# Second International Conference on Analysis and Applied Mathematics

# ICAAM 2014

## ABSTRACT BOOK

M. Auezov South Kazakhstan State University, Kazakhstan
Fatih University & Gumushane University, Turkey

Institute of Mathematics and Mathematical Modelling Kazakhstan

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## FOREWORD

The aim of the **International Conference on Analysis and Applied Mathematics** (**ICAAM**) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why we planned to found the conference series to provide a forum for researchers and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics.

The conference is organized biannually. The first conference was held in Gumushane, Turkey in 2012. The proceedings of ICAAM 2012 were published in AIP (American Institute of Physics) Conference Proceedings (http://scitation.aip.org/content/aip/proceeding/aipcp/1470).

The Second International Conference on Analysis and Applied Mathematics, ICAAM 2014, is jointly organized by M. Auezov South Kazakhstan State University (Kazakhstan), Institute of Mathematics and Mathematical Modelling (Kazakhstan), Fatih University (Turkey) and Gumushane University (Turkey). The meeting is held on September 11-13, 2014 in M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan. We also proudly announce that the Proceedings of ICAAM 2014 will be published in the world-renowned AIP Conference Proceeding Series (http://scitation.aip.org/content/aip/proceeding/aipcp/1611).

The conference will consist of **plenary lectures**, **mini symposiums** and **contributed oral presentations**.

Selected full papers of this conference will be published in the following peer-reviewed journals:

- **Boundary Value Problems;**
- **Contemporary Analysis and Applied Mathematics;**
- **Abstract and Applied Analysis;**
- **Numerical Functional Analysis and Optimization;**
- **Journal of Applied Mathematics;**
- **Mediterranean Journal for Research in Mathematics Education.**

We would like to thank M. Auezov South Kazakhstan State University (host university), Institute of Mathematics and Mathematical Modelling, Fatih University, Gumushane University for their support. We also would like to thank to all invited speakers, International Advisory Board, International Organizing Committee, and Technical Program Committee Members.

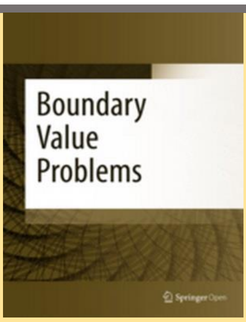
With our best wishes and warm regards,

**Chairs:**

**Prof. Allaberen Ashyralyev**

**Prof. Abdizhahan Sarsenbi**

# JOURNALS FOR FULL PAPERS:

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http://www.boundaryvalueproblems.com/about/update/PDEAdvances
*TS: Recent Advances in Partial Differential Equations and Their Applications.* The present thematic series is devoted to the publication of high-quality research papers presented in ICAAM-2014 in the fields of the construction and investigation of solutions of well-posed and ill-posed boundary value problems for partial differential equations and their related applications.

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http://caam.fatih.edu.tr
CAAM is an international journal dedicated to the latest advancement in analysis and applied mathematics.
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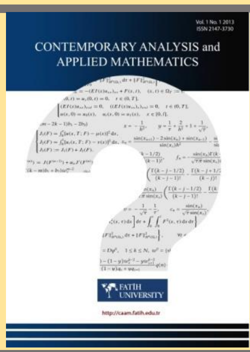


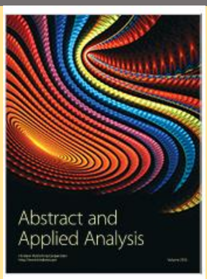


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**Mediterranean Journal for Research in Mathematics Education**
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| Prof. **Mohammad Mursaleen,** | Department of Mathematics, Aligarh Muslim University, Aligarh, India |
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# ANALYSIS

# ANALYSIS 1

## Fuzzy soft set over Fuzzy topological spaces

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**Abstract:** The aim of this article is to combine the fuzzy soft set and fuzzy topological space. We introduce fuzzy soft set over common universe of fuzzy topological space. We defined quasi-open fuzzy soft set over fuzzy topological space and we study the properties of extended union of two quasi-open fuzzy soft set over fuzzy topological space. We study "OR" and "AND" operations of quasi-open fuzzy soft set over fuzzy topological space. We also define open fuzzy soft set, b-open fuzzy soft set and b-closed fuzzy soft set over fuzzy topological space. We give some characterizations of these concepts.
**Keywords:** Fuzzy soft set, quasi-open fuzzy soft set, Fuzzy topological space.
**References:**
[1] D. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, 37, pp. 19-31, 1999.
[2] L.Z. Zadeh, *Fuzzy sets*, Information and Control, 8, pp. 338-353, 1965.

## Hardy type inequalities involving suprema

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**Abstract:** Let $I=(a;b)$, $-\infty \leq a < b \leq \infty$, $1\leq p$; $q\leq\infty$, $w$ is a nonnegative and continuous function on $I$, and $u^q$, $v^p$ are nonnegative locally integrable functions on $I$, and the function $v^{-p'}$ is also locally integrable on $I$.
Introduce the operators

$$R_{\infty}^{+}f(x) = \sup_{a<t<x} w(t)|P_x^{+}f(t)|,\ \ R_{\infty}^{-}f(x) = \sup_{x<t<b} w(t)|P_x^{-}f(t)|,$$

where $P_z^{+}f(t) = \int_t^z f(s)ds$, $P_z^{-}f(t) = \int_z^t f(s)ds$.

Consider the inequality

$$\|R_{\infty}^{\pm}f\|_{q,u} \le C^{\pm}\|f\|_{p,v} \qquad (1)$$

Inequality of type (1) has been studied in [1, 2, 3, 4], where the operators $P_b^{+}$, $P_a^{-}$ were considered instead of the operators $P_z^{+}$, $P_z^{-}$, respectively. The more general case has been considered in [4], when the integral operators with the kernel satisfying "Oinarov condition" were considered instead of the operators $P_b^{+}$, $P_a^{-}$. Impact of the operators $P_z^{+}$, $P_z^{-}$ can be written as integral operators considered in [4] but their kernels do not satisfy the "Oinarov condition".

We denote by

$$\Delta_w^{+}g(x) = \sup_{a<t<x} w(t)|g(x) - g(t)|, \qquad \Delta_w^{-}g(x) = \sup_{x<t<b} w(t)|g(x) - g(t)|$$

the estimates with a weight $w$, the values deviation of the function $g$ from $g(x)$, respectively, on the intervals $(a; x)$, $(x; b)$. If $g$ is a locally absolutely continuous function on $I$ with a derivative $g' \in L_{p,v}$, then the inequality (1) is equivalent to the inequality

$$\|\Delta_w^{\pm}g\|_{q,u} \le C^{\pm}\|g'\|_{p,v}\ . \qquad (2)$$

Here the necessary and sufficient conditions are obtained for the inequality (1), (2) when $1 \le p \le \le q \le \infty$, supplementing results of [5].

**Keywords:** Hardy type inequalities, Oinarov condition, integral operators.

**References:**

[1] A. Gogatishvili, B. Opic, L. Pick, *Weighted inequalities for Hardy - type operators involving suprema*, Collect. Math., vol. 57, no 3, pp. 227 – 255, 2006.

[2] A. Gogatishvili, L. Pick, *A reduction theorem for supremum operators*, S. Comput. And Appl. Math., vol. 208, pp. 270 – 279, 2007.

[3] D.V. Prokhorov, *Lorentz norm inequalities for the Hardy operator involving suprema*, Proc. Amer. Math. Soc. vol. 40, no 5, pp. 1585 – 1592, 2012.

[4] D.V. Prokhorov, *On the boundedness and compactness of an integral operator involving suprema*, Preprint 2012/180, Computer Center FEB RAS, Khabarovsk, 2012.

# Loewner evolution and stochastic differential equations

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**Abstract:** The classical Loewner equation in the unit disc is the following differential equation

$$\begin{cases} \frac{d\Phi_t(z)}{dt} = G(\Phi_t(z), t), \text{ for almost every } t \in [0, \infty), \\ \Phi_0(z) = z, \end{cases} \quad (1)$$

where $G(w,t) = -wp(w,t)$ with the function $p: D \times [0,\infty) \to \mathbb{C}$ measurable in $t$, holomorphic in $z$, $p(0,t) = 1$ for all $t \geq 0$ and $\mathrm{Re}\, p(z,t) \geq 0$.
Recently Georgy Ivanov and Alexander Vasil'ev [5] considered version of this differential equation with

$$G(w,t) = \frac{(\tau(t)-w)^2 p(w,t)}{\tau(t)}, \text{for } \tau(t,w) = \exp\big(ikB_t(w)\big). \quad (2)$$

They found a substitution which transform the Loewner equation to an Ito diffusion.

In this study we prove that under rather general substitutions on $\tau(t, B_t)$, it is a possible to find a substitution which transform (1) to an Itô diffusion if and only if $\tau(t, B_t)$ is given by (2).

**Keywords:** Loewner equation, diffusion, Loewner evolution.

**References:**


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[2] P. Duren, *Univalent Functions*, Springer –Verlag, 1983.
[3] G.M. Goluzin, *Geometrical theory of functions of a complex variable*, 2nd edit., Nauka, Moskow, 1966.
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[6] B. Oksendal, *Stochastic Differential Equations*, 6th edition, Springer, Verlag, New York, 2003.
[7] J.M. Steele, *Stochastic calculus and financial applications*, Springer –Verlag, New York, 2001.


# Generalized monotonicities and their applications


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**Abstract:** In this talk several versions of generalized monotone mappings are introduced i.e., $H(\cdot,\cdot)$ - co-coercive mappings, relaxed monotone mappings etc. Further their applications are given to solve variational inclusion problems and equilibrium problems.


**Keywords:** Co-coercive mappings, relaxed monotone mappings, equilibrium problems.

**References:**
[1] R. Ahmad, M. Dilshad, M-M Wong, J.C. Yao, *H(.,.) – co-coercive operator and an application for solving generalized variational inclusions*, Abstract and Applied Analysis, vol. 2011, Article ID 261534, doi: 10.1155/2011/261534, 2011.
[2] A. Kilicman, R. Ahmad, *Generalized equilibrium problem with mixed relaxed monotonicity*, H.A. Rizvi, The Scientific World Journal (accepted to publication).

# Mean value multipoint multivariate Padé approximations

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**Abstract:** Mean value interpolation by polynomials is one of the most natural generalizations of the univariate Lagrange interpolation to the multivariate case. Surprisingly, analogous generalizations of rational interpolation were not constructed. We present multipoint multivariate Padé approximation which generalizes both multivariate (in the scale of interpolations [1, pp.203] it corresponds to $m = k - 1$) and univariate (multipoint Padé approximation) cases.
**Keywords:** multipoint Padé approximation, rational interpolation, mean value interpolation, Lagrange interpolation.
**References:**
[1] B.D. Bojanov, H. Hakopian, B. Sahakian, *Spline Functions and Multivariate Interpolations*, Vol. 248 of Mathematics and Its Applications, Kluwer Academic Publishers, 1993.

# The estimates of widths of classes in the Lorentz space

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**Abstract:** In this talk, Lorentz space with anisotropic norm (see [1]), of periodic functions of many variables is considered. The estimate of the Kolmogorov's widths $d_M(F,X)$, Nikol'ski-Besov-Amanov classes in the space Lorentz is obtained. Let $\bar{p}=(p_1,\dots,p_m)$, $\bar{\theta}=(\theta_1,\dots,\theta_m)$ and $\theta_j, p_j \in (1,+\infty)$, $j=1,\dots,m$, $L^*_{\bar{p},\bar{\theta}}(I^m)$ - Lorentz space with anisotropic norm. $a_{\bar{n}}(f)$. Fourier coefficients of function $f\in L_1(I^m)$ with respect to system $\left\{e^{i\langle \bar{n},\bar{x}\rangle}\right\}_{\bar{n}\in Z^m}$. Suppose that $\langle \bar{y},\bar{x}\rangle=\sum_{j=1}^{m} y_j x_j$, $s_j=1,2,\dots$, $\rho(\bar{s})=\{\bar{k}=(k_1,\dots,k_m)\in Z^m : 2^{s_j-1}\le |k_j|<2^{s_j}, j=1,\dots,m\}$, $\delta_{\bar{s}}(f,\bar{x})=\sum_{\bar{n}\in\rho(\bar{s})} a_{\bar{n}}(f)e^{i\langle \bar{n},\bar{x}\rangle}$. In the Lorentz space $\overset{\circ}{L}{}^*_{\bar{p},\bar{\theta}}(I^m)$, consider Nikol'ski - Besov - Amanov class:

$$^{\circ}S^{\bar{r}}_{\bar{p},\bar{\theta},\bar{\tau}}B=\left\{ f\in \overset{\circ}{L}{}^*_{\bar{p},\bar{\theta}}(I^m): \left\| \left\{ 2^{\langle \bar{s},\bar{r}\rangle}\left\|\delta_{\bar{s}}(f)\right\|^*_{\bar{p},\bar{\theta}}\right\}_{\bar{s}\in Z^m_+}\right\|_{l_{\bar{\tau}}}\le 1\right\},$$

where $\bar{\tau}=(\tau_1,\dots,\tau_m)$, $1<p_j<\infty$, $1\le\theta_j,\tau_j<+\infty$, $j=1,\dots,m$.

*Theorem. Let* $1<\theta^{(1)}_j,\theta^{(2)}_j$, $1\le\tau_j\le\infty$, $j=1,\dots,m$ *and* $0<r_1+\frac{1}{q_1}-\frac{1}{p_1}=\dots=r_\nu+\frac{1}{q_\nu}-\frac{1}{p_\nu}<r_{\nu+1}+\frac{1}{q_{\nu+1}}-\frac{1}{p_{\nu+1}}\le\dots\le r_m+\frac{1}{q_m}-\frac{1}{p_m}$.

