hölder space $C^\alpha\left(R^2\right)$ and Slobodeckij space $W_p^\alpha\left(R^2\right)$ is proved.

Keywords: neutron transport operator, positive operator, fractional spaces, Hölder spaces, Slobodeckij spaces

2015 Mathematics Subject Classification: 94B05, 94B15

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Green's function of the second order differential operator with involution

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Abstract: In the present paper, we are interested in the Green's function of the differential operator L defined by formula

$$Lu = \alpha u''(x) - u''(-x) - \lambda u(x), -1 < x < 1 \tag{1}$$

with Dirichlet conditions

$$u(-1) = 0, u(1) = 0 \tag{2}$$

Note that a function $u(x)$ is called a *solution* of boundary value problem with the positive parameter λ for the differential equation

$$Lu = f(x), -1 < x < 1 \tag{3}$$

with Dirichlet conditions (2), if it satisfies this equation (3) and boundary conditions (2). Here $f(x)$ is a continuous function defined on [-1,1].

A function $G(x,t;\lambda)$ is called the *Green's function* of the differential operator L defined by formula (1) and conditions (2), if the function

$$\int_{-1}^{1} G(x,t;\lambda) f(t)\, dt$$

is a solution of boundary value problem (2)-(3).

The main findings of the report: we have the following point wise estimates for the Green's function $G(x,t;\lambda)$:

$$|G(x,t;\lambda)| \leq \frac{M(\alpha)}{\sqrt{\lambda}} \begin{cases} e^{-c_0(\alpha)\sqrt{\lambda}(-t-|x|)}, -1 \leq t \leq -|x|, \\ e^{-c_0(\alpha)\sqrt{\lambda}(|x|-|t|)}, -|x| \leq t \leq |x|, \\ e^{-c_0(\alpha)\sqrt{\lambda}(t-|x|)}, |x| \leq t \leq 1. \end{cases}$$

Keywords: differential equation with involution, Green's function

2010 Mathematics Subject Classification: 34B27, 34L10, 34L15

Bounded perturbations of the correct restrictions and extensions

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Abstract: This paper considers the correct restrictions of the maximal operator. We obtain a criterion of relative-boundedness of the perturbations of the inverse operator to a fixed correct restrictions. Given an applications of this criterion to some differential operators.

Under a weak perturbation of the compact operator T will be called the operator $T(I+S)$, where S is such a compact operator that the operator $I+S$ is continuously invertible. A lot of work (see [1], [2], [3]– [7]) is devoted to the spectral properties of a weak perturbation of a compact operator T.

If S is a bounded operator, then the perturbed operator will be called the bounded perturbation. This work is devoted to describe among the correct restrictions and extensions of the bounded perturbations of the fixed operator.

Keywords: bounded perturbations, correct restrictions, correct extensions

2010 Mathematics Subject Classification: 46Nxx, 47Fxx, 35P05

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On spectrum of perturbed two-dimensional harmonic oscillator in a strip

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Abstract:

Let $\Pi = \{(x,y), x \in \mathbb{R}, 0 \le y \le \pi\}$, $L_0 = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + x^2$. We denote by V the operator of multiplication by bounded measurable real-valued function $V(x,y)$, which is finite on the variable x. That is, there exists $r > 0$ with the property: $V(x,y) \equiv 0$, for any x, $|x| \ge r$. In our further considerations such functions $V(x,y)$ are said to be *finite in the strip* Π. We denote $\operatorname{supp} V$ by $\Pi_0(r)$. Here, we deal with the Dirichlet problem operator $L = L_0 + V$ in the space $L^2(\Pi)$. Notice that the title of the paper is just prompted by the possibility to consider the expression $(\widetilde{V}(x,y) + y^2)$, $y \in [0,\pi]$, instead of $V(x,y)$, without loss of generality.

Let $P_s^{(1)}$ $P_l^{(2)}$ be orthogonal projectors onto eigensubspaces of one-dimensional Laplace operators of Dirichlet problem and harmonic oscillator corresponding to the eigenvalues s^2, $s = 1, 2...$, and $2l+1$, $l = 0, 1, 2...$, correspondingly. That is, $P_s^{(1)} f = (f, f_s) f_s$, $f_s(y) = \sqrt{\frac{2}{\pi}} \sin sy$ $\;P_l^{(2)} f = (f, \varphi_l)\varphi_l$, $\varphi_l(x) = (2^l l! \sqrt{\pi})^{-\frac{1}{2}} e^{-\frac{x_2}{2}} H_l(x)$, where $H_l(x)$ are Hermitian polynomials.

Then, we have the following assertion on the eigenvalues $\lambda_{sl} = s^2 + 2l + 1$, $s = 1, 2, ..., l = 0, 1, 2, ...$ of the operator L_0

Theorem 1. The spectrum of the operator L_0 consists of the eigenvalues $\lambda_n = n$, $n \in \mathbb{N} \setminus \{1, 3\}$, with multiplicities ν_n

$$\nu_n = \begin{cases} [\frac{\sqrt{n}}{2}], & \lambda_n \in \left((2[\frac{\sqrt{n}}{2}])^2; (2[\frac{\sqrt{n}}{2}]+1)^2\right) \\ [\frac{\sqrt{n}}{2}] + \frac{(-1)^n + 1}{2}, & \lambda_n \in \left((2[\frac{\sqrt{n}}{2}]+1)^2; (2[\frac{\sqrt{n}}{2}]+2)^2\right). \end{cases}$$

The corresponding projector operators are $P_n = \sum\limits_{s=1}^{\nu_n} P_s^{(1)} \otimes P^{(2)}_{(\frac{n}{2} - \frac{s^2+1}{2})}$.

For each $n \in \mathbb{N} \setminus \{1, 3\}$ we denote by $\mu_s^{(n)}$, $s = 1, \dots, \nu_n$, the eigenvalues of the operator L corresponding to the eigenvalue λ_n of the operator L_0. We have established that there exists $d > 0$ with the property: for $n >> 1$ the inequalities $\left|\mu_s^{(n)} - \lambda_n\right| \le dn^{-\frac{1}{2}}$ hold for any $s = 1, \dots, \nu_n$.

Using the method of our paper [1] we get our main result:

Theorem 2. Assume that $V(x,y) \in C^4(\Pi_0(r))$. Then,

$$\sum_{k\in\mathbb{N}\setminus\{1,3\}}\left[\sum_{s=1}^{\nu_k}\left(\lambda_k-\mu_s^{(k)}\right)+\operatorname{sp}P_kV\right]=\frac{1}{12\pi}\iint\limits_{\Pi}V^2(x,y)dxdy.$$

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Dirac systems that contain discontinuity conditions

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Abstract: Boundary value problems which contain transmission conditions inside a finite interval have been studied in [1–3]. This type of problems may have discontinuities in the solution or its derivative at an inner point of the interval and arise in many areas of natural sciences. For example, heat and mass transfer, mechanics, electronics, radio, geophysics. Direct and inverse problems for Dirac differential equation with transmission conditions have been studied in [4–6]. The aim of this paper is to study a Dirac system which contain transmission conditions inside a finite interval.