*1. If* $1<p_j\le 2<q_j, r_j>\frac{1}{p_j}$, $j=1,\dots,m,$ *then*

$$d_M\left(S^{\bar{r}}_{\bar{p},\bar{\theta}^{(1)},\bar{\tau}}B, L^*_{\bar{q},\bar{\theta}^{(2)}}\right)\le C\left(\frac{\log^{\nu-1}M}{M}\right)^{\left(r_1+\frac{1}{2}-\frac{1}{p_1}\right)}(\log M)^{\sum_{j=2}^{\nu}\left(\frac{1}{2}-\frac{1}{\tau_j}\right)_+}.$$

*2. If,* $2\le p_j<q<\infty$, $j=1,\dots,m, r_1>\beta=\dfrac{\frac{1}{p_1}-\frac{1}{q}}{1-\frac{2}{q}}$, *then*

$$d_M\left(S^{\bar{r}}_{\bar{p},\bar{\theta}^{(1)},\bar{\tau}}B, L^*_{\bar{q},\bar{\theta}^{(2)}}\right)\le C\left(\frac{\log^{\nu-1}M}{M}\right)^{r_1}(\log M)^{\sum_{j=2}^{\nu}\left(\frac{1}{2}-\frac{1}{\tau_j}\right)_+}.$$

**Keywords**: Lorentz space, Besov class, the Kolmogorov width.

**References:**

[1] A.P. Blozinski, *Multivariate rearragements and Banach function spaces with mixed norms*, Transac. Amer. math. soc. , vol. 263, 1, pp. 146-167, 1981.

# Markov-type inequalities for rational functions on several intervals

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**Abstract:** In this study, we give Markov-type inequalities for rational functions on several intervals.

**Keywords:** inequalities in approximation, approximation by rational functions, rational functions.

**References:**

[1] P. Borwein, T. Erdelyi, *Polynomials and Polynomial Inequalities*, Springer, New York, 1995.
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[8] P. Borwein, *Markov's and Bernstein's Inequalities on Disjoint Intervals*, Can. J. Math., XXXIII(1), 201-209, 1981.

# Exact constants for best approximation on the group SU(2)

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**Abstract:** In the present work, we study the properties of the least upper bounds of the best approximation by algebraic polynomials in metrics $L_1$ and $L_\infty$ for classes of convolutions defined on the group SU(2). The exact constants for best approximation by trigonometric polynomials in $L_\infty\ (-\pi,\pi)$ are studied by many authors. Finally, in this paper, we prove that for group SU(2) analog of the Favard-Akhiezer-Krein theorem does not hold.

**Keywords:** best approximation, group SU(2), trigonometric polynomials.

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# Certain definition of half-Hartley transform in the space of generalized functions

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**Abstract:** In this article, we present the definition of the half-Hartley transform in a certain space of generalized functions. We derive operations and obtain some spaces of Boehmians where the transform is well defined.

**Keywords:** Half-Hartley transform, Hartley transform, signal processing, generalized Hartley, Boehmian space.

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# On an approach to the theory of some nonlinear integral equations

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**Abstract:** We are going to discuss some important classes of nonlinear integral equations such as integral equations of Volterra-Chandrasekhar type, quadratic integral equations of fractional orders, nonlinear integral equations of Volterra-Wiener-Hopf type and nonlinear integral equations of Erdelyi-Kober type. Those integral equations play very significant role in applications to the description of numerous real world events. Our aim is to show that the mentioned integral equations can be treated from the view point of nonlinear Volterra-Stieltjes integral equations. The Riemann-Stieltjes integral appeared in those integral equations is generated by a function of two variables. The choice of a suitable generating function enables us to obtain various kinds of integral equations. The results, which will be presented in our talk, come from the papers [2-4] and the book [1].

**Keywords:** integral equation, integral equation of fractional order, Wiener-Hopf integral equation, Erdelyi-Kober integral equation, Riemann-Stieltjes integral, function of bounded variation.

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# Estimating exponential sums associated with polynomial forms

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**Abstract:** Let $f(\underset{\sim}{x})$ be a polynomial in $Z[\underset{\sim}{x}]$, where $\underset{\sim}{x} = (x_1, x_2 \ldots x_n)$ and $p$ is a prime. In this paper, we give an estimate of the multiple exponential sums of the form $S(f; p^{\alpha}) = \Sigma \exp 2\pi i f(x)/p^{\alpha}$. We apply the method of $p$-adic Newton polyhedron to arrive at an explicit estimate of the sum associated with polynomials in terms of the coefficients of their dominant parts. This is done by first estimating the cardinality of common zeros of the partial derivatives of $f(\underset{\sim}{x})$ with respect to the components of $\underset{\sim}{x}$ whose information can be found from the combination of indicator diagrams associated with the Newton polyhedra of the partial derivative polynomials. The estimate of the exponential sums is shown to be dependent on this cardinality. We illustrate our results associated with some lower degree polynomials.

# A new three-step iteration for generalized nonexpansive mappings in a CAT(0) space

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**Abstract:** Suzuki [1] introduced a condition on mappings.
Let $T$ be a self mapping on a subset $K$ of a metric space $(X, d)$. Then, $T$ is said to satisfy Condition (C) (sometimes $T$ is called generalized nonexpansive mapping) if

$$\frac{1}{2} d(x, Tx) \leq d(x, y) \text{ implies } d(Tx, Ty) \leq d(x, y), \tag{C}$$

for all $x, y \in K$.
Karakaya et. al. [2] established a new three-step iteration method in a Banach space as follows.

$x_1 \in K$,
$z_n = (1 - c_n)x_n + c_n T x_n$,

$$y_n = (1 - a_n - b_n)z_n + a_n T z_n + b_n T x_n, \quad (1)$$
$$x_{n+1} = (1 - \alpha_n - \beta_n)y_n + \alpha_n T y_n + \beta_n T z_n,$$
where $\{ a_n + b_n \}, \{\alpha_n + \beta_n\}, \{ c_n \} \subset [0,1]$.
In this study, we apply the new three-step iteration process into a CAT(0) space and prove some theorems on the strong and Δ-convergence of the new three-step iteration for generalized nonexpansive mappings in a CAT(0) space.
**Keywords:** iteration methods, fixed point, CAT(0) space, nonexpansive mapping.
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# On statistical type convergence in uniform spaces

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**Abstract:** The concepts of $\mathscr{F}$-convergence and $\mathscr{F}$-fundamentality are introduced in uniform spaces, generated by some filter $\mathscr{F}$. Its equivalence to the concept of $\mathscr{F}$-convergence in uniform spaces is proved. This convergence generalizes many kinds of convergence, including the well-known statistical convergence.
**Keywords:** *F*-convergence, F-fundamentality, statistical convergence, a uniform space.
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# On a Hardy's inequality

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**Abstract:** For fulfillment of three-weighted Hardy's integral inequalities, the necessary and sufficient conditions on the weighted functions are received.

We consider the following inequality

$$\sup_{x<b}\left(u(x)\int_x^b f(s)ds\right)\le C\left\{\left(\int_J|\rho f|^p ds\right)^{1/p}+\left(\int_J v^p(x)\left(\int_x^b G(s,x)f(s)ds\right)^p dx\right)^{1/p}\right\}, \quad (1)$$

where $f\ge 0$, and the functions $u(\cdot),\rho(\cdot),v(\cdot)$ are non-negative and continuous on $J=(a,b)$, $-\infty\le a<b\le\infty$, which are called as weighted. Let $G$ be integral operator

$$Gf(x)=\int_x^b G(s,x)f(s)ds \quad (2)$$

with $G(s,x)\ge 0$ and $G(s,x)\le G(t,x)$ under $s\le t$. (3)

We denote by $R_p=R_p(J,\rho,v,K)$ a space of measurable functions $f\ge 0$ on $J$, for which the functional

$$\|f\|_{R_p}\equiv\|\rho f\|_p+\|vGf\|_p \quad (4)$$

is finite, where $\|\cdot\|_p$ is norm of the space $L_p$, $1<p<\infty$.

From (3) and (4) it follows that

$$\rho^{-1}(\cdot)\equiv\frac{1}{\rho(\cdot)}\in L_{p'}^{loc},\ v(\cdot)\in L_p(a,t),\ v(\cdot)G(\cdot,t)\in L_p(a,t),\ \forall t\in J.$$

Introduce the following relations: we said that $B<<D$ is true, if a constant $c>0$ exists which may depend on $p$, and inequality $B<cD$ is valid. The relation $B \overset{\cup}{\underset{\cap}{}} D$ means that $B<<D<<B$.

***Theorem 1.*** *Let $1<p<\infty$ and $G$ be the operator of type (2) with non-negative kernel $G(s,x)$, and for it condition (3) is satisfied. Then, inequality (1) is valid if and only if $B<\infty$. Here $A \overset{\cup}{\underset{\cap}{}} C$, where $C$ is the least constant in (1).*

**Keywords:** weighted function, integral operator, fractional derivative, operator of fractional differentiation, Hardy's inequality.

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# Paranormed sequence space of non-absolute type founded using generalized difference matrix

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The present article mainly dwells on introducing the generalized Riesz difference sequence space $r^q(B_u^p)$ that consists of all sequences whose $R_u^q B$-transforms are in the space $l_p$, where $B$ stands for generalized difference matrix. Some topological properties of the new brand sequence space have been investigated as well as $\alpha-$, $\beta-$ and $\gamma-$duals. In addition to this, we have also constructed the basis of $r^q(B_u^p)$. At the end of the article, we characterize a matrix class on the sequence space. These results are more general and more comprehensive than the corresponding results in the literature.

**Keywords**: Paranormed sequence space, alpha-, beta-and gamma-duals and matrix mappings.

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# Some fixed point results of single-valued mapping for c-distance in TVS-cone metric spaces

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**Abstract:** The purpose of this paper is to prove some fixed point theorems in a tvs-cone metric space using c-distances. The presented theorems extend, generalize and improve the corresponding results of Fadail et al. [10], Dordevic et al [8] and the results cited therein. In the last section, we give an example in support of our theorems.
**Keywords:** TVS-cone metric Spaces, c-distance, w-distance.
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# Stability and convergence results for modified Jungck-Picard-S iterative method for asymptotically nonexpansive mappings

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**Abstract:** will denote the set of natural numbers including zero over the course of this study. Let $C$ be a nonempty subset of a Banach space $B$ and $S, T : C \to C$ be two mappings with $T(C) \subseteq S(C)$. Then, we denote the set of all fixed points of $T$ by $F_T$ and the set of common fixed points of $S$ and $T$ by $F$.
It is now considered indisputable that iteration methods are among the most important and useful mathematical tools which are used in solving a wide variety of problems arise in mathematics and other branches of science.
In this study, we introduce a Modified Jungck-Picard-S hybrid type iterative methods as follows:

$$\begin{cases} x_0 \in C, \\ Sx_{n+1} = T^n y_n, \\ Sy_n = (1-\alpha_n^0)T^n x_n + \alpha_n^0 T^n z_n, \\ Sz_n = (1-\alpha_n^1)Sx_n + \alpha_n^1 T^n x_n, n \in \text{¥}, \end{cases} \tag{1}$$

where $\{\alpha_n^i\}_{n=0}^{\infty}, i = \overline{0,1}$, are real sequences in $[0,1]$ satisfying certain control condition(s).
We establish weak stability, weak convergence and strong convergence results for a pair of Jungck asymptotically nonexpansive mappings with the help of the Modified Jungck-Picard-S iterative procedure. An illustrative example is also discussed to show that the iterative procedure (1) converges faster than some iterative procedures in the existing literature.

**Keywords:** modified Jungck-Picard-S iteration method, Jungck asymptotically nonexpansive mappings, fixed points, weak and strong convergence, weak stability.
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# On $\widetilde{X}$-frames and conjugate systems in Banach spaces

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**Abstract:** The generalization of p-frame in Banach spaces is considered in this paper. The concepts of an $\tilde{X}$ -frame and a system conjugate to $\tilde{X}$ -frame are introduced. Analogues of the results on the existence of conjugate system are obtained. The stability of $\tilde{X}$ -frame having a conjugate system is studied.
**Keywords:** p-frames, $\tilde{X}$ -frames, conjugate systems to $\tilde{X}$ .
**References:**
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# Approximation problems in the variable exponent Lebesgue spaces

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**Abstract:** The variable exponent Lebesgue spaces are generalization of the classical Lebesgue spaces replacing the constant exponent $p$ with a variable exponent function $p(\cdot)$. Interest in the variable exponent Lebesgue spaces has increased since 1990s, because of their use in the different application problems in mechanic, especially in fluid dynamic for the modeling of electrorheological fluids and also in the study of image processing and some physical problems (see, for example [3]). Nowadays there are sufficiently wide investigations relating to the fundamental problems of these spaces, in view of potential theory, maximal and singular integral operator theory and others. But the approximation problems in the variable exponent Lebesgue spaces, specially in the complex plane aren't investigated sufficiently wide.
In this study, we consider the direct and inverse problems of approximation theory in the variable exponent Lebesgue spaces of $2\pi$ periodic functions and also in the Smirnov classes of analytic functions defined on the simple connected domains of the complex plane. First, we obtain the direct and inverse theorems in the variable exponent Lebesgue spaces and later using the method developed in the papers [1] and [2] we investigate the similar problems in the variable exponent Smirnov classes and obtain the appropriate results in these spaces. For a given function $f$ the approximation polynomials are constructed by using its series representation with respect to the Faber polynomials of the simple connected domain $G$, where $f$ is defined. For the estimation of the approximation error are used the modulus of smoothness constructed via Steklov means of $f$. Using this modulus are also defined the generalized Lipschitz classes of analytic functions and in these classes the constructive characterization problems are investigated.
**Keywords:** direct theorem, inverse theorem, modulus of smoothness, Faber series, variable exponent Lebesgue spaces, Smirnov classes.
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# On Jensen's inequality involving averages of convex functions

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**Abstract:** Using Wulbert's result from [1], we deduce a method for constructing exponentially convex functions. These functions have monotonicity property with respect to defining parameters. Then constructed exponentially convex functions are used for construction of two and three parameters Cauchy means. Particularly, we generalize recent results on Cauchy means which were proposed in [2], [3].

**Keywords:** Jensen's inequality, log-convex function, exponential convexity.

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# Definition of number of the valid roots of the polynom

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**Abstract:** At calculation of members of a number of Sturm on the basis of computers loss of the importance of number and exponent disappearance are possible. It leads to loss of signs in a number of Sturm that attracts to the wrong definition of number of the valid roots of a polynom.
Problem: to find number of the valid roots of a polynom with the real coefficients.

$$f(p) = a_0 p^n + a_1 p^{n-1} + \ldots + a_{n-1} p + a_n \qquad (1)$$

Use the following ranges of change of a variable in system (1) $-\infty \le p \le \infty$, $-\infty \le p \le 0$, $0 \le p \le \infty$ and $V(p)$ - number of changes of signs in Sturm system interest. More effective and less labor-consuming method of creation of system of Sturm is offered. We will put $f_1(p) = f'(p)$. Then divide $f(p)$ on $f'(p)$ and the rest from this division, taken with the return sign, we accept for $f_2(p)$:

$$f_{k-2}(p) = f_{k-1}(p) g_{k-1}(p) - f_k(p), \tag{2}$$

then from the rest we take a derivative

$$f_{k+1}(p) = -f_k{}'(p), \tag{3}$$

where $f_0(p) = f(p)$, $f_1(p) = f'(p)$.