Keywords: Dirac systems, transmission conditions, discontinuous boundary-value problems, eigenvalues, eigen-vector functions

2010 Mathematics Subject Classification: 34L05, 34L40.

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Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

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Abstract: This report is devoted to a spectral problem for a multiple differentiation operator with an integral perturbation of boundary conditions of one type which are regular, but not strongly regular:

$$l(u) \equiv -u''(x) = \lambda u(x),\ 0 < x < 1, \tag{2}$$

$$U_1(u) \equiv u'(0) - u'(1) + \alpha u(0) = \int_0^1 \overline{p(x)} u(x) dx,\ p(x) \in L_2(0,1), \tag{3}$$

$$U_2(u) \equiv u(0) - u(1) = 0. \tag{4}$$

Here $\alpha \neq 0$ is an arbitrary complex number.

The unperturbed problem ($p(x) \equiv 0$) has an asymptotically simple spectrum, and its system of normalized eigenfunctions creates the Riesz basis. We construct the characteristic determinant of the spectral problem with an integral perturbation of the boundary conditions. The perturbed problem can have any finite number of multiple eigenvalues. Therefore, its root subspaces consist of its eigen and (maybe) adjoint functions. It is shown that the Riesz basis property of a system of eigen and adjoint functions is stable with respect to integral perturbations of the boundary condition.

Throughout this note we mainly use techniques from our works [1].

Keywords: Riesz basis, regular boundary conditions, eigenvalues, root functions, spectral problem, integral perturbation of boundary condition, characteristic determinant

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25, 35P05

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On a homogeneous parabolic problem in an infinite corner domain

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Abstract: It is considered the homogeneous boundary problem

$$\frac{\partial u(x,t)}{\partial t} - a^2\frac{\partial^2 u(x,t)}{\partial x^2} = 0,\ \{x,t\} \in G = \{x,t:\ 0<x<t,\ t>0\}; \tag{1}$$

$$\frac{\partial u(x,t)}{\partial x}|_{x=0} = 0,\ \frac{\partial u(x,t)}{\partial x}|_{x=t} + \frac{d\tilde{u(t)}}{dt} = 0; \tag{2}$$

where $\tilde{u}(t) = u(t,t)$.

Note that problem (1)–(2) is homogeneous case of the problem, studied in [1], which stated that the case of non-homogeneous boundary value problem "... it appears to be useful in the study of some free boundary value problems. "In the paper [1] was obtain a theorem on the unique solvability of the non-homogeneous boundary value problem in weighted Holder spaces.

In this paper, we set in class of essentially bounded functions with a given weight the existence of a nontrivial solution for a constant factor and a constant term. We introduce the class as follows:

$$(x+t^{1/2})^{-1}\,u(x,t) \in L_\infty(G),\ \text{i.e. } u(x,t) \in L_\infty(G;\,(x+t^{1/2})^{-1}).$$

THEOREM 1. *The boundary value problem (1)–(2) has a nontrivial solution* $u(x,t) = C_2\,\tilde{u}(x,t) + C_1$, *where* $\tilde{u}(x,t) \in L_\infty(G;\,(x+\sqrt{t})^{-1})$, $C_1,\,C_2 = const.$

THEOREM 2. *In the class of functions* $L_\infty\left(G;\,[x^{1+\alpha} + t^{(1+\alpha)/2}]^{-1}\right)$ *the boundary value problem (1)–(2) has only the trivial solution* $u(x,t) \equiv 0$.

Keywords: parabolic equation, boundary value problem, integral equation

2010 Mathematics Subject Classification: 94B05, 94B15

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Spectral geometry inequalities for Schatten p-norms of compact operators

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Abstract: In this talk we give answers to some spectral geometry questions of Schatten p-norms of convolution type compact operators on complete, connected, simply connected Riemannian manifolds of constant sectional curvature. For example, we show that among all domains of a given measure the geodesic ball is a maximizer of Schatten p-norms of some convolution type integral operators. The main reason why the results are useful, beyond the intrinsic interest of geometric extremum problems, is that they produce a priori bounds for spectral invariants of operators on arbitrary domains. This talk is mainly based on the papers [1]- [5].

Keywords: Convolution operators, Schatten p-norm, n-sphere, real hyperbolic space, Rayleigh-Faber-Krahn inequality, Hong-Krahn-Szego inequality

2010 Mathematics Subject Classification: 35P99, 47G40, 35S15

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On a heat transfer model for the locally inhomogeneous initial data

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Abstract: We consider a model case of the problem of heat diffusion in a homogeneous body with a special initial state. The peculiarity of this initial state is its local inhomogeneity. That is, there is a closed domain Ω inside a body, the initial state is constant out of the domain. Mathematical modeling leads to the problem for a homogeneous multi-dimensional diffusion equation.

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with sufficiently smooth boundary $\partial\Omega$. Through D we denote a cylindrical domain $D = \Omega \times (0, T)$. In D we consider a surface heat potential

$$u = \int_{\Omega} \varepsilon_n(x - \xi, t) u_0(\xi) d\xi, \tag{1}$$

where $\varepsilon_n(x, t) = \theta(t)(2a\sqrt{\pi t})^{-n} e^{-\frac{|x|^2}{4a^2 t}}$ is a fundamental solution of the heat equation

$$\Diamond u \equiv \left(\frac{\partial}{\partial t} - a^2 \triangle_x \right) u = 0. \tag{2}$$

We construct the boundary conditions on the boundary of the domain Ω, which can be characterized as "transparent" boundary conditions . We separately consider a special case – a model of redistribution of heat in a uniform linear rod, the side surface of which is insulated in the absence of (internal and external) sources of heat and of locally inhomogeneous initial state.

Throughout this note we mainly use techniques from our works [1, 2].

Keywords: diffusion equation, homogeneous body, initial state, local inhomogeneity, transparent boundary conditions

2010 Mathematics Subject Classification: 35Q79, 35K05, 35K20

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A uniqueness theorem of a boundary inverse problem of a differential operator on an interval with integro–differential boundary conditions

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Abstract: In this work we research a boundary inverse problem of spectral analysis of a differential operator with integral boundary conditions in the functional space $L_2(0, b)$, where $b < \infty$. A uniqueness theorem of the inverse boundary problem in $L_2(0, b)$ is proved. Note that a boundary inverse problem of spectral analysis is the problem of recovering boundary conditions of the operator by its spectrum and some additional data.

In this paper we investigate the inverse problem of spectral analysis of high order differential operators with integro–differential boundary conditions. In this case, it is necessary to find from the spectral data not only coefficients of the differential expression, also, we need to find boundary functions of the integro–differential boundary conditions. Coefficient inverse problems are well studied. Therefore in this paper we study the issue of reconstruction of boundary functions.

Keywords: boundary inverse problem, uniqueness theorem, differential operator, integro–differential boundary condition

2010 Mathematics Subject Classification: 47A55, 35P25

On identities for eigenvalues of a well-posed pertubation of the Laplace operator in a punctured domain

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Abstract: In this paper we consider a class of finite perturbed well-posed problems for the Laplace operator. And, identities for eigenvalues of the considering problems are found.

In this paper we are going to continue research of the chain of papers [1–5], where a class of well-posed problems in a Hilbert space for the m-Laplace operator in a punctured domain is considered and an analog of the Green formula and a class of self-adjoint problems are obtained by the first author and his co–authors. Our aim is to obtain some identities for eigenvalues of well-posed problems for the Laplace operator in a punctured domain.

Keywords: Laplace operator, well-posed solvable boundary value problem, punctured domain, Dirichlet boundary condition, finite perturbation, identities for eigenvalues, resolvent, non-local type boundary value problem, nuclear operator

2010 Mathematics Subject Classification: 35P15

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On first eigenvalue of Laplace operator

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Abstract: We consider a self-adjoint differential operator in a Hilbert space. Then the domain of the operator is changed by the perturbation of the boundary conditions so that a given neighborhood "there is no eigenvalues on neighborhood of zero" from the points of the spectrum of the perturbed operator. For the Sturm-Liouville operator on the segment and the Laplace operator on the square such a possibility is achieved through integral perturbations of boundary conditions. These statements are given with full proves and with possible extension. The G.G. Islamov's paper [1] solves the following problem. Suppose that the closed operator A of the Banach space X has eigenvalues in a "prohibited" area of Ω the complex plane. The requirement is for a K finite-dimensional $V = A - K$ operator, wherein the operator will not have points in the spectrum in Ω area. Our main result is following theorem:

Theorem 1. If the parameter α chosen so, that inequality holds

$$(\lambda_2 - \lambda_1) \leq \alpha \int_0^\pi \frac{\partial v_1(x_1, \pi)}{\partial x_2} dx_1, \tag{1}$$

the eigenvalues $\{\eta_k\}_{k=1}^{\infty}$ of the operator B_1 defined by the formula

$$\eta_k = \lambda_k \ \text{ when } \ k \geq 2,$$

and η_1 is the only real root of the equation.

Keywords: Laplace operator, eigenfunction, eigenvalue.

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25, 35P05

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Estimates of the eigenvalues of operator arising in swelling pressure model

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Abstract: Swelling pressures from materials confined by structures can cause structural deformations and instability. Due to the complexity of interactions between expansive solid and solid-liquid equilibrium, the forces exerting on retaining structures from swelling are highly nonlinear. In 1994, Mesri and etc. [1], also [2], and [3], developed a simplistic equation for swelling pressure as a function of mobilized volume strain which can be used to show ([4]) that the pressure p acting on a wall can be modeled by $p(x) = p_{sl}e^{-\alpha v(x)}$, where p_{sl} is the swelling pressure against an unyielding wall, α a constant depending upon the solid-liquid equilibrium, and $v(x)$ the deflection of the wall modeled as a cantilever beam at location x along the beam from the fixed end. We consider the initial/boundary value problem of an Euler-Bernoulli elastic beam subject to the swelling pressure with one end clamped and another end free.

In this paper two sided estimates of eigenvalues of the operator corresponding to some nonlinear equation, which is the model of expansive clay are established.

Keywords: Euler-elastic beam equation, Swelling Pressure Model, Estimates of the eigenvalues

2010 Mathematics Subject Classification: 31B20, 35P20

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The resolvent equation of nonlinear Fredholm integral equations

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Abstract: It is considered the new method solving of nonlinear Fredholm integral equation. It is found the sufficient condition for the existence of a solution of the nonlinear integral equation and it is proved uniqueness of the found solution.

In the article we consider the nonlinear Fredholm integral equation [1, 2]

$$\varphi(x) = \lambda \int_a^b K\left(x, t, \varphi(t)\right) dt + f(x), \tag{1}$$

where $f(x)$ is known continuous function defined on the $[a, b]$; Kernel $K\left(x, t, \varphi(t)\right)$ nonlinearly depends on the unknown function $\varphi(x)$ and continuous on set of arguments $x \in [a, b]$, $t \in [a, b]$,$\varphi \in (c, h)$, and it has a continuous derivative with respect to functional variable φ; λ is a parameter.

By the method described in [3, 4] we have established that if the kernel $K\left(x, t, \varphi(t)\right)$ satisfies

$$0 \equiv \int_a^b \int_a^b K'_\varphi\left[x, t, \psi(t, \lambda)\right] u(t)\, dt\, dx, \tag{2}$$

the equation (1) admits a solution of the form

$$\varphi(x) = \varphi_0(x) + \lambda u(x), \tag{3}$$

where $\varphi_0(x)$, $u(x)$, $\psi(t, \lambda)$ are known functions . This method is the most effective in the case where identity (2) can be represented in the form

$$0 \equiv A(x) \int_a^b G\left[t, \psi(t, \lambda)\right] \bar{u}(t)\, dt. \tag{4}$$

In this case, if the parameter λ is *the resolvent number*, i.e. is the root of *the resolvent algebraic equation*

$$B(\lambda) = \int_a^b G\left[t, \psi(t, \lambda)\right] \bar{u}(t)\, dt = 0, \tag{5}$$

correlation (2) holds for any $x \in [a, b]$.

Substituting the found value for resolvent number $\lambda_1, ..., \lambda_p$, $p \leq \infty$ into (3) we obtain the solution of equation (1). We note that if λ is not a characteristic number of kernels $K'_\varphi\left[x, t, \psi(t, \lambda)\right]$, the solution is unique.

Keywords: nonlinear integral equation, uniqueness of solution, characteristic number, orthogonality

2010 Mathematics Subject Classification: 45B05

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On Spectral Properties of Perturbations of the Operator $-u''(-x)$ with Initial Data

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Abstract: In the recent years there appears a rising interest to the problems that contain involution in the main terms of its differential operations [1–3].

The peculiarity of the initial value problem

$$L_0 u = -u''(-x), \ -1 < x < 1, \quad u(-1) = u'(-1) = 0,$$

for the simplest second-order differential operation with involution (reflection) is the non-emptiness of its spectrum. As the related spectral expansion could be constructed explicitly, it is interesting to study the spectral expansion for the perturbed operator $L = L_0 + Q$ where $Qu = q(x)u(x)$ is the multiplication on a square integrable complex-valued potential $q(x)$.

Let $S_m(x, f)$ and $\sigma_m(x, f)$ be m-th partial sums of the spectral expansions for L_0 and L respectively. Using asymptotics of the Green's function for L, the equiconvergence property is obtained, i.e.

$$S_m(x, f) - \sigma_m(x, f) = o(1), \quad m \to \infty,$$

uniformly for $x \in [-1, 1]$ and any $f(x) \in L_1(-1, 1)$. It directly provides the claim that $\sigma_m(x, f)$ converges to $f(x)$ in the L_2-metric for any $f(x) \in L_2(-1, 1)$, or, in other words, the root functions of L constitute basis.

The research is supported by the grant 0971/GF4 and by the target program 0085/PTSF-14 from the Ministry for Education and Science of the Republic of Kazakhstan.

Keywords: ODE with involution, spectral expansion, equiconvergence

2010 Mathematics Subject Classification: 34K08, 34L10

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Well-posed problems for the Laplace operator in the multiply connected domains

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Abstract: We give a complete description of well-posed solvable boundary-value problems for the Laplace operator in the multiply connected domains.

In [1] are studied the spectral properties of the Schrödinger operator with point-like interactions. In [2] are given of well-posed solvable boundary-value problems for the Laplace operators in the disk and in the punctured disk. The practical application of these kinds of operators can be found in [3], [4]. In [5] are obtained an analog of the Green formula and a class of self-adjoint extensions of the Laplacian. In [6] are studied of the first regularized of the Laplace operator in a plane bounded domain.

This report is a continuation of the paper [2].

Keywords: Laplace operator, interior boundary problem, Green function.