Steps (2), (3) we repeat for finding of polynoms $f_{k-1}$, $f_k$, $k = 2,3,\ldots$ and so on until we will receive. In an existing method [1] operation (2) repeats until then yet we won't receive a constant $f_m(p) = const$. In an existing method [1] operation (2) repeats until then yet we won't receive a constant. The system of polynoms constructed by us meets all requirements of the theorem of Sturm

$$f(p) = f_0(p), f'(p) = f_1(p), f_2(p), \ldots, f_m(p) \tag{4}$$

**Lemma.** Differences $V(-\infty)$ - $V(\infty)$, $V(-\infty)$ - $V(0)$, $V(0)$ - $V(\infty)$ are equal to number of the valid, negative and positive roots of a polynom (1) respectively. Less labor-consuming new system (4) by definition of number of the valid roots of a polynom has smaller number of changes of signs, than in an existing method of Sturm.

**Keywords:** polynom, number of Sturm, roots, theorem of Sturm.

**References:**

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# Weighted inequalities of *n*-multiple discrete Hardy operators on cones of monotone functions

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**Abstract:** Let $1 < p, q < \infty$, $n \ge 1$ and $\frac{1}{p} + \frac{1}{p'} = 1$. Let $v = \{v_k\}_{k=1}^{\infty}$ be positive, $\omega_i = \{\omega_{i,k}\}_{k=1}^{\infty}$, $i = 1,2,\ldots,n-1$ and $u = \{u_k\}_{k=1}^{\infty}$ non-negative number sequences, which we in the sequel call weighted sequences.

Let $l_{p,v}$ be the space of sequences $f = \{f_k\}_{k=1}^{\infty}$ for which the following norm is finite:

$$\|f\|_{p,v} = \left( \sum_{i=1}^{\infty} |v_i f_i|^p \right)^{\frac{1}{p}}.$$

Here we investigate the following weighted estimate

$$\|S_{n-1}f\|_{q.u} \le C\|f\|_{p,v}, \tag{1}$$

for all non-negative non-increasing sequences $f=\{f_k\}_{k=1}^{\infty}$, when $1<q<p<\infty$, where $S_{n-1}$ - $n$-multiple discrete Hardy operator:

$$(S_{n-1}f)_i = \sum_{k_1=1}^{i}\omega_{1,k_1}\sum_{k_2=1}^{k_1}\omega_{2,k_2}\dots\sum_{k_{n-1}=1}^{k_{n-2}}\omega_{n-1,k_{n-1}}\sum_{j=1}^{k_{n-1}}f_j,\ i\ge 1.$$

In paper [1] necessary and sufficient conditions of validity inequality (1) are obtained for the case $1<p\le q<\infty$.

**Keywords:** weighted inequality, discrete Hardy operator, monotone functions.

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# On estimates of M-term approximations of classes in a symmetric space

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**Abstract:** Symmetric space of periodic functions of many variables and Nikol`ski–Besov class in this space are considered. Inequalities of different metrics for trigonometric polynomials are proved. The exact order of the best M −term approximation of Nikol`ski–Besov class in symmetric space is obtained.

Let $I^m = \{\bar{x}\ (x_1,\dots,x_m)\in R^m;\ 0\le x_j\le 1;\ j=1,\dots,m\}$, $X(\varphi)$ −separable, symmetric space of Lebesgue with fundamental function $\varphi$ defined in $I^m$. $\|f\|_X$ is the norm of element $f\in X(\varphi)$[1]. For function $\varphi(t)$, $t\in[0,1]$, suppose that

$$\alpha_\varphi = \underline{\lim}_{t\to 0}\frac{\varphi(2t)}{\varphi(t)},\quad \beta_\varphi = \overline{\lim}_{t\to 0}\frac{\varphi(2t)}{\varphi(t)}.$$

Let $1<\alpha_\varphi\le\beta_\varphi<2$, $1\le\theta\le\infty$ and $r>0$. In the symmetric space Nikol`ski–Besov class $B^r_{X,\theta}$ is considered. The quantity $e_M(f)_X$ is the best M −term approximation of a function $f\in X(\varphi)$ [2]. For a given class $F\subset X(\varphi)$ let $e_M(F)_X = \sup_{f\in F} e_M(f)_X$. So, the following statement is true.

***Theorem 1.*** *Let* $X(\varphi)$ − *symmetric space and*
$0<\frac{1}{q}<\frac{1}{2}\le log_2\,\alpha_\varphi\le log_2\,\beta_\varphi<1,\quad 1<\tau<\infty,\quad 1\le\theta\le\infty.$ *If*

$\frac{1}{2} \le \frac{1}{q} < log_2\, \alpha_\varphi \le log_2\, \beta_\varphi < 1 \quad and\ log_2\, \beta_\varphi - \frac{1}{q} < \frac{r}{m},\ then$

$$e_M\left(B^r_{X,\theta}\right)_{L_{q,\tau}} \asymp \frac{M^{-\left(\frac{r}{m}+\frac{1}{q}\right)}}{\varphi(M^{-1})}.$$

$If\ \ 0 < \frac{1}{q} < log_2\, \alpha_\varphi \le log_2\, \beta_\varphi \le \frac{1}{2} \quad and\ \ \frac{r}{m} > \frac{1}{2},\ then\, e_M\left(B^r_{X,\theta}\right)_{L_{q,\tau}} \asymp M^{-\frac{r}{m}}.$


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# Matrix transformations on matix domains of triangles


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**Abstract:** Most matrix methods of summability are given by triangles. So it is of interest to study matrix transformations on matrix domains of triangles. We present a unified approach to the solution of the main problems that arise in this context, namely the determination of the duals of those spaces, characterizations of matrix transformations and compact matrix operators on them.


# Oscillatory conditions of the nonlinear second order differential equation


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**Abstract:** In this study, we consider the nonlinear second order differential equation

$$(\rho(t)|y'(t)|^{p-2}\, y'(t))' + \vartheta(t)|y(t)|^{p-2}\, y(t) = 0, \quad t > 0 \qquad (1)$$

Let $I = (0,\infty)$, $1 < \rho < \infty$ and $\rho(\cdot)$ be positive continuous function.

The function $y: I \to R$ is called a solution of the equation (1), if $y(t)$ and $\rho(t)|y'(t)|^{p-2}y'(t)$ are continuously differentiable functions, and it satisfies the equation (1) for all $t>0$.

For $p=2$, the equation (1) becomes the Sturm-Liouville linear equation

$$(\rho(t)y'(t))' + \vartheta(t)y(t) = 0, \tag{2}$$

There are a lot of studies on the qualitative properties of equations (1), (2) such as conjugate, disconjugate in a given interval and the oscillatory, nonoscillation of equations (1) and (2) for $t=\infty$ . For nonnegative , these properties of the equations (1) and (2), were studied in [1].

In this paper, we investigate the questions of conjugacy, oscillatory of the equations (1) and (2) for the function which sign changes.

**Keywords:** conjugate, disconjugate, oscillatory, nonoscillation, conjugate points.

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# Weighted inequalities of a class of semiadditive operators and their applications

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**Abstract:** Let $0<\theta, q\le\infty, \frac{1}{p}+\frac{1}{p'}=1$ and $I=(0,\infty)$. Let $u(\cdot)$, $w(\cdot)$ and $v(\cdot)$ be positive functions locally summable on I.

We introduce the following operators:

$$T_\theta^+ f(x) = \left(\int_0^x w(t)\left|\int_t^x f(s)ds\right|^\theta dt\right)^{\frac{1}{\theta}}, \quad T_\theta^- f(x) = \left(\int_x^\infty w(t)\left|\int_x^t f(s)ds\right|^\theta dt\right)^{\frac{1}{\theta}}.$$

The operators $T_\theta^+$ and $T_\theta^-$ are sublinear for $\theta>1$ and superlinear for $0<\theta<1$. These operators become linear for $\theta=1$.

We consider the weighted inequality:

$$\left(\int_0^\infty u(x)\left|T_\theta^\pm f(x)\right|^q dx\right)^{\frac{1}{q}} \le C^\pm\left(\int_0^\infty v(t)|f(t)|^p dt\right)^{\frac{1}{p}} \tag{1}$$

Let $\Delta_\theta^+ g(x) = \left( \int_0^x w(t) |g(x) - g(t)|^\theta dt \right)^{\frac{1}{\theta}}$ and $\Delta_\theta^- g(x) = \left( \int_x^\infty w(t) |g(t) - g(x)|^\theta dt \right)^{\frac{1}{\theta}}$ be the $\theta$ - mean deviations with the weight $w$ of the value of a function $g$ from $g(x)$ on the intervals $(0,x)$ and $(x,\infty)$, respectively. Then inequality (1) is equivalent to the following inequality:

$$\left( \int_0^\infty u(x) |\Delta_\theta^\pm g(x)|^q dx \right)^{\frac{1}{q}} \le C^\pm \left( \int_0^\infty v(t) |g'(t)|^p dt \right)^{\frac{1}{p}}. \qquad (2)$$

Inequalities (1), (2) are studied in paper [1] for $1 \le p \le q < \infty$ and $0 < \theta < \infty$. In this work, we investigate inequalities (1), (2) for $\{\theta, q\} < p$ and present some applications which expand the results of [2].

**Keywords:** weighted inequality, superlinear operator, sublinear operator.

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# Hardy type inequalities on the cone of monotone sequences

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**Abstract:** Weighted Hardy type inequalities restricted to the cones of monotone functions and sequences, as well as their applications in the estimation of maximal functions, in the theory of interpolation of operators, in the embedding theory and their relations with the stochastic inequalities have been extensively studied within the last two decades. We study Hardy type inequality on the cone of non-negative and non-increasing sequences under weaker conditions than those studied before in the literature. We introduce a general class of matrices, which includes well-known classical operators such as the operator of multiple summation, Holder's operator, Cesaro's operator and others. Such classes of matrices are wider than those previously studied in the theory of discrete Hardy type inequalities. We investigate two-sided estimates for matrix operators on the cone of non-negative and non-increasing sequences, when the corresponding matrices belong

to such classes. We obtain new results, which generalize the known results of this subject.

**Keywords:** inequalities, weighted matrix inequalities, weights, matrix operators, summable matrix, summation methods.

# A new sequence space and A-statistical convergence

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**Abstract:** In the present paper, we introduce and study an idea of $\lambda$- strong (A, $\varphi$)- convergence with respect to a modulus function.
We also study some connections between $\lambda$- strong (A$\varphi$)-convergence with respect to a modulus and $\lambda$- statistical convergence.

**Keywords :** modulus function, $\lambda$-statistical convergence , $\varphi$-function, statistical convergence.

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# One step iteration scheme for multivalued mappings in CAT(0) spaces

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**Abstract:** In this paper, we introduce one step iteration scheme involving two multivalued nonexpansive mappings in CAT(0) spaces and utilize the same to prove Δ-convergence as well as strong convergence theorems with and without end point conditions. Our results generalize and extend several relevant results in the literature.
**Keywords:** CAT(0) space, fixed point, Δ-convergence and Opial's property.
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# ANALYSIS 2

## Spectral decompositions related to the polyharmonic operators

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**Abstract:** In this work, we study the localization problems of the differential operators in closed domain. Riesz means of the eigenfunction expansions are investigated. The properties of the functions from Nikolskii classes are used.
**Keywords:** polyharmonic operator, spectral expansions, eigenvalues, eigenfunctions, convergence.
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# Conditions for uniformly convergence of expansions of continuous functions from Nikolskii classes in eigenfunction expansions of the Schrodinger operator

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**Abstract:** In this work, the spectral expansions of the Schrodinger operator on closed domain are investigated. Using the asymptotical estimation of spectral function of Schrodinger operator on closed domain uniformly convergence of eigenfunction expansions of continuous functions from Nikolskii classes is proved. Properties of the continuous functions from Nikolskii classes are used.
**Keywords:** Schrodinger operator, eigenfunction expansions, Riesz means, uniformly convergent.

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# On the spectrum of Volterra integral equation with the "incompressible" kernel

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**Abstract:** We consider the singular Volterra integral equation of the second kind which has the "incompressible" kernel

$$\varphi(t) - \lambda\int_0^t K(t,\tau)\,\varphi(\tau)\,d\tau = f(t), \quad t > 0, \tag{1}$$

where

$$K(t,\tau) = \frac{1}{2a\sqrt{\pi}}\left\{\frac{t+\tau}{(t-\tau)^{3/2}}\exp\left(-\frac{(t+\tau)^2}{4a^2(t-\tau)}\right) + \frac{1}{(t-\tau)^{1/2}}\exp\left(-\frac{t-\tau}{4a^2}\right)\right\}.$$

The feature of equation (1) follows from the limit relations for the kernel $K(t,\tau)$

$$\lim_{t\to 0}\int_0^t K(t,\tau)\,d\tau = 1, \ \lim_{t\to +\infty}\int_0^t K(t,\tau)\,d\tau = 1.$$

We assume that $|\lambda| > 1$. Case $|\lambda| = 1$ was considered in [1] and [2]. After allocating the characteristic part, we present equation as an equation with a difference kernel [3]. Further, the initial equation is reduced to the Abel equation. As a result, it is proved the solvability of equation (1) for any function $f(t):\ \sqrt{t}\,f(t) \in L(0;\infty) \cap C(0;\infty)$ and the presence of eigenfunctions, which are found in explicit form. The equation has a continuous spectrum, and the multiplicity of the characteristic numbers grows with increasing $|\lambda|$.

**Keywords:** incompressible kernel, eigenfunction, Abel equation.

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# Fractional spaces generated by the positive differential operator in the half-line $\mathbb{R}^+$ and their applications

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**Abstract:** In this study, we investigate the structure of fractional spaces generated by the $2m$-th order differential operator

$$A^x = (-1)^m a(x)\frac{d^{2m}}{dx^{2m}} + \delta I$$

acting in the space of all continuous functions defined on the half-line $\mathbb{R}^+ = [0,\infty)$. Here, $\delta>0$ is the sufficiently large number and $a(x)$ is a continuous function defined on $\mathbb{R}^+$ with $0 < s_1 \leq a(x) \leq s_2 < \infty$.
The domain of this operator is

$$D(A^x) = \{u \in C^{2m}(\mathbb{R}^+): u(0) = u'(0) = \cdots = u^{(m-1)}(0) = 0\}.$$

**Keywords:** fractional spaces, positive operators, Green function.

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# The positivity of the second order difference operator with periodic conditions in Holder spaces and its applications

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**Abstract:** In this study, the second order of approximation of difference operator $A_h^x$ approximates the second order differential operator $A^x$ defined by the formula
$A^x = -u_{xx}(x) + \delta u(x), \delta > 0$
with domain
$$D(A^x) = \left\{u(x): u(x), u'(x), u''(x) \in C(R^1), u(x) = u(x+2\pi), x \in R^1, \int_0^{2\pi} u(x)\,dx = 0\right\}$$
is considered. The Green's function of the difference operator $A_h^x$ is constructed. The estimates for the Green's function are obtained. The positivity of operator $A_h^x$ in the Banach space $C(R_{1h})$ of periodic mesh functions defined on $R_{1h}$ is established. Here, $R_{1h} = \{x_k = kh, k = 0, \pm 1, \pm 2 \ldots\}$. It is proved that for any $\alpha \in \left(0, \frac{1}{2}\right)$, the norms in spaces $E_\alpha = E_\alpha(C(R_{1h}), A_h^x)$ and $C^{2\alpha}(R_{1h})$ are equivalent. The positivity of the operator $A_h^x$ in the Holder spaces of $C^{2\alpha}(R_{1h}), \alpha \in \left(0, \frac{1}{2}\right)$ is proved.

**Keywords:** positivity of difference operators, fractional space, nonlocal boundary conditions, Green's function.

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# Self-adjoint extensions of singular third-order differential operator and applications

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**Abstract:** In this work, a selfadjoint extensions of the minimal operator generated by symmetric linear differential expressions for third order with selfadjoint operator coefficients in the Hilbert space of Hilbert space valued functions defined in right and separated left-right semi-axes in terms of boundary values are investigated. The obtained results are supplied by the applications in both cases.
**Keywords:** selfadjoint and normal extensions.

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# On the basis properties of root functions of two generalized eigenvalue problems

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**Abstract:** We consider the spectral problem for a model second-order differential operator with an involution. The operator is given by the differential expression $Lu = -u''(-x)$ and boundary conditions. We obtain a criterion for the basis property of the systems of Eigen functions of this operator in terms of the coefficients in the boundary conditions.
Convergence issues for spectral expansions associated with nonself-adjoint differential operators are one of the key fields of spectral theory. A plethora of results recently obtained with the use of V.A. Il'in's theory for classical and nonclassical differential operators can be found in the survey [1]. Lomov's results [2] for loaded differential operators and Tikhomirov's results [3] for differential operators with deviating arguments are the closest to the topic of the present paper.
In the space $L_2(-1, 1)$ consider the nonclassical operator $L$ given by the differential expression

$$-u''(x) = \lambda u(-x), -1 < x < 1, \tag{1}$$

and the boundary conditions of the form

$$u'(-1) = \alpha u'(1), u(-1) = u(1), \tag{2}$$

or the boundary conditions of the form

$$u'(-1) = u'(1), u(-1) + \alpha u(1) = 0. \tag{3}$$

We prove that in the case $\alpha^2 \neq 1$, the system of root functions of the boundary eigenvalue problems (1), (2) and (1), (3) form a Riesz basis in $L_2(-1, 1)$.
**Keywords:** eigenfunctions, basis, differential operator with involution.

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# Nonselfadjoint correct restrictions and extensions with real spectrum

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**Abstract:** In this work, we obtain sufficient conditions for the reality of the spectrum of some correct restrictions and extensions of operators. It is shown, that the system of eigenvectors forms a Riesz basis in the case of discrete spectrum.
We consider the densely defined minimal operator $L_0$ in Hilbert space. Let $L_0$ be a symmetric operator. Then $\hat{L} = L_0^*$ is the maximal operator (see [2]).

Using a known fixed boundary correct restriction $L$ of maximal operator to describe all possible correct restrictions $L_K$ of a maximal operator $\hat{L}$ in terms of an inverse operator was described in [2] as following

$$L_K^{-1} f = L^{-1} f + Kf, \quad \text{for every } f \in H, \tag{1}$$

where $K$ is an arbitrary bounded operator in Hilbert space $H$, satisfying the condition $R(K) \subset \ker \hat{L}$.

***Theorem 1.*** *Let $L_0$ be a symmetric linear operator, $L$ - be a fixed selfadjoint correct extension of the operator $L_0$ in Hilbert space $H$. Then, if $KL$ is a bounded operator and $I + KL > 0$ or $I + KL < 0$, then the correct restrictions $L_K$ have only real spectrum where $K$ is from (1).*

**Keywords:** correct restriction, correct extension, Riesz basis.

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# Functions under a linear operator

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**Abstract:** The aim of this paper is to establish certain inclusion relationships for some subclasses of analytic functions using subordination properties. Some applications of our work to the Bernardi integral operator is also given.
**Keywords:** analytic function, conic domains, unit disc.

# Characteristic determinant of the spectral problem for the ordinary differential operator with the boundary load

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**Abstract:** We consider problem for the differential equation:

$$l(u) = u^{(n)}(x) + q_2(x)v^{(n-2)}(x) + \cdots + q_n(x)u(x) = \lambda u(x), \quad 0 < x < 1 \qquad (1)$$

with the integral boundary value conditions:

$$U_j(y) \equiv \sum_{k=0}^{n\text{-}1}\left[\alpha_{jk}u^{(k)}(0) + \beta_{jk}u^{(k)}(1)\right] + \int_0^1 \overline{p_0(x)}u(x)dx = 0, \quad j = \overline{1,n}, \qquad (2)$$

where $p_{0j}(x) \in L_2(0,1)$.

We assume that coefficients of the equation: $q_k(x) \in C^{n\text{-}k}[0,1], \quad k = \overline{2,n}$; and forms $U_j(u)$ are linear independent and refer to strongly regular boundary value conditions.

Let $L_1$ be an operator in $L_2(0,1)$, given by the expression (1) and «perturbed» boundary value conditions:

$$U_j(u) = 0, \quad j = \overline{1,n}, \quad j \neq m,$$
$$U_m(u) = \int_0^1 \overline{p_m(x)} u(x) dx, \quad p_m(x) \in L_2(0,1). \tag{3}$$

This direction is closely connected with the study of operators with potential containing a delta function, but also has its peculiarities. Nowadays this direction is under the stage of accumulation of primary information, for which it is necessary to obtain formulas of the explicit form to compute eigenvalues and study their asymptotics. An important step in this direction is to construct explicitly or identify structure of the characteristic determinant of the spectral problem.

**Keywords:** spectral problem, operator with an integral perturbation, systems of root functions, strongly regular boundary conditions.

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# Weyl numbers of diagonal matrices

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**Abstract:** In this work, a connection between Weyl numbers of infinite block diagonal matrix with linear bounded operator elements in the direct sum of Banach spaces and its coordinate operators has been investigated.

**Keywords:** direct sum of operators, spectrum and resolvent sets, approximation and Weyl numbers of operator.

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# Structure of spectrum of solvable pantograph differential operators for the first order

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**Abstract:** In this work, all solvable extensions of the minimal operator generated by linear pantograph-type delay differential-operator expression for first order in Hilbert space of vector-functions at finite interval in terms of boundary values have been described based on M.I. Visik's result. Later on, the spectrum structure of these extensions has been researched.
**Keywords**: pantograph differential operators, solvable extension, spectrum, resolvent operator.

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# Weak normal classes of substitution operators

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**Abstract:** In this article, we discuss measure theoretic characterizations for weighted composition operators in some operator classes on $L_2(\Sigma)$ such as, n-power normal, n-power quasi-normal, k-quasi-paranormal and quasi-class A. Then we show that weighted composition operators can separate these classes.
**Keywords:** Conditional expectation, weighted composition operator, n-power normal, *k*-quasi-paranormal.

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# On spectral zeta functions for a non-local boundary value problem of the Laplacian

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**Abstract:** By using the Feynmann-Kac formula and spherical rearrangement, Luttinger proved that a ball is the maximizer of the partition function of the Dirichlet Laplacian among all domains of the given measure for all positive values of time [1]. From the Luttinger's result it follows that the real valued spectral zeta function of the Dirichlet Laplacian is also maximized in a ball among all domains of the given measure.
In this talk using methods of probability theory [2], we generalize above results for a non-local Laplacian [3]. We mainly concern ourself with bounded domains in three-dimensional Euclidian space for the real spectral zeta function of a non-local boundary value problem of the Laplacian.
**Keywords:** spectral zeta functions, Laplacian, non-local boundary value problems.
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# Fourier transform and Pseudo-differential operators

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**Abstract:** We consider the development of the Fourier analysis based on a boundary value problem for the derivative operator on a segment. In particular, we derive an explicit formula for the convolution generated by the problem. We start in

direction of discrete analysis based on elliptic boundary value problems, continuing, in a sense, the analysis on the torus that introduced by M. Ruzhansky and V. Turunen [1], in which case one may think of a problem having periodic boundary conditions.

**Keywords:** Fourier transform, pseudo-differential operator.

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# Some properties of the Schur complement of a block operator matrix

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**Abstract:** Let $T^d$ be the $d$-dimensional torus, $L_2(T^d)$ be the Hilbert space of square integrable functions defined on $T^d$ and $L_2^s((T^d)^2)$ be the Hilbert space of square integrable symmetric functions defined on $(T^d)^2$. Denote by $H$ the direct sum of Hilbert spaces $H_1 := L_2(T^d)$ and $H_2 := L_2^s((T^d)^2)$.
In the Hilbert space $H$, we consider the following block operator matrix

$$A := \begin{pmatrix} A_{11} & A_{12} \\ A_{12}^* & A_{22} \end{pmatrix} \qquad (1)$$

with the entries $A_{ij} : H_j \to H_i, i \leq j, i, j = 1,2$:

$$(A_{11} f_1)(p) = u(p) f_1(p), \ \ (A_{12} f_2)(p) = \int_{T^d} v(s) f_2(p,s) ds,$$

$$(A_{22} f_2)(p,q) = w(p,q) f_2(p,q), \ \ f_i \in H_i, \ i = 1,2.$$

Here $u(\cdot)$ and $v(\cdot)$ are real-valued continuous functions on $T^d$, and $w(\cdot,\cdot)$ is a real-valued continuous symmetric function on $(T^d)^2$.

For a $2 \times 2$ block operator matrix (1), there exist two Schur complements, the first of them is formally given by:

$$S_1 : C \setminus \sigma(A_{22}) \to H_1, \ \ S_1(\lambda) := A_{11} - \lambda - A_{12}(A_{22} - \lambda)^{-1} A_{12}^*, \ \ \lambda \in \rho(A_{22}).$$

$S_1(\cdot)$ is an analytic operator function defined outside of the spectrum $A_{22}$ and were first used in the theory of matrices [1]. We recall that in the theory of bounded and unbounded block operator matrices, Schur complements are powerful tools to study the spectrum and various spectral properties [2, 3].

In this work, the several spectral properties of the first Schur complement $S_1(\lambda)$ related with the number of eigenvalues are studied. In one dimensional case

($d=1$) for special class of parameter functions of $A$ the existence of the infinitely many eigenvalues of $S_1(\lambda)$ is established. The eigenvalues, its multiplicities and the corresponding eigenvectors are founded.

**Keywords:** block operator matrix, Schur complement, eigenvalues, and spectrum.

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# Spectral properties of some non strongly regular boundary value problems for fourth order differential operators

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**Abstract:** In this paper, we consider some class of not strongly regular [1] boundary value problems for fourth order differential operator, which is defined by the differential expression

$$y^{IV}(x)=\lambda y(x) \tag{1}$$

and with the two-point boundary conditions on the interval (0,1):

$$\begin{aligned}
&y'''(0)+a_{13}y''(0)+a_{14}y''(1)+a_{15}y'(0)+a_{16}y'(1)+a_{17}y(0)+a_{18}y(1)=0,\\
&y'''(1)+a_{23}y''(0)+a_{24}y''(1)+a_{25}y'(0)+a_{26}y'(1)+a_{27}y(0)+a_{28}y(1)=0,\\
&y''(0)+a_{35}y'(0)+a_{36}y'(1)+a_{37}y(0)+a_{38}y(1)=0,\\
&\sqrt{2}y'(0)+y'(1)+a_{47}y(0)+a_{48}y(1)=0.
\end{aligned} \tag{2}$$

***Theorem 1.*** *Eigenvalues of the problem (1)-(2) form two series* $\lambda_{k,1},\lambda_{k,2}:|\lambda_{k,1}-\lambda_{k,2}|\to 0, k\to\infty$*, and eigenfunctions which correspond to these eigenvalues have the form*

$$y_{k,1}(x) = \cos(\sqrt[4]{\lambda_{k,1}}x) - \sin(\sqrt[4]{\lambda_{k,1}}x) + o(\frac{1}{k}),$$

$$y_{k,2}(x) = \cos(\sqrt[4]{\lambda_{k,2}}x) - \sin(\sqrt[4]{\lambda_{k,2}}x) + o(\frac{1}{k}),$$

*where k is sufficiently large integer.*

***Theorem 2.*** *System of eigenfunctions* $\{y_{k,1}(x), y_{k,2}(x)\}$ *of the boundary-value problem (1)-(2) does not form a basis in* $L_2(0,1)$.

The proof is based on the application of the necessary conditions for the basis in the Hilbert space [2].

**Keywords:** asymptotic formulas, eigenvalue, eigenfunction, not strongly regular boundary conditions.

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# About an approach to the fourth-order operator bundles

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**Abstract:** We consider the bundle form [1]

$$Au \equiv (\alpha^2 - 1)u^{(4)}(x) - 2\alpha\lambda^2 u''(x) + \lambda^4 u(x), \quad -1 < x < 1 \tag{1}$$

in the space $L_2(-1,1)$, where α is a complex number different from zero and $\pm 1$, λ is spectral parameter. The expression (1) should be considered together with the spectral problem [2]

$$-u''(-x) + \alpha u''(x) = \lambda^2 u(x), \quad -1 < x < 1 \tag{2}$$

Here, $\alpha \neq 0$, $\alpha \neq \pm 1$ is a complex constant.

The first term in equation (2) contains involution. We assume that the spectral problem (2) is considered with some boundary conditions, the form of which we

have not listed yet. There is following relation between the eigenfunctions of the spectral problem (2) and the operator bundle (1).