2010 Mathematics Subject Classification: 35A2

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Very weak solutions to wave equations

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Abstract: In this talk we discuss a notion of a very weak solution to hyperbolic equations with irregular coefficients which was introduced for the case of the wave equation in [1]. Using methods of nonharmonic analysis developed in [2] this allows one to prove existence and uniqueness for Cauchy problems of the type

$$u''(t) - a(t)Au(t) = f(t), \quad u(0) = u_0, \quad u'(0) = u_1,$$

where A is a densely defined positive operator on some Hilbert space, and $a(t)$ is a non-negative distribution. Examples will be given for the classical wave equation (based on joint work with Claudia Garetto), and for the wave equation for the Landau Hamiltonian describing the movement of the particle in irregular electromagnetic field (based on joint work with Niyaz Tokmagambetov).

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An embedding theorem of Sobolev type spaces on stratified Lie groups

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Abstract: In this talk, we present a version of horizontal weighted Hardy-Rellich type inequality on stratified Lie groups and study some of their consequences. Our results reflect on many results previously known in special cases. Moreover, new Sobalev type spaces are defined on stratified Lie groups and proved an embedding theorem for these functional spaces.

Keywords: Stratified Lie groups, Hardy-Rellich type inequality, Sobolev space

2010 Mathematics Subject Classification: 94B05, 94B15

An isoperimetric inequality for heat potential and heat equation

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Abstract: In this paper we prove that the ball is a minimizer of the first characteristic number of heat potential and heat operator among all Euclidean domains of a given measure.

Historically, for the first time of scientific literature, in Rayleigh's famous book "The Theory of Sound" (first published in 1877), by using some explicit computation and physical interpretations, he stated that a disk minimizes (among all domains of same area) the first eigenvalue of the Dirichlet Laplacian and the norm of an inverse operator is maximized in a disk among all Euclidean domains of a given area. The proof of this conjecture was obtained after about 50 years later, simultaneously (and independently) by G. Faber and E. Krahn. Nowadays, the Rayleigh-Faber-Krahn inequality has been extended to many other operators. In this paper we prove the Rayleigh-Faber-Krahn theorem for the heat potential operator H, i.e. it is proved that the ball is a minimizer of the first characteristic number of the integral operator H among all domains of a given measure in R^d. The main reason why the result is useful, beyond the intrinsic interest of geometric extremum problems, is that it gives a priori bound for operator norm of the heat potential on arbitrary cylindric domains.

Keywords: heat potential, heat equation, Rayleigh-Faber-Krahn inequalty, Schatten p-norm, isoperimetric inequality.

2010 Mathematics Subject Classification: 35K10, 31B35, 47G10.

On some spectral inequalities for a nonlocal elliptic problem

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Abstract: Let $\Omega \subset R^2$ be a bounded domain, symmetric with respect to the origin and with a smooth boundary $\partial\Omega$. This symmetry means that alongside with a point (x_1, x_2) her "opposite" point $x* = (-x_1, -x_2)$ also belongs to the domain. Let us denote $\partial\Omega_+ = \partial\Omega \cap \{x_1 \geq 0\}$, $\partial\Omega_- = \partial\Omega \cap \{x_1 < 0\}$.

In this paper we consider the following problem:

$$-\Delta u(x) = f(x), x \in \Omega,$$

in the domain Ω, and satisfying the following boundary conditions

$$u(x) = -u(x*), \frac{\partial u(x)}{\partial n_x} = \frac{\partial u(x*)}{\partial n_x}, x \in \partial\Omega_+.$$

Here n_x is a derivative in the direction of an outer normal to $\partial\Omega$.

Investigated problem is an analogue of the classical periodic boundary value problems in the case of non-rectangular region. Note that the problem P in the case of a circle was first formulated and investigated in [1].

We prove self-adjointness of the problem and show a method of constructing eigenfunctions.

We obtain an analogue of the Rayleigh type inequality and some spectral inequality for the first eigenvalue of the nonlocal problem.

Keywords: Laplace operator, nonlocal problem, eigenvalue, eigenfunctions, Rayleigh type inequality

2010 Mathematics Subject Classification: 35P15, 35J05, 49R50

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Spectral properties of a Laplace operator with Samarskii–Ionkin type boundary conditions in a disk

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Abstract: In this paper we consider a spectral problem for a model differential operator of the second order with involution. The operator is given by a differential expression

(1) $$Lu= -u''(-x), \ -1 < x<1$$

and boundary conditions of the general form:

(2) $$\alpha_j u'(-1) + \beta_j u'(1) + \alpha_{j1} u(-1) + \beta_{j1} u(1) = 0, \ \ j = 1, 2.$$

Coefficients of the boundary condition are numbers, which can be complex.

A criterion of the basis property of systems of eigenfunctions of the operator is set in the sense of coefficients of the boundary conditions. The interest to studying the basis properties of operators of the form (1), (2) has arose relatively recently. In works of T.Sh. Kal'menov and his disciples the *self-adjointness* property of the Cauchy operator:

(3) $$L_K u = -u''(-x), \ -1 < x < 1, \quad u'(-1) = 0, \ \ u(-1) = 0$$

is successfully applied in solving initial-boundary value problems for a heat equation with inverse time direction, in finding a criterion of well-posedness of the Cauchy problem for the Poisson operator, in formulating an asymptotic solution to problems with a small parameter.

Keywords: generalized spectral problem, involution, eigenfunctions, eigenvalues, basis property

2010 Mathematics Subject Classification: 34L10, 34L15

Spectral properties of a Laplace operator with Samarskii–Ionkin type boundary conditions in a disk

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Abstract: One of the most important problems in the theory of harmonic functions are Dirichlet and Neumann boundary value problems. In the one-dimensional case, or when considering the problem in a multidimensional parallelepiped, the main problems include also the periodic boundary value problems.

In this paper we consider spectral properties of a Laplace operator with boundary conditions of Samarskii-Ionkin type in a disk and prove the completeness of eigenfunctions.

Let $\Omega = \{z = (x, y) = x + iy \in C : |z| < 1\}$ be a unit disk, $r = |z|, \varphi = \arctan(y/x)$, $\Omega^+ = \Omega \cap \{y > 0\}$, and $\Omega^- = \Omega \cap \{y < 0\}$. We consider the spectral problem corresponding to the Laplace operator

$$-\Delta u(z) = \lambda u(z), |z| < 1$$

with local boundary conditions

$$u(1, \varphi) = 0, \;\; 0 \leq \varphi \leq \pi,$$

and with one of the nonlocal boundary conditions

$$\frac{\partial u}{\partial r}(1, \varphi) - \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0, \;\; 0 \leq \varphi \leq \pi,$$

or

$$\frac{\partial u}{\partial r}(1, \varphi) + \frac{\partial u}{\partial r}(1, 2\pi - \varphi) = 0, \;\; 0 \leq \varphi \leq \pi.$$

We note that unlike the one-dimensional case the system of root functions of the problems consists only of eigenfunctions.