***Theorem 1.*** *The eigenfunctions of the spectral problem (2) are the eigenfunctions of the operator bundle (1).*

We consider the fourth-order operator bundle (1) with boundary conditions

$$u(-1)=0, \qquad u(1)=0, \quad u''(-1)=0, \quad u''(1)=0. \tag{3}$$

***Theorem 2.*** *The system eigenfunctions of the bundle (1), (3) contains subsystem*

$$u_{k1}(x)=cos\left(\frac{\pi}{2}+k\pi\right)x, \;\; k=0,1,2,\dots,u_{k2}=sink\pi x, \qquad k=1,2,\dots,$$

*which forms Riesz basis in the space* $L_2(-1,1)$.

**Keywords:** operator bundle, eigenfunctions, basis, differential operator with involution.

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# The third-order polynomial bundle and eigenfunctions

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**Abstract:** We consider the bundle [1] of the third order with involution

$$Au=(\alpha^2-1)u'''(x)-\alpha\lambda^2u'(-x)+\lambda^3u(-x), \quad \alpha\neq 0, \alpha\neq \pm 1$$

in the space $L_2(-1,1)$ and the corresponding it equation

$$(\alpha^2-1)u'''(x)-\alpha\lambda^2u'(-x)+\lambda^3u(-x)=0, -1<x<1. \tag{1}$$

Together with equation (1), we study the following spectral problem [2]

$$-u'(-x)+\alpha u'(x)=\lambda u(x), \; -1<x<1, \; \alpha\neq 0, \alpha\neq \pm 1 \tag{2}$$

with some boundary conditions. The equation (2) contains involution.

The connection between the eigenfunctions of the bundle A and the spectral problem (2) is established in the following theorem.

***Theorem 1.*** *The eigenfunctions of the spectral problem (2) are the eigenfunctions of the bundle A and satisfies the equation (1).*

The basis property of subsystem eigenfunctions of the operator bundle A with boundary conditions is established in the following theorem.

$u(-1) = u(1),\ u'(-1) = u'(1),\ u''(-1) = u''(1).$ (3)

***Theorem 2****. The following functions*

$$u_k(x) = -\sqrt{\frac{1-\alpha}{\alpha+1}} cosk\pi x + sink\pi x, k = 0, \pm 1, \pm 2, \ldots,$$

*are eigenfunctions of the operator bundle (1), (3), corresponding to the* $\lambda_k = -\sqrt{1-\alpha^2} k\pi,\ k = 0, \pm 1, \pm 2, \ldots,$ *and form a Riesz basis in* $L_2(-1,1)$*.*

**Keywords:** operator bundle, eigenfunctions, basis, differential operator with involution.

**References:**

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# Periodic problem for first-order equations with deviating argument

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**Abstract:** In the present work, we study spectral characteristic marginal problem:

$$y'(x) = \lambda y(\alpha - x), y(0) = y(2\pi),$$

where $0 \ll \alpha \leq 2\pi, \lambda$ is a spectral parameter.

We consider in the space $L^2(0,2\pi)$ periodic problem

$$y'(x) = \lambda y(\alpha - x), \quad (1)$$

$$y(0) = y(2\pi), \quad (2)$$

where α is real value satisfying condition $0 \leq \alpha < 2\pi$, and $\lambda$ is a spectral parameter.

Similar problem was considered in earlier work [1]

$$y'(x) = \lambda y(1 - x),$$

$$\alpha y(0) + \beta y(1) = 0,$$

where α, β are complex numbers, λ is spectral parameter. In our case, a magnitude α does not coincide with the end of the interval, i.e. α ≠ 2π and this significantly affects the results. It turns out that the problem (1) - (2) has a complete and orthogonal system of eigenvectors and also another series of eigenvectors, which form an incomplete system in $L^2(0,2\pi)$. When $\alpha = 2\pi$ second series disappear and the results coincide with the results of work [1].

It is known that [2], the self-adjoint and quite continuous operator has a complete and orthogonal system of eigenvectors and it does not have other eigenvectors. Not completely continuous, but self-adjoint operator can have a complete orthogonal system of eigenvectors and real eigenvalues, corresponding to them.

**Keywords:** eigenvalues, eigenfunctions, boundary conditions, adjoined function.

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# About the strong solvability of mixed problems for first order equations with deviating argument

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**Abstract:** In this study, we have obtained the criteria for strong solvability of a mixed problem for functional differential equations, by methods of spectral theory of operators.

***Statement of the problem.***

Let $\Omega \subset R^2$ is a quadrangle bounded by segments: $AB: 0 \le y \le 1,\ x = 0$; $BC: 0 \le x \le 1$, $y = 1$; $CD: 0 \le y \le 1,\ x = 1$; $DA: 0 \le x \le 1,\ y = 0$.

The set of functions $u(x,y)$ denoted by $C^{1,1}(\Omega)$, which are continuously differentiable in $\Omega$, by variables $x$ and $y$. The set of segments explains the boundary of domain $\Omega$ [1].

We consider the following mixed boundary value problem

$$Lu = iu_x(x,y) + u_y(x,1-y) = f(x,y), (x,y) \in \Omega, \quad (1)$$

$$u\big|_{y=0} = 0, 0 \le x \le 1, \quad (2)$$

$$u\big|_{x=0} = \alpha u\big|_{x=1}, \quad 0 \le y \le 1, \quad (3)$$

where $|\alpha|=1$, $f \in L^2(\Omega)$.

Investigation task corresponds to some linear operator in Hilbert space [2]. Strong solvability of the mixed problem is reduced to the investigation of a limit convertibility of this operator.

**Keywords:** continuous spectrum, self-conjugate operator, orthonormal base.

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# The deficiency indices of singular differential operators in vector-valued functions space

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**Abstract:** In this work, we consider the minimal differential operator $L_0$ in the space $H = L^2(0,\infty) \oplus L^2(0,\infty)$ which is generated by the following differential expression:

$$L_0(y) = y^{(4)} + Q(x)y. \tag{1}$$

Here $y(x)=(y_1(x),y_2(x))$, $0<x<\infty$ and $Q(x)=\|q_{ij}\|^2_{i,j=1}$ – is a real symmetric matrix, whose eigenvalues $\mu_1(x) \to +\infty$, $\mu_2(x) \to -\infty$, for $x \to \infty$.

Introduce $\varphi(x) = \frac{1}{2} arctg \frac{q_{22}-q_{11}}{2q_{12}}$. The function $\varphi(x)$ is called as the speed of the rotation of eigenvectors of the matrix $Q(x)$.

***Theorem 1.*** *Assume that for sufficiently large* $x_0$ *and* $x > x_0$ *the following inequalities*

1) $|\varphi'(x)| < const,$

2) $0 < A \le \left|\frac{\mu_i(x)}{\mu_j(x)}\right| \le B, \ i,j = 1,\ 2,$

3) $\int_{x_0}^{\infty} \left|\mu_i^{-\frac{1}{4}}(x)\right| dx < \infty, \ \int_{x_0}^{\infty} \left|\frac{\mu_i'^2(x)}{\mu_i^{\frac{9}{4}}(x)} + \frac{\mu_i''(x)}{\mu_i^{\frac{5}{4}}(x)}\right| dx < \infty, \ \int_{x_0}^{\infty} \left|\frac{\varphi''(x)}{\mu_i^{\frac{1}{4}}(x)}\right| dx < \infty, \ i = 1,2$

4) $|\mu_i'(x)| \le C\,|\mu_i(x)|^{\alpha}, \ C = const, \ i = 1,2, \quad 0 < \alpha < \frac{5}{4}$

*are satisfied. Then, system (1) has eight linearly independent solutions* $y_j(x,\lambda)$ *for* $x \to \infty$*, such that*

$$y_1 = \varphi_1(x,\lambda)\exp\left\{\int_0^x (\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),\ y_2 = \varphi_1(x,\lambda)\exp\left\{-\int_0^x (\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),$$

$$y_3 = \varphi_1(x,\lambda)\exp\left\{i\int_0^x (\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),\ y_4 = \varphi_1(x,\lambda)\exp\left\{-i\int_0^x (\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),$$

$$y_5 = \varphi_2(x,\lambda)\exp\left\{\int_0^x (\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),\ y_6 = \varphi_2(x,\lambda)\exp\left\{-\int_0^x (\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),$$

$$y_7 = \varphi_2(x,\lambda)\exp\left\{i\int_0^x (\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),\ y_8 = \varphi_2(x,\lambda)\exp\left\{-i\int_0^x (\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),$$

*where*

$$\varphi_1(x,\lambda) = \frac{1}{\sqrt[8]{(\lambda-\mu_1(x))^3}}\begin{pmatrix}\cos\varphi(x)\\ -\sin\varphi(x)\end{pmatrix},\ \varphi_2(x,\lambda) = \frac{1}{\sqrt[8]{(\lambda-\mu_2(x))^3}}\begin{pmatrix}\sin\varphi(x)\\ \cos\varphi(x)\end{pmatrix}.$$

***Theorem 2.*** *Assume that conditions of Theorem 1 are satisfied. Then, the deficiency indices of operator* $L_0$ *are equal to (6,6).*

**Keywords:** differential operator, distribution of eigenvalues, indices of deficiency, asymptotic of the spectrum

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# Solution of a singularly perturbed Cauchy problem by similitude method

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**Abstract:** About receiving a boundary layer expansion of the solution of a singularly perturbed Cauchy problem:

$$L_\varepsilon y = \varepsilon y' + q(x)y(x) = f(x) 0 < x \le 1, y(0) = 0, \tag{1}$$

where $f(x) \in L^2(0,1)$, $q(x) \in C[0,1]$, $\varepsilon > 0$ is small parameter.

In this paper the method of spectral expansion and similarity method were used. The main result of this work is the following theorem

**Theorem.** *If* $f(x) \in W_2^n[0,1], q(x) \in W_2^n[0,1]$; $q(x) \ge \alpha > 0$, *then a strong solution of a singularly perturbed Cauchy problem (1) - (2) belongs to* $W_2^{n+1}[0,1]$ *and satisfies the estimate*

$$\left\| y(x,\varepsilon,f) - \sum_{k=0}^{n-1}(-1)^k\left[\frac{J^k f(x)}{q(x)} - \frac{J^k f(0)}{q(0)} \cdot e^{-\frac{1}{\varepsilon}\int_0^x q(t)dt}\right]\varepsilon^k \right\| \le$$

$\leq \frac{\varepsilon^n}{\alpha} \|J^n f\|, \ Jf(x) = \frac{d}{dx}\frac{f(x)}{q(x)}, \quad J^0 = I$ – *unit operator*.

**Keywords:** spectral expansion, self - adjoint operators, singularly perturbed differential equations.

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# Duality theorems for the noncommutative spaces

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**Abstract:** In this work, we introduce the noncommutative $H_p(A, l_\infty)$ and $H_p(A, l_1)$ spaces, and give results on duality.

Let $M$ be a finite von Neumann algebra equipped with a normal faithful tracial state $\tau$ . Let $D$ be a von Neumann subalgebra of $M$ , and let $\phi : M \to D$ be the unique normal faithful conditional expectation such that finite subdiagonal algebra of $M$ with respect to $\phi$ is a w* -closed subalgebra $A$ of $M$ satisfying the following conditions:

(i) $A + A^*$ is w *-dense in $M$ ;
(ii) $\phi$ is multiplicative on $A$;
(iii) $A \cap A^* = D$,

where $A^*$ is the family of all adjoint elements of the element of $A$. The algebra $D$ is called the diagonal of $A$ (see [1], [3]). We define $H_p(A, l_\infty)$ and $H_p(A, l_1)$ spaces by a similar way as in [3].

***Theorem.*** *Let* $1 \leq p < \infty$ *such that* $1/p + 1/p' = 1$. *Then*

*(i)* $H_p(A; l_1)^* = L_{p'}(M; l_\infty) / H^0_{p'}(A; l_\infty)^*$ *and* $(L_p(M; l_1)/H^0_p(A; l_1))^* = H_{p'}(A; l_\infty)$

*isometrically via the following duality bracket*

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

*for* $x \in H_p(A; l_1)$ *and* $y \in H_{p'}(A; l_\infty)$.

**Keywords:** von Neumann algebra, subdiagonal algebras, vector valued noncommutative Hardy spaces, duality theorems.

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# ANALYSIS 3

## On boundary value problem with nonlocal conditions for the system of partial differential equations

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**Abstract:** In $\overline{\Omega}=\{(x,t): t\leq x\leq t+\omega, 0\leq t\leq T\}$, $T>0$, $\omega>0$, we consider nonlocal boundary value problem for the system of partial differential equations

$$D\left[\frac{\partial}{\partial x}u(x,t)\right]=A(x,t)\frac{\partial u}{\partial x}(x,t)+S(x,t)u(x,t)+f(x,t),\ (x,t)\in\overline{\Omega}, \tag{1}$$

$$B(x)\frac{\partial u}{\partial x}(x,0)+C(x)\frac{\partial u}{\partial x}(x+T,T)=d(x),\ x\in[0,\omega], \tag{2}$$

$$u(t,t)=\Psi_1(t),\ \ t\in[0,T], \tag{3}$$

$$Du(t,t)=\Psi_2(t), t\in[0,T]. \tag{4}$$

Here, $D=\frac{\partial}{\partial t}+\frac{\partial}{\partial x}$; $u(x,t)$ is unknown $n$ – vector function on $\overline{\Omega}$; $A(x,t), S(x,t)$ are given $(n\times n)$ matrices functions, $f(x,t)$ is a known $n$ – vector function on $\overline{\Omega}$; $B(x), C(x)$ are given $(n\times n)$ matrices functions, $d(x)$ is a known $n$ - vector – function on $[0,\omega]$; $\Psi_1(t), \Psi_2(t)$ are given continuous functions on $[0,T]$.

A method of parameterization for the system of hyperbolic equations with mixed derivative was investigated in [1], [2].

In this work, new functions $v(x,t)=\frac{\partial u}{\partial x}(x,t)$, $w(x,t)=Du$ are introduced. Hence, the problem is reduced to the equivalent problem for the system of hyperbolic first – order equations with identical main part on Courant.

Sufficient conditions for the correct solvability of the problem in the terms of invertibility of the matrix, and boundary condition are obtained. In this work, an algorithm to find the solution of the problem is offered. Existence of the solution of this problem is established in the sense of Fridrihsu.