Keywords: Poisson equation, Samarskii–Ionkin type problem, eigenfunctions, eigenvalues

2010 Mathematics Subject Classification: 35J05, 35J25, 35P10

The theorem on the basis property of eigenfunctions of second order differential operators with involution

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Abstract: In the works [1-2] was shown that the eigenfunction system

$$\left\{\cos k\pi x,\, \sin(k+\frac{1}{2})\pi x,\ k=0,1,2,...\right\}$$

of the spectral problem

$$-u''(-x)=\lambda u(x),\ -1<x<1, \tag{1}$$

$$u'(-1)=0,\, u'(1)=0. \tag{2}$$

forms a Riesz basis of the space $L_2(-1,1)$. In this connection, naturally arises the question, will the eigenfunctions system of the spectral problem

$$-u''(-x)+q(x)u(x)=\lambda u(x),\ -1<x<1, \tag{3}$$

with boundary conditions (2) be a basis in space $L_2(-1,1)$? Let $\sigma_m(f)$ partial sum of partial sum of eigenfunctions expansion of the spectral problem (1), (2), and $S_m(f)$ partial sum of eigenfunctions expansion of the spectral problem (3), (2). Sequence $S_m(f)$ call equconvergent with sequence $\sigma_m(f)$ on the interval $-1\le x\le 1$, if $S_m-\sigma_m\to 0$ uniformly converges on this interval at $m\to\infty$. It is possible to show the validity of the theorem.

Theorem 1. *Eigenfunction system of the spectral problem (3), (2) with continuous coefficient $q(x)$ forms basis of the space $L_2(-1,1)$.*

Keywords: differential equation with involution, Green's function, eigenfunction expansions, basis

2010 Mathematics Subject Classification: 34K08, 34L10, 46B15

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Solution of a singularly perturbed Cauchy problem for linear systems of ordinary differential equations by the method of spectral decomposition

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Abstract: In this paper a completely new method, which is essentially an operational method [1] and is based on the spectral theory of equations with deviating argument [2-7], is proposed. The method consists in the fact that the spectral decompositions of Cauchy problem solutions are constructed by the system of eigenfunctions of the self-adjoint differential operators with deviating argument. It turned out that so-called fundamental solutions of a system of linear differential equations and equations of higher order play a special role. At the asymptotic analysis of solutions of singularly perturbed problems the so-called regulatory functions of S.A. Lomov occupy the important place. For the first time it is proved that the fundamental solutions of linear differential equations act as the regularizing functions of S.A. Lomov.

Keywords: Spectrum, spectral decomposition, deviating argument, singular perturbation

2010 Mathematics Subject Classification: 34D15, 34K08

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Differential operators with singular coefficients

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Abstract: Classical theory of symmetric ordinary differential operators allows to work with differential expressions of the form

$$(4) \quad \ell_{2n}[y] := \sum_{k=0}^{n} (-1)^{n-k} (p_k\, y^{(n-k)})^{(n-k)} + \\ + i \sum_{k=0}^{n-1} (-1)^{n-k-1} \{(q_k\, y^{(n-k-1)})^{(n-k)} + (q_k\, y^{(n-k)})^{(n-k-1)}\},$$

provided that the coefficients p_k, q_k and $(p_0)^{-1}$ are locally integrable functions. This theory was developed in the works of D. Shin, N. Glazman, A. Zettle, N. Everitt et al.

Our goal is to extend the frames of the classical theory and to define operators associated with non-symmetric differential expressions

$$(5) \qquad \tau(y) = \sum_{k,s=0}^{n} \left(r_{ks} y^{(n-k)} \right)^{(n-s)},$$

whose coefficients r_{ks} are distributions of finite order singularity (depending on the indices k, s).

We shall discuss several approaches to this problem. The most important one is based on the so-called regularization procedure. This procedure provides several advantages. In particular, one can write out useful asymptotic formulae for the solutions of the equation

$$\ell_{2n}[y] - \lambda y = 0.$$

We shall also discuss similar problems for partial differential operators.

The talk is based on the joint papers with prof. K.A. Mirzoev and A.M. Savchuk.

Keywords: differential operators with distribution coefficients, differential operators with singular coefficients, regularization method

2010 Mathematics Subject Classification: 34E15

Existence of eigenvalues of problem with shift for an equation of parabolic-hyperbolic type

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Abstract: In the paper a spectral problem for an operator of parabolic-hyperbolic type of I kind with non-classical boundary conditions is considered. The problem is considered in a standard domain. The parabolic part of the space is a rectangle. And the hyperbolic part of the space coincides with a characteristic triangle. We consider a problem with the local boundary condition in the domain of parabolicity and with the boundary condition with displacement in the domain of hyperbolicity.

Let $\Omega \in R^2$ be a finite domain bounded for $y > 0$ by the segments AA_0, A_0B_0, B_0B, $A = (0,0)$, $A_0 = (0,1)$, $B_0 = (1,1)$, $B = (1,0)$, and for $y < 0$ by the characteristics $AC: \ x + y = 0$ and $BC: \ x - y = 1$ of an equation of the mixed parabolic-hyperbolic type

$$Lu = \left\{ \begin{array}{c} u_x - u_{yy}, y > 0 \\ u_{xx} - u_{yy}, y < 0 \end{array} \right\} = f(x,y). \tag{1}$$

Problem S. *Find a solution to Eq.* (1) *satisfying boundary conditions*

$$u|_{AA_0 \cup A_0B_0} = 0, \tag{2}$$

$$\alpha u\left(\theta_0\left(t\right)\right) = \beta u\left(\theta_1\left(t\right)\right), \quad 0 \le t \le 1, \tag{3}$$

where $\theta_0\left(t\right) = \left(\frac{t}{2}, -\frac{t}{2}\right)$, $\theta_1\left(t\right) = \left(\frac{t+1}{2}, \frac{t-1}{2}\right)$.

We prove the strong solvability of the considered problem. The main aim of the paper is the research of spectral properties of the problem. The existence of eigenvalues of the problem is proved.

Keywords: spectral problem, equation of parabolic-hyperbolic type, boundary condition with shift

2010 Mathematics Subject Classification: 35M10, 35M12, 35P

Duality property of the noncommutative ℓ_∞ and ℓ_1 valued symmetric Hardy spaces

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Abstract: In this paper we consider the noncommutative $H_E(\mathcal{A};\ell_\infty)$ and $H_E(\mathcal{A};\ell_1)$ spaces and obtain some result on duality for these spaces, i.e we obtain the following result:

Theorem. Let E be an r-convex symmetric Banach function space on $[0;1]$ for some $0 < r < \infty$ and E do not contain c_0 or separable. Then

$$(i)\ \ (H_E(\mathcal{A};\ell_1))^* = L_{E^\times}(\mathcal{M};\ell_\infty)/J(H^0_{E^\times}(\mathcal{A};\ell_\infty))$$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_E(\mathcal{A};\ell_1)$ and $y \in H_{E^\times}(\mathcal{A};\ell_\infty)$, where $J(H^0_{E^\times}(\mathcal{A};\ell_\infty)) = \{x^* : x \in H^0_{E^\times}(\mathcal{A};\ell_\infty)\}$.

$$(ii)\ \ (L_E(\mathcal{M};\ell_1)/J(H^0_p(\mathcal{A};\ell_1)))^* = H_{E^\times}(\mathcal{A};\ell_\infty)$$

isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_E(\mathcal{A};\ell_1)$ and $y \in H_{E^\times}(\mathcal{A};\ell_\infty)$, where $J(H^0_{E^\times}(\mathcal{A};\ell_1)) = \{x^* : x \in H^0_{E^\times}(\mathcal{A};\ell_1)\}$.

This theorem is analogue of Theorem 5 in [1].

Keywords: von Neumann algebra, subdiagonal algebras, noncommutative vector valued symmetric Hardy spaces, duality.