**Keywords:** nonlocal, correct solvability, hyperbolic, Courant, Fridrihsu.

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# On multiperiodicity and almost periodicity of solutions of boundary value problem for system of parabolic type equation

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**Abstract:** Consider the linear parabolic type equation with multivariate time

$$Lu \equiv \frac{\partial u}{\partial \tau} + \sum_{j=1}^{m} \frac{\partial u}{\partial t_j} - \Delta u - \frac{\partial^2 u}{\partial y^2} + \gamma u = f(\tau, t, x, y)\,, \qquad (1)$$

where $(\tau, t) \epsilon E_{1+m}$ - space of time variable, $y \in E_1^+ = [0, +\infty)$, $E_n$ – $n$ - measure Euclid space of vectors $x = (x_1, x_2, .., x_n)$; $\gamma = const > 0$;
$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \cdots + \frac{\partial^2}{\partial x_n^2}$ –Laplace operator; $f(\tau, t, x, y)$ – set function.
Assume that the $f(\tau, t, x, y)$ has $(\theta, \omega)$ - periodicity on $\tau, t$ and almost periodic on $x$ with $\varepsilon$ - almost period $\vartheta$ even with respect to $y$, also satisfying on $\tau, t$ and $x, y$ condition of Gelder with exponents $\frac{\alpha}{2}$ and $\alpha \in (0,1)$ accordingly.

***Problem.*** *Find sufficient conditions of existence and uniqueness multi periodical on* $\tau, t$ *and almost periodic on* $x$ *solution of parabolic type equation (1), which satisfy boundary condition*

$$u(\tau, t, x, 0) = \Psi(\tau, t, x), \qquad (2)$$

*where function* $\Psi(\tau, t, x)$ *-* $(\theta, \omega)$ *- periodic on* $\tau, t$ *and almost periodic on* $x$.
Among of the boundary value problem which is setting all along the space, considerable interest represents half-space boundary value problem which bring to study periodic and almost periodic solutions of parabolic equation [1], [2].
Developing idea of work [3] is finding some condition of quality bound with multi periodicity and almost periodicity of variable part of the fundamental solution and getting sufficient conditions of existence and uniqueness of multi periodicity on $\tau, t$ and almost periodicity on $x$ solution of the first boundary value problem (1) – (2).

**Keywords:** multi periodic, almost periodic, parabolic, multivariate time.
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# On a stability of a solution of the loaded heat equation

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**Abstract:** In this study, we consider the stabilization problem of finding $\{y(x,t),\ u_1(t), u_2(t)\}$ for the loaded heat equation

$$\begin{cases} y_t(x,t) - y_{xx}(x,t) + \alpha \cdot y(0,t) = 0, \quad \{x,t\} \in Q, \\ y\left(-\frac{\pi}{2},t\right) = u_1(t),\ \ y\left(\frac{\pi}{2},t\right) = u_2(t),\ \ y(x,0) = y_0(x), \end{cases} \tag{1}$$

where solution $y(x,t)$ tends to zero when $t \to \infty$, such that

$$\|y(x,t)\|_{L_2\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} \le C_0\, e^{-\sigma_0 t}. \tag{2}$$

Here $Q = \left\{x,t \middle| -\frac{\pi}{2} < x < \frac{\pi}{2},\ \ t > 0\right\}$, $\alpha \in C$, $\sigma_0$ is a known positive number, $y_0(x) \in L_2(-\pi/2,\ \pi/2)$ is a given function. The equation (1) is called loaded ([1] – [3]).

For the boundary value problem (1) on the semibar of width $\pi$ with the nonhomogeneous Dirichlet boundary conditions and initial condition on the segment $(-\pi/2,\ \pi/2)$ by the given function $y_0(x)$, we consider the auxiliary boundary value problem on the extended semibar of width $2\pi$ with the periodicity conditions (instead of Dirichlet conditions) and initial function $z_0(x)$ on the segment $(-\pi,\ \pi)$. Further, we determine a function $z_0(x)$ as continuation of the given function $y_0(x)$.

Theorem on solvability of the inverse problem (1) – (2) is proved and the algorithm of approximately construction of boundary controls is developed. Numerical calculations show efficiency of the offered algorithm.
**Keywords:** inverse problem, loaded heat equation, stabilization problem.

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# The Cauchy problem for a doubly nonlinear parabolic equation with inhomogeneous density and sources

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**Abstract:** In $Q_T = R_+^N \times (0,T), \quad T>0, \quad N \geq 1$, we consider the Cauchy problem for doubly nonlinear parabolic equation with a source and an inhomogeneous density:

$$|x|^m \frac{\partial u}{\partial t} = div\left(|x|^k \left|\nabla u^l\right|^{p-2} \nabla u^l\right) + |x|^m u^q, \tag{1}$$

$$u(x,0) = u_0(x) \geq 0, \ x \in R_+^N, \tag{2}$$

where $m$, $k$, $l > 0$, $p > 2$ are given numerical parameters.
Equation (1) arises in the Mathematical Modelling of diffusion processes in nonlinear media. The problems of fluid flow through porous layers, in the dynamics of biological populations, and in some other areas are studied in [1, 2]. Various properties of the solutions of the Cauchy problem and boundary value problems for (1) are intensively investigated by many authors [2-7].
On the basis of the self-analysis, the global solvability and nosolvability (Lions' problem) of the Cauchy problem (1), (2) is proved, including in the case of $l = p$, the asymptotic behavior of solutions with compact support and vanishing at infinity of solutions are obtained. The initial approximations for numerical calculations is proposed. It leads to the exact solution with rapid convergence.
**Keywords:** global solvability, nosolvability, asymptotic, numerical analysis.
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# Solvability of the nonlocal multi-point boundary value problem for system of quasilinear hyperbolic equations

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**Abstract:** In this study, we consider the multi-point boundary value problem on $\overline{\Omega}=[0,T]\times[0,\omega]$ for the second-order system of quasilinear hyperbolic equations

$$\begin{cases} \frac{\partial^2 u}{\partial x\partial t}=A(t,x)\frac{\partial u}{\partial x}+f\left(t,x,u,\frac{\partial u}{\partial t}\right), \\ u=(u_1,\dots,u_n)\in R^n,\ (t,x)\in\overline{\Omega}, \\ \sum_{i=0}^m P_i(x)u(t_i,x)=\varphi(x), \qquad x\in[0,\omega], \\ u(t,0)=\psi(t), \quad t\in[0,T], \end{cases} \tag{1}$$

where $n\times n$ matrix $A(t,x)$ is continuous on $\overline{\Omega}$, $n\times n$ matrices $P_i(x)$, $n$ vector-function $\varphi(x)$ are continuously differentiable on $[0,\omega]$, $i=\overline{0,m}$, $0=t_0<t_1<t_2<\dots<t_m=T$, $n$ vector-function $\psi(t)$ is continuously differentiable on $[0,T]$. The initial data meet a coordination condition: $\sum_{i=0}^m P_i(0)\psi(t_i)=\varphi(0)$.
The necessary and sufficient conditions of a well-posed solvability of the linear problem corresponding to (1) in terms of the initial data were found in [1]. The

main results of [1] based on equivalence of the well-posed solvability of the non-local boundary value problem for a system of hyperbolic equations and a family of multi-point boundary value problem for systems of ordinary differential equations. In the present work, the sufficient condition coefficients of the unique solvability of problem (1) are established by introducing additional functions [2] and applying results of concerning families of multi-point boundary value problems for system of ordinary differential equations, we suggest a solution algorithm.

**Keywords:** multi-point boundary value problem, hyperbolic equation, solvability.

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# A note on the stability of a time-dependent source identification problem

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**Abstract:** In this study, the following time-dependent source identification problem subject to an integral overdetermined condition

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} - \frac{\partial}{\partial x}\left(a(x)\frac{\partial u(t,x)}{\partial x}\right) + \delta u(t,x) = f(t,x) \\ +p(t)q(x), x \in (0,l), t \in (0,T), \\ u(0,x) = \varphi(x), x \in [0,l], \\ u_x(t,0) = u(t,l) = 0, t \in [0,T], \\ \int_0^l u(t,x)dx = \psi(t), t \in [0,T] \end{cases}$$

is considered. Here, $(u,p)$ is the solution pair of the problem, $a(x)>0$, $f(t,x)$, $q(x)$, $\psi(t)$ and $\varphi(x)$ are given sufficiently smooth functions assuming $q'(0)=q(l)=0$, and $\int_0^l q(x)dx \neq 0$.

The stability inequalities for the solution of this problem are presented. Rothe and Crank-Nicholson difference schemes are constructed for the numerical solution

of this problem. Moreover, stability and almost coercive stability inequalities for these difference schemes are given.
**Keywords:** finite difference method, integral condition, stability, almost coercivity.

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# On a first order partial differential equation with the nonlocal boundary condition

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**Abstract:** In this study, we consider the initial value problem

$$\begin{cases} u_t(t,x) + a(x)u_x(t,x) + \delta u(t,x) = f(t,x), 0 < x < l, 0 < t < T, \\ u(t,0) = u(t,l), 0 \le t \le T, \\ u(0,x) = u_0(x), 0 \le x \le l \end{cases} \tag{1}$$

for the first order partial differential equation with the nonlocal boundary condition. Here, $b \ge a(x) \ge a > 0$ and $u_0(x)\ (x \in (0,l))$ and $f(t,x)\ ((t,x) \in (0,T) \times (0,l))$ are given smooth functions and they satisfy every compatibility conditions which guarantees problem (1) has a smooth solution $u(t,x)$. The positivity of the space operator $A$ generated by problem (1) in the space C with maximum norm is established. The structure interpolation spaces of the space operator $A$ are investigated. The positivity of this space operator in Holder spaces is established.
The finite difference method for the approximate solution of problem (1) is presented. The positivity of the difference analogy of the space operator generated by this problem in the space C with maximum norm is established. The structure interpolation spaces generated by this difference operator are studied. The positivity of this difference operator in Holder spaces is established.
Following some mathematical models [1-3], in application the Mathematical Modelling of lake pollution is described and an example is given.

**Keywords:** stability analysis, difference schemes, pollution model.
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# On the nonlocal boundary value problem for semilinear hyperbolic equation

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**Abstract:** In this work, we study the nonlocal boundary value problem for a semilinear hyperbolic equation. Under the compatibility conditions and sufficiently smooth assumption for given data, the theorem on existence and uniqueness is established. The first and second orders of accuracy difference schemes for the approximate solution of these problems are presented. The convergence estimates for the solution of these difference schemes are obtained. Finally, these difference schemes are applied to the one dimensional nonlinear hyperbolic equation.
**Keywords:** difference scheme, semilinear hyperbolic equations.
**References:**
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# Inverse Neumann problem for an equation of elliptic type

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**Abstract:** In this study, we consider inverse problem of finding a function $u$ and an element $p$ for an elliptic equation

$$\begin{cases} -u_{tt}(t) + Au(t) = f(t) + pt, 0 < t < T, \\ u_t(0) = \varphi,\ u_t(T) = \psi,\ u_t(\lambda) = \xi \end{cases} \tag{1}$$

in an arbitrary Hilbert space $H$ with the self-adjoint positive definite operator $A$. Stability and coercive stability estimates for the solution of inverse problem (1) are obtained. The first and second orders of accuracy difference schemes are presented. Stability and coercive stability inequalities for these difference schemes are given. In application, inverse problem for the multidimensional elliptic equation is studied. The first and second orders of accuracy difference schemes for the multidimensional inverse problem with overdetermination are presented. Well-posedness of both difference problems are established. The results are supported by numerical example for the two-dimensional elliptic equation.

**Keywords:** difference scheme, overdetermination, well-posedness, stability, coercive stability.

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# A generalized fractional sub-equation method for nonlinear fractional differential equations

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**Abstract:** This letter studies some nonlinear fractional differential equations [1,2,3,4]. The sub-equation method is used for finding exact solutions of these equations [5,6]. Meanwhile, the traveling wave transformation method has been used to convert fractional order partial differential equation to fractional order ordinary differential equation. Calculations in this method are simple and effective mathematical tool for solving fractional differential equations in science and engineering. The power of this manageable method is presented by applying it to several examples. This approach can also be applied to other nonlinear fractional differential equations.

**Keywords:** exact solutions, traveling wave transform, sub-equation method, space-time fractional differential equation.

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# On a boundary-value problem for the parabolic-hyperbolic equation with the fractional derivative and the sewing condition of the integral form

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**Abstract:** In this study, we consider a boundary problem with sewing condition of integral form for mixed parabolic-hyperbolic type equation with the Caputo fractional derivative

$$0=\begin{cases} u_{xx}-{}_C D_{0t}^{\alpha}u, & t>0, \\ u_{xx}-u_{tt}, & t<0 \end{cases} \qquad (1)$$

in the domain $\Omega=\Omega^{+}\cup\Omega^{-}\cup AB$, where $0<\alpha\le 1$, $AB=\{(x,t):0<x<1,t=0\}$, $\Omega^{+}=\{(x,t):0<x<1,0<t<1\}$, $\Omega^{-}=\{(x,t):-t<x<t+1,-1/2<t<0\}$, ${}_C D_{0t}^{\alpha}f=\frac{1}{\Gamma(1-\alpha)}\int_0^t (t-z)^{-\alpha}\, f'(z)dz$ is the Caputo fractional differential operator of the order $\alpha(0<\alpha\le 1)$, $\Gamma(\cdot)$ is Euler's gamma-function.

We prove the uniqueness of the solution for considered problem by the method of energy integrals. The existence of the solution have been proved by reducing the considered problem to the Fredholm integral equation. We represent solution in an explicit form using Green's function.

Boundary problems with sewing conditions of integral form were studied in works [1-3].

**Keywords:** fractional derivative, mixed type equation, sewing condition, Green's function.

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# Solvability of the nonlinear boundary value problem for Fredholm integro-differential equation with impulse effect

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**Abstract:** Consider the nonlinear two-point boundary value problem for Fredholm integro-differential equation with impulse effect

$$\frac{dx}{dt} = f_0(t,x) + \int_0^T f_1(t,s,x(s))ds, \quad t \in (0,T)\backslash\{\theta\}, \quad x \in R^n, \tag{1}$$

$$A_0 \lim_{t\to\theta-0} x(t) + B_0 \lim_{t\to\theta+0} x(t) = d_0, \tag{2}$$

$$A_1 x(0) + B_1 x(T) = d_1. \tag{3}$$

In [1, 2], solvability of problem (1) – (3) was investigated by the parameterization method. Approximation methods for finding its solutions are constructed. The problem of choice of initial approximation to the solution and algorithms for solving are studied.

Algorithms for finding solution to problem (1) – (3) needs solving the special Cauchy problem for integro-differential equations with parameters.

Sufficient conditions for existence of unique solution to the special Cauchy problem and the estimate of difference of its solutions are obtained in [3]. The solvability conditions for the linear boundary value problem with parameter for the Fredholm integro-differential equation with impulse effect is given in [4].

**Keywords:** integro-differential equation, nonlinear boundary value problem, parameterization method, solvability, impulse effect.

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# Parameterization method for the second-order nonlinear three-point boundary value problem

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**Abstract:** In this study, we consider the three-point boundary value problem for second-order nonlinear differential equations

$$\begin{cases} x''(t) = f\big(t, x(t)\big), \qquad 0 < t < 1, \\ x'(0) = a \\ \beta_1 x(1) + \beta_2 x'(1) = \delta_1 x(\eta) + \delta_2 x'(\eta) + b, \\ 0 < \eta < 1, \end{cases} \qquad (1)$$

where $f \in C([0,1] \times \mathbb{R}, \mathbb{R})$, and $a, b, \beta_1, \beta_2, \delta_1, \delta_2$ are real numbers.
Second-order three-point problems were arising at Mathematical Modelling of various processes of physics, chemistry, biology, ecology, economy, etc. Problem (1) was object of studying of many authors. For finding conditions of existence of solution the method of fixed points, the upper and lower solutions and monotone iterative method are applied (see [1-2] and the references therein). Despite numerous studies the questions of existence and qualitative properties of solutions of the three-point problems remain open. This leads to the development of effective methods for studying nonlinear three-point boundary value problems and construction of algorithms for finding their solutions.
In the present work, the problem (1) is investigated by parameterization method [3].
The conditions of existence of solutions to problem (1) are established by the terms of initial data. Algorithms of finding solution to problem (1) are proposed.
**Keywords:** second-order differential equation, three-point boundary value problem, parameterization method, solvability.
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# Ill-posed problem for the biharmonic equation

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**Abstract:** Recently, among the experts on the equations of mathematical physics, there is considerably increased interest to problems that are ill-posed by J. Hadamard [1]. Problems of this kind are always attracted the attention of researchers. First of all, this is due not only to their importance in theory, but also to the fact that they have to be faced in many applications in various fields of science and technology. Due to the ill-posed problems it can be noted classical works of J. Hadamard [1], A.N. Tikhonov [2], M.M. Lavrent'ev [3] and many others, who paid the attention of researchers to the ill-posed problems and brought a significant contribution to the development of this important area of mathematics.

Formulation of the boundary value problem for the biharmonic equation is as follows. We consider the boundary value problem

$$\Delta^2 u = f,\ \{x,t\} \in Q; \tag{1}$$

$$u(0,y) = u_x(0,y) = 0,\ u(2\pi,y) = u_x(2\pi,y) = 0; \tag{2}$$

$$u(x,0) = 0,\ u_y(x,0) = \varphi_1(x),\ u_{yy}(x,0) = 0,\ u_{yy}(x,1) = 0, \tag{3}$$

in the domain $Q = \{x, y \mid 0 < x < 2\pi,\ 0 < y < 1\}$ with the additional condition

$$u(x,1) = \psi(x) \in U_g \text{ is closed convex set of } H_0^{3/2}(0,2\pi). \tag{4}$$

It is assumed that the data in problem (1)–(3) satisfies the following conditions

$$f \in (\widetilde{H}^2(Q))',\ \varphi_1 \in H_0^{1/2}(0,2\pi),\ \psi \in H_0^{3/2}(0,2\pi), \tag{5}$$

here

$$\widetilde{H}^2(Q) = \left\{ u \mid u \in L_2(0,1;\ H_0^2(0,2\pi)),\ \frac{\partial^2 u}{\partial y^2} \in L_2(Q) \right\}.$$

In this paper, we use optimal control methods to solve problem (1)–(5).

**Keywords:** biharmonic equation, ill-posed problem, inverse problem, optimal control.

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# On the solvability of boundary value problems for the nonhomogeneous polyharmonic equation in a ball

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**Abstract:** Let $m$ be a positive integer. We consider in $n$-dimensional unit ball $\Omega = \{x : |x| < 1\} \subseteq R^n$ the nonhomogeneous polyharmonic equation (PHE)

$$\Delta_x^m u(x) = f(x), \tag{1}$$

with the boundary conditions

$$\left.\frac{\partial^{k_j}}{\partial n_x^{k_j}} u\right|_{x\in\partial\Omega} = \varphi_j(x),\ \ x \in \partial\Omega,\ \ j = \overline{1,m};\ \ 0 \le k_1 < k_2 < \ldots < k_m \le 2m-1. \tag{2}$$

Usually, the existence of regular solutions to the original data $f(x)$, $\varphi_1(x), \varphi_2(x), \ldots, \varphi_m(x)$ imposes limitations of two types [1,2]: (i) some loss of the smoothness; (ii) certain conditions such as orthogonality to the solutions of the homogeneous adjoint equation.

In this paper, we describe criterion for the solvability of the problem (1) - (2) in the initial terms [3] in particular, Neumann problem (when $k_1 = 1, k_2 = 2, ..k_m = m$). We give representation of the Green function of the Dirichlet problem (when $k_1 = 0, k_2 = 1, ..k_m = m-1$) for the PHE in a ball without restrictions on the number of spatial variables and the order of the equation [4], as well as built some classes correct boundary value problems in bounded domains.

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**Keywords:** polyharmonic equation, regular solution, Dirichlet problems, solvability of boundary value problems.

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# Existence of positive solutions for the fourth-order nonlinear ordinary differential systems with two parameters

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**Abstract:** In this work, we study the existence of positive solutions of the following boundary value problem for the fourth order nonlinear differential systems with two parameters:

$$\begin{cases} u^{(4)}(t)+\beta_1 u''(t)-\alpha_1 u(t)-\gamma_1 v(t)-\mu_1 h_1(t) f\big(u(t),v(t)\big)=0, \\ \\ v^{(4)}(t)+\beta_2 v''(t)-\alpha_2 v(t)-\gamma_2 u(t)-\mu_2 h_2(t) g\big(u(t),v(t)\big)=0, \end{cases} \tag{1}$$

$$\begin{cases} \boldsymbol{u(0)=u(1)=u''(0)=u''(1)=0}, \\ \\ \boldsymbol{v(0)=v(1)=v''(0)=v''(1)=0}, \end{cases} \tag{2}$$

where $\boldsymbol{\beta_i,\alpha_i,\gamma_i}$ are three positive constants and $\boldsymbol{\mu_i>0}$ with $\boldsymbol{i=1,2}$, $\boldsymbol{f,g}\in \boldsymbol{C(R^+.R^+,R^+)}$ and $\boldsymbol{h_i}\in \boldsymbol{C([0,1],R^+)}$

**Keywords:** positive solution, system of fourth order boundary value problem, Krasnosel'skii fixed point theorem, Green's function, variable parameters

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# Coercive estimates for solutions of one singular equation with the third-order partial derivative

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**Abstract:** Sufficient conditions of boundedness for the highest derivative of partial differential equations of the third order are established and coercive estimate in the norm of the space $C_*(\overline{\Omega},R)$ is obtained.

Let $\overline{\Omega}=[0,\omega]\times(-\infty,+\infty)$. We consider the equation

$$u_{xtt}=a_0(x,t)u_{xt}+a_1(x,t)u_x+a_2(x,t)u_t+a_3(x,t)u+f(x,t), \tag{1}$$

where the functions $a_i(x,t)\,(i=\overline{0,3})$, $f(x,t)$ are continuous and, generally speaking, are unbounded on $\overline{\Omega}$.

We denote by $C_*(\overline{\Omega},R)$ the space of bounded functions, which continuous with $t\in R$ at $x\in[0,\omega]$ and uniformly with respect $t\in R$ continuous for $x\in[0,\omega]$.

Let $\|V(x,\cdot)\|_1=\sup\limits_{t\in R}\|V(x,t)\|$, where $\|V(x,t)\|=\max\limits_{i=\overline{1,n}}|V_i(x,t)|$. We investigate the properties for solutions $u(x,t)$ of equation (1) satisfying the conditions

$$u(0,t)=\psi(t),\ u(x,t),u_x(x,t),u_t(x,t),u_{xt}(x,t)\in C_*(\overline{\Omega},R). \tag{2}$$

Assume that $P_{\alpha,\beta}(x,t)=\dfrac{\alpha(x,t)}{\sqrt{\beta(x,t)}}$, $\theta(x,t)=\dfrac{1}{d}\displaystyle\int_t^{t+d}a_1(x,\tau)d\tau.$

***Theorem 1.*** *Let the functions $a_i(x,t)$ $(i=\overline{0,3})$ of equation (1) be continuous on $\overline{\Omega}$ and $\psi$ ,$\psi^*$,$\psi^{**}$ be continuous and bounded on $R$ and the following conditions are satisfied:*

*a)* $a_1(x,t)\geq\gamma>0$, $\gamma$ *- const.,* $\dfrac{a_1(x,t)}{a_1(x,\bar t)}\leq c$ *at* $t,\bar t\in R:|t-\bar t|<d,\ c,d$ *- const.;*

*b) for every* $\varepsilon>0$ *there exists a number* $\delta>0$ *such that for all t from R and* $x',x''\in[0,\omega]:|x'-x''|<\delta$ *the inequality* $\left|\dfrac{a_1(x',t)-a_1(x'',t)}{a_1(x'',t)}\right|<\varepsilon$ *holds;*

*c)* $P_{a_0,a_1}(x,t)\leq K, P_{a_2,a_1}(x,t),P_{a_3,a_1}(x,t),P_{f,a_1}(x,t),$ $f(x,t),\sqrt{\theta(x,t)}\psi(t),\ \sqrt{\theta(x,t)}\psi^*(t)\in C_*(\overline{\Omega},R)$.

*Then, there exists a unique solution* $u(x,t)$ *of problem (1), (2), moreover* $u_{xtt}\in C_*(\overline{\Omega},R)$ *and the following estimate*

$$\|u_{xtt}\|_1+\|a_0u_{xt}\|_1+\|a_1u_x\|_1+\|a_2u_t\|_1+\|a_3u\|_1\leq C.$$

*holds. Here, C depends on norms of the functions* $f,\psi,$ *and constants* $\gamma,K,c,d,\varepsilon$.

# Differentiable solutions of the second order elliptic equation

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**Abstract:** Let $n \geq 3$ and $Q$ be a bounded set in $R^n$ with a smooth boundary and $\Omega$ be an arbitrary open subset of $Q$. We study the second order elliptic equation with discontinuous intermediate coefficients:

$$-\Delta u + \sum_{k=1}^{n} a_k(x) u = f \ , \qquad (1)$$

where $x = (x_1, x_2, ..., x_n) \in \Omega$.

It is known that if $p > n$ and $a_k, f \in L_p(\Omega)$, $k = 1,2,...,n$, then there is a generalized solution $u \in W_p^1(\Omega)$ of equation (1) such that it has continuous first order partial derivatives in $\Omega$ (see [1], Ch. 3). This result can not be improved. Since if at least one of $a_k$ ($k = 1,2,...,n$) and $f$ belongs to $L_n(\Omega) \setminus L_p(\Omega)$, $p > n$, then the solution $u$ of equation (1) does not belong to $C_{loc}^{(1)}(\Omega)$. So the following question arises:

***Question:*** Are there other spaces such that the above precise result about differentiable solutions of equation (1) holds for any $a_k$ ($k = 1,2,...,n$) and $f$ in this spaces?

The purpose of this work is to answer this question.

Suppose that the variable coefficients $a_k$ ($k = 1,2,...,n$) and the right-hand side $f$ of equation (1) belong to some space $M$. We find the necessary and sufficient conditions on $M$ for continuous differentiability of the solution to equation (1), when $M$ is a space of type $F$ or a symmetric space, or one of a Sobolev and Besov space.

**Keywords:** elliptic equation, intermediate coefficient, Lorentz space.

**References:**

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# A note on nonlocal boundary value problems for parabolic-Schrodinger equations

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**Abstract:** The nonlocal boundary value problem

$$\begin{cases} \frac{du(t)}{dt} + Au(t) = f(t) \ (0 \leq t \leq 1), \\ i\frac{du(t)}{dt} + Au(t) = g(t) \ (-1 \leq t \leq 0) \\ u(-1) = \alpha u(\mu) + \varphi, \ \ 0 \leq \mu \leq 1, \end{cases} \tag{1}$$

for parabolic-Schrodinger equations in a Hilbert space H, with the self-adjoint positive definite operator A is considered. The stability estimates for the solution of (1) are established. In practice, the stability estimates for solutions of the mixed type boundary value problems for parabolic-Schrodinger equations are obtained. A numerical analysis is given for the parabolic-Schrodinger differential equation. For appoximate solutions of problem (1) the first and second orders of accuracy difference schemes are presented.

**Keywords:** nonlocal boundary value problem, parabolic-Schrodinger equation, difference scheme, stability.

**References:**

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# On solvability of some nonlocal boundary value problems with the Hadamard boundary operator

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**Abstract:** In the present work, questions of solvability of nonlocal problems for the Laplace equation in a $\Omega=\{x\in R^n : |x|<1\}$ - ball are studied.

*Problem.* To find a harmonic function $u(x)\in C(\bar{\Omega})$ such that the function $D^\alpha[u](x)$ is continuous in $\bar{\Omega}$ and satisfies the conditions:

$$u(x)+(-1)^k u(x^*)=f(x),\ x\in\partial\Omega_+,$$

$$D^\alpha[u](x)-(-1)^k D^\alpha[u](x^*)=g(x),\ x\in\partial\Omega_+.$$

Here, $k=0,1$, $f(x)\in C^{\lambda+\alpha}(\bar{\Omega}), g(x)\in C^{\lambda}(\bar{\Omega}), 0<\lambda,\ \lambda+\alpha$ is noninteger, $D^\alpha$ - operators of a fractional order in the Hadamard sense, any point $x=(x_1,x_2,...,x_n)\in\Omega$ we associate the "opposite" point $x^*=(\alpha_1 x_1,\alpha_2 x_2,...,\alpha_n x_n)\in\Omega$, here $\alpha_1=-1$, and $\alpha_j, j=2,...,n$ take one of the values $\pm 1$.

It is proved correctness of the considered problem. Smoothness of the solution of considered problem is also studied in the Holder classes depending on the order of boundary operators.

It should be noted that problem is proposed and first studied in [1,2] for the case of integer $\alpha=1$.