2010 Mathematics Subject Classification: 46L51, 46L52.

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Lateral boundary conditions for the Klein-Gordon-Fock equation

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Abstract: In this paper we consider an initial-boundary value problem for the Klein-Gordon-Fock equation. In [**?**], Authors considered one dimentional potential

$$u(x,t) = \int_{\Omega} (-\frac{1}{2}|x-y|) f(y) dy \tag{1}$$

in $\Omega = (0,1)$ and the equation

$$-u^{"}(x) = f(x), \ \ x \in \Omega, \tag{2}$$

and they showed that if they are solved the equation (2) with the boundary conditions

$$u^{'}(0) + u^{'}(1) = 0, \ \ -u^{'}(1) + u(0) + u(1) = 0. \tag{3}$$

then they were finded an unique solution of that boundary value problem in the form (1).

This simple method finds equivalent boundary value problems of ODE for one dimensional potential integrals. However this task becomes tedious for PDE, and in works [1, 2] are obtained boundary conditions of the volume potentials for elliptic and hyperbolic equations and showed some their applications. In particular, in [2], by using a new non-local boundary value problem, which is equivalent to the Newton potential, Authors found explicitly all eigenvalues and eigenfunctions of the Newton potential in the 2-disk and the 3-ball. The aim of this work is to find lateral boundary conditions for the Klein-Gordon-Fock equation as shown in [**?**].

Keywords: Potential, fundamental solution, Klein-Gordon-Fock equation

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25, 35P05

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Euler-Hilbert-Sobolev spaces on homogeneous groups

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Abstract: In this talk we define Euler-Hilbert-Sobolev spaces and obtain embedding results on homogeneous groups using Euler operators, which are homogeneous differential operators of order zero. Sharp remainder terms of L^p and weighted Sobolev and Sobolev-Rellich inequalities on homogeneous groups are given. As consequences, we obtain analogues of the generalised classical Sobolev and Sobolev-Rellich inequalities. We also discuss applications of logarithmic Hardy inequalities to Sobolev-Lorentz-Zygmund spaces.

Keywords: Sobolev inequality, Hardy inequality, weighted Sobolev inequality, Rellich inequality, Euler-Hilbert-Sobolev space, Sobolev-Lorentz-Zygmund space, homogeneous Lie group

2010 Mathematics Subject Classification: 22E30, 43A80

MS3
DINAMICAL
SYSTEMS

On a dynamical system relating to the Ricci flow on Aloff-Wallach spaces

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Abstract: The talk is devoted to the normalized Ricci flow equation

$$\dot{\mathbf{g}}(t) = -2\mathrm{Ric}_{\mathbf{g}} + 2n^{-1}\mathbf{g}(t)S_{\mathbf{g}}$$

on Aloff-Wallach spaces $\mathrm{SU}(3)/\,\mathrm{SO}(2)$, where $\mathbf{g}(t)$ means a 1-parameter family of Riemannian metrics, $\mathrm{Ric}_{\mathbf{g}}$ is the Ricci tensor and $S_{\mathbf{g}}$ is the scalar curvature of $\mathbf{g}$. On Aloff-Wallach spaces the normalized Ricci flow equation can be reduced to a system of four ODE's of the form

$$\dot{x_i} = -2\,x_i\left(\mathbf{r_i} - \frac{\mathbf{r_4} + 2(\mathbf{r_1} + \mathbf{r_2} + \mathbf{r_3})}{7}\right), \tag{4}$$

where $\mathbf{r_i}$, $i = 1, \dots, 4$, are the principal Ricci curvatures (see [3,4] for explicit expressions of $\mathbf{r_i}$). The main result is contained in the following two Lemmas.

Lemma 1. *The system* (4) *has exactly two singular (equilibrium) points.*

Lemma 2. *The volume function* $V := x_1^2 x_2^2 x_3^2 x_4$ *is a first integral of the system (4).*

It should be noted that similar questions were studied in [1,2] in the case of generalized Wallach spaces.

Acknowledgements. The project was supported by Grant 1452/GF4 of MES of the Republic of Kazakhstan for 2015-2017.

Keywords: Ricci flow, Riemannian metric, Ricci curvature, Aloff-Wallach space, generalized Wallach space.

2010 Mathematics Subject Classification: 53C30, 53C44, 34C05, 37C10.

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Investigation on absolute stability of regulated systems in a simple critical case

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Abstract: A new method for studying of absolute stability of the equilibrium position of nonlinear regulated systems in a simple critical case by evaluating improper integrals along the solutions of the system is proposed. Sectors are found, where the equilibrium position of the system is absolutely stable and the Aizerman problem has a positive solution. The effectiveness of the method is shown in the example.

Keywords: Absolute stability, non-singular transformation, properties of solutions, improper integrals, Aizerman problem, absolute stability sectors

2010 Mathematics Subject Classification: 02.30.Hq, 02.30.Yy

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Controllability of process described by system of linear integro-differential equations with resrtictions

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Abstract: Studied control process described by linear integro-differential equation $\dot{x} = A(t)x + B(t)u(t) + C(t)\int\limits_a^b K(t,\tau)w(\tau)d\tau + \mu(t),\ t \in I = [t_0, t_1]$ with boundary, phase, integral restrictions and restrictions on the control value.

For this problem offered method of solution based on passing from solving of initial problem to solving of some class of Fredholm integral equation of the first kind. For initial problem obtained necessary and sufficient conditions of it's solvability. Offered numerical method for solving control problem by constructing minimizing sequences.

This work is a continuation of researches conducted in the works [1, 2]

Keywords: optimal control, linear integro-differential equations

2010 Mathematics Subject Classification: 49N05

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Localization of an air target by means of GNSS-based multistatic radar

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Abstract: The research of employing GNSS satellites as transmitters of opportunity for passive bistatic and multistatic radars to detect [1, 2], track [3] and localize [4] an aircraft has aroused some interest lately. A main benefit of utilizing GNSS signals for such application is because of its global coverage. Furthermore, the presence of several GNSS satellites at various positions provides an advantage for using this application, as such geometrical arrangements enable the radar to receive reflections of the target from different angles.

In this work, we offer a contribution to multistatic GNSS-based radar methods, which allows for positioning of air targets provided that pseudoranges of reflected signals have been preliminarily measured. This novel computational method consists from decomposition of non-linear function in a Taylor series and the Least Squares method. Employing pseudorange equitions based on reflected signals from multiple satellites should make it possible to determine a position estimate of the aircraft more accurately than by means of the bistatic radar.

Keywords: GNSS, multistatic radar, GPS, air target, reflected signals

2010 Mathematics Subject Classification: 65D15, 94A12

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Gradient in the optimal control problem described by the differential equations

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Abstract: A gradient is calculated in the optimal control problem with terminal functional described by the second order ordinary differential equation. Using the calculated gradient of the optimal control problem a problem of numerical solving of the initial problem is considered.

Consider the minimization of the functional

$$J(u) = |R(r_1, u) - z|^2 \tag{1}$$

subject to

$$-a\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + (Q(r) - Er^2)R(r) = u(r), \quad r_0 < r < r_1, \quad r_0 > 0 \tag{2}$$

$$R(r_0) = z_0, \ \dot{R}(r_1) = z_1. \tag{3}$$

where $R(\cdot) \in H'[r_0, r_1]$, $u(\cdot) \in L_2[r_0, r_1]$, $\tilde{H}^1[r_0, r_1] = \{\psi(\cdot) \in H^1[r_0, r_1] : \psi(r_0) = 0\}$.

The following theorem is proved.

Theorem 1. If there exists $\alpha > 0$ such that $Q(r) - Er^2 \geq \alpha$ for $r \in [r_0, r_1]$, where $Q(\cdot) \in C[r_0, r_1]$ then for any $h(\cdot) \in L_2[r_0, r_1]$ the adjoint to the initial one problem has the only generalized solution and

$$\|\Delta R(\cdot)\| \leq \|h\|_{L_2[r_0, r_1]}.$$

On the base of these results a numerical algorithm is developed for the solution of the considered problem.

Keywords: gradient, optimal control, differential equation.

2010 Mathematics Subject Classification: 31A25, 65L10, 65M32, 80A23

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Time frequency method of solving one boundary value problem for a hyperbolic system and its application to the oil extraction

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Abstract: We consider the boundary value problem, where the motion of the object is described by the two-dimensional linear system of partial differential equations of hyperbolic type [1, 2], where a discontinuity is at a point within the interval that defines the phase coordinate x. Using the method of series and Laplace transformation in time t (time-frequency method), an analytical solution is found for the determination of debit $Q(2l, t)$ and pressure $P(2l, t)$, which can be effective in the calculating of the coefficient of the hydraulic resistance in the lift at oil extraction by gas lift method [3], where $l-$ is the well depth. For the case, where the boundary functions are exponential form, the formulas for the $P(2l, t)$ and $Q(2l, t)$ depending only on t are obtained. It is shown that at constant boundary functions, these formulas allow us to determine the coefficient of hydraulic resistance in the lift of the gas lift wells, which determines the change in the dynamics of pollution [4].

Keywords: hyperbolic equation, boundary problems, method of series, Laplace transformation, time-frequency method, gas lift, coefficient of hydraulic resistance

2010 Mathematics Subject Classification: 35L04, 35L20, 65N99

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On Numerical Solutions of Telegraph Equations with the Dirichlet Boundary Condition

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Abstract: In this study, the Cauchy problem for telegraph equations in a Hilbert space is considered. Stability estimates for the solution of this problem are presented. The third order of accuracy difference scheme is constructed for approximate solutions of the problem. Stability estimates for the solution of this difference scheme are established. As a test problem to support theoretical results, one-dimensional telegraph equation with the Dirichlet boundary condition is considered. Numerical solutions of this equation are obtained by first, second and third order of accuracy difference schemes..

Keywords: telegraph equations, numerical solutions, error analysis

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Optimal control problem for the degenerate phase field system of equations

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Abstract:

In this work, the optimal control problem for the degenerate linear system of the phase field is considered

$$v(x,0) = \varphi(x), \quad x \in \Omega, \tag{1}$$

$$\theta \frac{\partial v}{\partial n}(x,t) + (1-\theta)v(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \tag{2}$$

$$\theta \frac{\partial w}{\partial n}(x,t) + (1-\theta)w(x,t) = 0, \quad (x,t) \in \partial\Omega \times (0,T), \tag{3}$$

$$v_t(x,t) + lw_t(x,t) = k\Delta v(x,t) + u_1(x,t), \quad x \in \Omega \times (0,T), \tag{4}$$

$$\Delta w(x,t) + \alpha w(x,t) + \beta v(x,t) + u_2(x,t) = 0, \quad x \in \Omega \times (0,T), \tag{5}$$

$$\|u_1\|^2_{L_2(0,T;L_2(\Omega))} + \|u_2\|^2_{L_2(0,T;L_2(\Omega))} \leq R^2, \tag{6}$$

$$J(v,w) = \frac{1}{2}\|v-\tilde{v}\|^2_{L_2(0,T;L_2(\Omega))} + \frac{1}{2}\|w-\tilde{w}\|^2_{L_2(0,T;L_2(\Omega))} \to \inf, \tag{7}$$

where $\theta, l, k, \alpha, \beta, R$ are constants, $k > 0$, $\tilde{v}, \tilde{w} \in L_2(0,T;L_2(\Omega))$ are given functions, v, w are unknown functions, $(u_1,\ u_2)$ is function of control. The goal is to minimize (7) when (u_1, u_2) satisfy condition (6). Here we use results on the existence of the degenerate initial value problem and the optimal control problem which presented in [1]. Simplified version of system (1)–(7) is considered in [2]. So, we are looking through the work in order to prove the conditional gradient numerical method convergence for the control problem. Besides, we've described the algorithm of numerical solution constructing.

Keywords: optimal control, degenerate evolution, numerical solution

2010 Mathematics Subject Classification: 49J20,49M05,64K10

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About the robustness of the middle stabilizing controller for quasi-linear state dependent coefficients discrete-time systems

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Abstract: This paper is dedicated to the robustness analysis of a stabilizing controller for quasi-linear state dependent coefficients discrete systems. Two nonlinear stabilizing regulator construction algorithms based on different versions of the discrete state dependent Riccati equation are discussed, the one from [1], [2] and the new one based on the discrete maximum principle. The robustness or fault-tolerance is the property of the regulator that allows it to maintain the control quality if the parameters of the real model differ from the computational model. There are different approaches for robustness analysis, for example the Afanas'ev minimax method, the robust stability properties of Schur polynomials analogous to Kharitonov theorems, linear matrix inequalities (LMI) approach [3] - [4] etc. Here the interval parametric uncertainties in the linear part of the system are investigated. The proposed nonlinear stabilizing regulator is calculated at the average values of the uncertainty parameters and is used for all realizations of the uncertain system. The basic idea is that the existence of only weak nonlinearity in the system allows us to study its robustness based on the robustness of the corresponding unperturbed discrete linear system. First we consider the linear part of the closed-loop system obtained along the so-called middle controller and then take into account the nonlinear part of the system, by introducing some conditions. The sufficient robustness conditions are formulated in the form of linear matrix inequalities. The results of the numerical experiments that demonstrate the robustness of the closed-loop system are analyzed [5] - [6].

Keywords: nonlinear discrete control systems, SDRE, regulator construction, robustness, stabilization, discrete maximum principle

2010 Mathematics Subject Classification: 93D05, 93D09, 93D15, 93D20, 93C10, 93C55

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On a controllability for a class of $2D$-linear modular dynamic systems with periodic parameters

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Abstract: The homogeneous $2D$- linear modular dynamic systems (2D-LMDS) with periodic parameters are described by the following equations (1)-(3):

$$x[n+1,c]=\sum_{\rho\in P}A[n,\rho]\,x[n,c+\rho]+\sum_{q\in Q}B[n,q]\,u[n,c+q],\ \ GF\,(p)\,, \tag{1}$$

$$y[n,c]=\sum_{\tau\in R}G[n,\tau]\,x[n,c+\tau],\ \ GF\,(p)\,. \tag{2}$$

$$A[n,\rho]=A[n+M,\rho],\ \ B[n,q]=B[n+M,q],\ \ G[n,\tau]=G[n+M,\tau]. \tag{3}$$

Based on the definitions (4)

$$q_i[k,c]=x[k\cdot M+i-1,c],\ \ z_i[k,c]=u[k\cdot M+i-1,c],$$
$$\gamma_i[k,c]=y[k\cdot M+i-1,c],\ \ w_i[k,c]=z_i[k+1,c],\ \ i=1,...,M;$$

$$q[k,c]=(q_1[k,c],...,q_M[k,c]\,)^T\,,\ \ z[k,c]=(z_1[k,c],...,z_M[k,c]\,)^T \tag{4}$$
$$\hat{q}\,(k,c)=(q_1[k,c],...,q_M[k,c],\ z_1[k,c],...,z_M[k,c])^T\,.$$

2D-LMDS (1)- (3) is written in the form called a block model

$$\hat{q}[k+1,c]=\sum_{\rho\in D}\tilde{A}[\rho]\,\hat{q}[k,c+\rho]+\sum_{q\in L}\tilde{B}[q]w[k,c+q],\ \ GF\,(p)\,, \tag{5}$$

$$\gamma[k,c]=\sum_{\tau\in\Phi}\tilde{G}[\tau]\,\hat{q}[k,c+\tau],\ \ GF\,(p)\,. \tag{6}$$

Theorem. In order to 2D-LMDS described by the (1-3) be fully N- controllable it is sufficient fulfillment of the condition: the corresponding block model (5), (6) have to be k_N controllable, where

$$k_N=k_1-k_0,\ \ k_1=int\,(n_1/N)\,,\ \ k_0=int\,(n_0/N)\,.$$

Keywords: modular dynamic systems with periodic parameters, a block model

2010 Mathematics Subject Classification: 37N35 , 11S31

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On the systems of rational difference equations

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Abstract: In this paper, we have investigated the solutions of the system of rational difference equations

$$\begin{cases} x_{n+1} = \frac{\alpha y_{n-2}}{\beta+\rho y_{n-2}x_{n-1}y_n} \\ y_{n+1} = \frac{\delta x_{n-2}}{\zeta+\eta x_{n-2}y_{n-1}x_n} \\ z_{n+1} = \frac{\theta z_{n-2}}{\lambda+\mu y_{n-2}x_{n-1}y_n} \end{cases}$$

where the parameters $\alpha, \beta, \rho, \delta, \zeta, \eta, \theta, \lambda, \mu$ and initial values $x_{-i},\ y_{-i},\ z_{-i}$, $i \in \{0, 1, 2\}$ are reel numbers.

Keywords: difference equation, difference equation systems, solutions

2010 Mathematics Subject Classification: Applied Mathematics, MS6:

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Design of PI regulators for dynamic systems with constrained control and fixed endpoints of trajectories

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Abstract: The construction of the automatic operation control for real and complex systems requires the use of new information technologies, it is actually necessary to develop new principles of design the systems with a high level of complexity. In the field of automatic control published works one can find various examples of mathematical formulation and methods for solving optimal control problems [1]. But still, the development of various methods of constructing the PI and PID controllers with the necessary properties is an urgent problem [2].

In this work, the problem of optimal control for time-varying linear systems with fixed endpoints of trajectories is considered. A correspondent quadratic objective functional depends on the control, the state of the object and on its integral. New technique of designing the PI controller for the automatic control systems with box constraints on values of control is proposed. The problem is solved by using Lagrange multipliers of a special type [3].

Keywords: Optimal control problem, box constraints, Lagrange multipliers, PI regulator

2010 Mathematics Subject Classification: 49J15, 49N35

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Optimal control problem for the three-sector economic model of a cluster

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Abstract: Problems of optimal control for dynamic systems with fixed ends of trajectories are often used in practical applications. Various mathematical formulations of optimal control problems and their classification are given in [1].

In this work the problem of optimal control for the three-sector economic model of a cluster is considered [2]. Problem statement is to determine the optimal distribution of investment and manpower in moving the system from a given initial state to desired final state. To solve the optimal control problem with finite-horizon planning, in case of fixed ends of trajectories, with box constraints, method of Lagrange multipliers of a special type is used [3]. This approach allows to represent the desired control in the form of synthesis control, depending on state of the system and current time. The results of numerical calculations for a three-sector economic model show the effectiveness of the proposed method.

Keywords: Economic model, cluster, investments, manpower, optimal control problem, method of Lagrange multipliers

2010 Mathematics Subject Classification: 49J15, 49N35

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The method of accelerated convergence for constructing conditional-periodical solutions

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Abstract: In the present paper we take quasi-linear system of differential equations

$$\frac{dx}{dt} = Ax + \varepsilon f(t,x), \tag{1}$$

where $x = colon(x_1, x_2)$, $A = (a_{jk}), j = k = 1, 2$,
$f(t,x) = colon(f_1(t,x_1,x_2), f_2(t,x_1,x_2))$
conditionally-periodic by t with frequency basis $\omega_1, \omega_2, ..., \omega_n$; analytical by t and x in the domain
$D = \left\{(t,x) \subset C^3 : ||x|| \leq h, ||Im\omega t|| \leq q\right\}$ function, $det|A - \lambda E| = 0$ has purely imaginary roots $i\sigma_1, i\sigma_2$, and rational numbers σ_1, σ_2 non-co-measurable with $\omega_1, \omega_2, ..., \omega_n$, ϵ is a small parameter.

In order to find a conditionally-periodic solutions of (1) the method of accelerated convergence [1] is applied. As an initial approximate conditionally-periodic solutions of the system (1) $x^{(0)}(t,\varepsilon) = 0 := colon(0,0)$ is chosen. Its residual denoted by $x^{(1)}(t,\varepsilon)$ and take this function as a first approximation to the original conditionally-periodic solutions of the system (1).

Keywords: quasi-linear system of differential equations, conditionally-periodic solution, small parameter

2010 Mathematics Subject Classification: 34B15

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Martinet-Ramis modulus for one Quadratic System

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Abstract: There are considered a quadratic system

$$\begin{cases} \dot{p} = p(1-v) \\ \dot{v} = v(p-v) \end{cases} \tag{1}$$

This system is in some sense, limit system for well known Jouanolou system [1]. The system (1) has a saddlenode singularity at the origin. In this work we calculate first coefficients of Martinet-Ramis' modulus [2].

Martinet-Ramis' modulus (C, ϕ) (for saddlenode singular point) [2] are constructed by transformations reducing initial system to its (orbital) formal normal form. Solutions of the system (1) with given initial conditions can be found as a series (with respect to initial condition). Using these solutions it is possible to find coefficients of the normalizing transformations, and then to determinate coefficients of modulus.

As a result we get

Theorem 1. *Let (C, ϕ) be Martinet-Ramis modulus for (1). Then: $C = 0$, $\phi(z) = z + 2\pi i z^2 + (2\pi i - 4\pi^2)z^3 + \ldots$*

Corollary 2. *The system (1) is not analytically orbital equivalent to its formal normal form.*

Keywords: Martinet-Ramis modulus, saddlenode, central manifold

2010 Mathematics Subject Classification: 34C20, 37F75

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Soliton solutions of the Hirota equation

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Abstract: It is well known that nonlinear integrable systems have attracted a lot of attention among researchers. This fascinating subject of nonlinear science has branched out in almost all areas of technology and science. In nonlinear science soliton solutions play an important role. There are many ways to obtain soliton solutions of the nonlinear evolution equations, such as the Painleve analysis [1], the Hirota's bilinear method [2], Darboux transformation (DT) [3] and so on. Among the various methods, the DT has been proved very successful in driving different kinds of solutions for many of the integrable equations from a trivial seed. In this work, we focus on the construction soliton solutions for the 2+1-dimensional Hirota equation [4], which is modified nonlinear Schrodinger equation. One-soliton solutions are obtained by means of the one-fold Darboux transformation for the 2+1-dimensional Hirota equation.

Keywords: Hirota equation, Darboux transformation, soliton solution

2010 Mathematics Subject Classification: 35C08, 35Q51, 37K40

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