**Keywords:** the Laplace equation, nonlocal problem, the Hadamard operator, solvability, smoothness.

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# A criterion for the strong solvability of the Neumann-Tricomi problem for the Lavrent'ev-Bitsadze equation in $L_p$

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**Abstract:** In this study, we consider the Neumann-Tricomi problem of finding $u(x,y)$ for the Lavrent'ev-Bitsadze equation with the following boundary value conditions

$$\begin{cases} Lu \equiv -sgny u_{xx} - u_{yy} = f(x,y), (x,y) \in \Omega, \\ \left.\frac{\partial u}{\partial n}\right|_{\sigma} = 0, \\ u|_{AC} = 0. \end{cases} \tag{1}$$

Here, $\Omega \subset \mathrm{R}^2$ is a finite domain bounded by a curve σ for $y > 0$ and by the characteristics AC: $x + y = 0$ and BC: $x - y = 1$ for $y < 0$, and the symbol $\frac{\partial}{\partial n}$ is the directional derivative in the outward normal to the σ.

The existence and uniqueness of regular solution of Neumann-Tricomi problem (1) was proved by Bitsadze [1]. The completeness of eigenfunctions of the Neumann-Tricomi problem for a degenerate equation of mixed type in the elliptic part of the domain was investigated by Moiseev and Mogimi [2].

***Theorem 1.*** *The Neumann-Tricomi problem (1) is strongly solvable for any right-hand side* $f \in L_p(\Omega)$ *if and only if*

$$\alpha \le \frac{\pi}{8}\frac{p}{p-1}, \beta \le \frac{3\pi}{4}\frac{p}{p-1}.$$

***Corollary 2.*** *The Neumann-Tricomi problem in the classical domain, when the elliptic part of the domain coincides with the semi-circle, is not strongly solvable in* $L_2(\Omega)$.

**Keywords:** Neumann-Tricomi problem, Lavrent'ev-Bitsadze equation, strong solution.

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# Gellerstedt equation
# with integral perturbation of Cauchy condition

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**Abstract:** For a degenerate hyperbolic equation of the first kind in the characteristic triangle, well-posedness of the problem with integral perturbation conditions of Cauchy problem on the degenerate line is proved.
**Keywords:** Gellerstedt operator.
**References:**
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# APPLIED MATHEMATICS

# APPLIED MATHEMATICS 1

## Well-posedness of difference scheme for elliptic--parabolic equations in Holder spaces without a weight

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**Abstract**: In the present paper, we are interested in studying the following second order of accuracy difference scheme for the solution of the elliptic-parabolic equation with nonlocal boundary condition

$$\begin{cases} -\frac{u_{k+1}-2u_k+u_{k-1}}{\tau^2}+Au_k=g_k, g_k=g(t_k), \\ t_k=k\tau, 1\le k\le N-1, N\tau=1, \\ \frac{u_k-u_{k-1}}{\tau}=-\frac{1}{2}(Au_k+Au_{k-1})=f_k, f_k=f\left(t_{k-\frac{1}{2}}\right), \\ t_{k-\frac{1}{2}}=\left(k-\frac{1}{2}\right)\tau, -(N-1)\le k\le 0, \\ \mathrm{u}_2-4\mathrm{u}_1+3\mathrm{u}_0=-3\mathrm{u}_0+4\mathrm{u}_{-1}-\mathrm{u}_{-2}, \mathrm{u}_\mathrm{N}=\mathrm{u}_{-\mathrm{N}}+\mu. \end{cases} \tag{1}$$

Theorem on well-posedness of this problem in Holder spaces without a weight is given. In an application, coercivity estimates in Holder norms for approximate solution of a nonlocal boundary value problem for elliptic-parabolic differential equation are obtained.

**Keywords**: difference scheme, elliptic-parabolic equation, coercivity inequalities, well-posedness.

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# On the numerical solution of a telegraph equation by difference schemes

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**Abstract:** In this study, the following initial-value problem for a telegraph equation

$$\frac{d^2u(t)}{dt^2} + \alpha\frac{du(t)}{dt} + Au(t) + \beta u(t) = f(t), \qquad 0 \le t \le T$$
$$u(0) = \phi, \ \frac{\partial}{\partial t}u(0) = \psi.$$

In a Hilbert space H with the self-adjoint positive operator A is investigated. The first and and second order of accuracy difference schemes are presented. The stability estimates for the solutions of these difference schemes are established. Some results of numerical experiments are presented in order to support theoretical statements. The obtained results are discussed by comparing with other existing numerical solutions.
**Keywords:** Initial value problem, telegraph equation, numerical solution.
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# Well-posedness of a third order of accuracy difference scheme for Bitsadze-Samarskii type multi-point NBVPs

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**Abstract:** A multi-point nonlocal boundary value problem

$$\begin{cases} -u_{tt} - \sum_{r=1}^{n}\left(a_r(x)u_{x_r}\right)_{x_r} + \delta u(t,x) = f(t,x), \\ 0 < t < 1,\ x = (x_1,\dots,x_n) \in \Omega, \\ u_t(0,x) = \varphi(x),\ u_t(1,x) = \beta_1 u_t(\lambda_1,x) + \beta_2 u_t(\lambda_2,x) + \psi(x),\ x \in \overline{\Omega}, \\ \beta_1 + \beta_2 \le 1,\ 0 \le \lambda_1 < \lambda_2 \le 1, \\ u(t,x) = 0,\ 0 \le t \le 1,\ x \in S,\ S = \partial\overline{\Omega} \end{cases} \quad (1)$$

for the multidimensional elliptic equation with Bitsadze-Samarskii condition is considered. Here, $\Omega$ is the unit open cube in
$R^n = \{x = (x_1,\dots,x_2): 0 < x_k < 1, 1 < k < n\}$
with boundary S, $\overline{\Omega} = \Omega \cup S$ and $a_r(x) \ge 0 > 0,\ (x \in \Omega)$, $\varphi(x),\ \psi(x)$ $(x \in$ ⍰$)$, $f(t,x)\ (t \in (0,1),\ x \in \Omega)$ are given smooth functions. δ is a large positive constant.
The third order of accuracy stable difference scheme for the approximate solution of (1) is presented. The stability, coercive stability and almost coercive stability estimates for the solution of this difference scheme are obtained. A numerical analysis is given to support the theoretical statements.
**Keywords:** Bitsadze-Samarskii problem, elliptic equation, nonlocal boundary value problems, difference scheme, well-posedness, stability, coercive stability.

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# Approximate solution of the two-dimensional singular integral equation

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**Abstract:** In this study, approximate quadrature formulas for numerical calculation of two-dimensional Vekua potential and singular integrals are obtained. The mechanical quadrature method for two-dimensional quasilinear singular integral equation with Vekua operators is described. The numerical results are compared with the exact solution of the integral equation.
**Keywords:** two-dimensional singular integral, singular integral equation, mechanical quadrature method.

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# Approximation of the inverse elliptic problem with mixed boundary value conditions

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**Abstract:** In this study, we consider inverse problem of finding $u(t,x)$ and $p(x)$ for the multidimensional elliptic equation with the following Dirichlet-Neumann boundary value conditions

$$\begin{cases} -u_{tt} - \sum_{r=1}^{n}\left(a_r(x)u_{x_r}\right)_{x_r} + \delta u(t,x) = f(t,x) + p(x), \\ \qquad x = (x_1, \ldots, x_n) \in \Omega,\ 0 < t < T, \\ u(0,x) = \varphi(x), u(T,x) = \psi(x), u(\lambda,x) = \xi(x), x \in \overline{\Omega} \\ \qquad \frac{\partial u(t,x)}{\partial \vec{n}} = 0, x \in S_1, u(t,x) = 0, x \in S_2. \end{cases} \tag{1}$$

Here $a_r(x)$ $(x \in \Omega)$, $\varphi(x)$, $\psi(x)$, $\xi(x)$ $(x \in \overline{\Omega})$, $f(t,x)$ $(t \in (0,T), x \in \Omega)$ are given smooth functions, $a_r(x) \geq 0, \Omega = (0,l) \times \ldots \times (0,l)$ is the open cube in the $n$-dimentional Euclidian space with boundary $S = S_1 \cup S_2, \overline{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants.

Well-posedness of the inverse problem (1) follows from results of abstract theorems [2]. In the present work, the first and second orders of accuracy difference schemes for the approximate solution of inverse problem (1) are presented. Stability, almost stability and coercive stability estimates for the solution of these difference schemes are obtained. The algorithm for approximate solution is tested in a two-dimensional case.

**Keywords:** difference scheme, overdetermination, well-posedness, stability, coercive stability.

**References:**

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# Finite difference method of fractional parabolic partial differential equations with variable coefficients

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**Abstract:** In this study, we consider the fractional parabolic partial differential equations with variable coefficients with the Caputo fractional derivative. We construct a difference scheme based on Crank- Nicholson method. Stability estimate for the solution of this difference scheme is obtained with a sufficient condition.
**Keywords:** difference scheme, stability, Caputo fractional derivative.
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# Hybridizable discontinuous Galerkin method for solving Caputo fractional boundary value problems

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**Abstract:** In this work, we introduce Hybridizable discontinuous Galerkin method for the numerical solution of Caputo fractional differential equations. The first time, we use the method for solving fractional order boundary value problems. The method possesses several unique features which distinguish itself from other

discontinuous Galerkin methods. At first, it reduces the number of globally coupled unknowns significantly when high order approximate polynomials are used. Secondly, it takes local solvers by using a Galerkin method to weakly enforce the equations on each element. We focus on the efficient implementation of the method and on the validation of its computational performance for one-dimensional fractional differential equations. By this way, we obtain some convergence results for this model problem.

**Keywords:** Hybridizable discontinuous Galerkin Method, Caputo fractional operator, local solver, stabilization parameter.

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# An optimal control problem for linearised Navier-Stokes equation

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**Abstract:** In this study, we consider the optimal control problem for Navier-Stokes equation where control function is square summable. We examine the existence and uniqueness of optimal solution and get differentiability of functional. Then, we obtain a necessary optimality condition in the form of a variational inequality using gradient.

**Keywords:** Navier-Stokes, optimal control problem, variational problem.

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# A note on the numerical solution of the semilinear elliptic equation

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**Abstract:** The study of boundary value problems for elliptic equations were considered in [1-6]. ,A. V Bitsadze and A. A. Samarskii had proved existence and uniqueness of a classical solution to an elliptic second-order equation in a rectangle with nonlocal boundary conditions [7]. Previously, the Bitsadze-Samarskii type nonlocal boundary value problems for semilinear elliptic equations were studied [3]. The unique solvability of Bitsadze-Samarskii type nonlocal boundary value problems for the semi-linear elliptic equations in a Hilbert space was investigated in that work. In this paper, we consider the same problem for approximately solving this problem in a Hilbert Space [3]. The difference scheme is used to overcome the difficulties in numerical procedure.

**Keywords:** semilinear elliptic equation, difference scheme, Bitsadze- Samarskii type problem.

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# On the numerical solution of hyperbolic equations with singular source terms

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**Abstract:** A numerical study is presented of hyperbolic equations having singular source terms, singular in the sense that within the spatial domain the source is defined by a Dirac delta function. Solutions of such problems will have discontinuities which form an obstacle for standard numerical methods. In this study, a fifth order flux implicit WENO method with non-uniform meshes is investigated for approximate solutions of hyperbolic equations having singular source terms. Numerical examples are provided.
**Keywords:** WENO method, non-uniform mesh, hyperbolic equations with singular source terms.
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# On fourth order of accuracy stable difference scheme for multipoint NBVP of the hyperbolic type

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**Abstract:** In this work, we consider the multipoint NBVP

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - \sum\limits_{r=1}^{m} (a_r(x) u_{x_r})_{x_r} = f(t,x), \\ x = (x_1, \dots, x_m) \in \Omega_h, \ 0 < t < 1, \\ u^h(0,x) = \sum\limits_{j=1}^{n} \alpha_j u^h\left(\lambda_j, x\right) + \phi^h(x), x \in \overline{\Omega}_h, \\ u_t^h(0,x) = \sum\limits_{j=1}^{n} \beta_j u_t^h(\lambda_j, x) + \psi^h(x), x \in \overline{\Omega}_h, \\ u^h(t,x) = 0, \ x \in S. \end{cases} \tag{1}$$

Here, $a_r(x)$, $(x \in \Omega)$, $\phi(x), \psi(x)$ $(x \in \overline{\Omega}_h)$ and $f(t,x)$ $(t \in (0,1),\ x \in \Omega_h)$ are given smooth functions and $a_r(x) \geq a > 0$, $\Omega = (0,l) \times \dots \times (0,l)$ is the open cube in the $m$-dimensional Euclidian space with the boundary $S = S_1 \cup S_2$, $\bar{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants.

The fourth order of accuracy difference scheme for approximately solving problem (1) generated by the integer power of the operator is presented. Stability estimates for the solution of these difference schemes are obtained. The theoretical statements are supported by the numerical results using MATLAB.

**Keywords:** multipoint nonlocal boundary value problem, difference schemes, convergence, stability, numerical analysis.

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# On third order of accuracy stable difference scheme for multipoint NBVP of the hyperbolic type

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**Abstract:** We consider the following multipoint nonlocal boundary value problem (NBVP)

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - \sum\limits_{r=1}^{m} (a_r(x) u_{x_r})_{x_r} = f(t,x), \\ x = (x_1, \dots, x_m) \in \Omega_h, \ 0 < t < 1, \\ u^h(0,x) = \sum\limits_{j=1}^{n} \alpha_j u^h\left(\lambda_j, x\right) + \phi^h(x), x \in \overline{\Omega}_h, \\ u_t^h(0,x) = \sum\limits_{j=1}^{n} \beta_j u_t^h(\lambda_j, x) + \psi^h(x), x \in \overline{\Omega}_h, \\ u^h(t,x) = 0, \ x \in S. \end{cases} \tag{1}$$

Here, $a_r(x)$, $(x \in \Omega)$, $\phi(x), \psi(x)$ $(x \in \overline{\Omega}_h)$ and $f(t,x)$ $(t \in (0,1),\ x \in \Omega_h)$ are given smooth functions and $a_r(x) \geq a > 0$, $\Omega = (0,l) \times \ldots \times (0,l)$ is the open cube in the $m$-dimensional Euclidean space with the boundary $S = S_1 \cup S_2$, $\bar{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants.

In this work, the third order of accuracy difference scheme for approximately solving problem (1) generated by the integer power of the operator is presented. Stability estimates for the solution of these difference schemes are obtained. The theoretical statements are supported by the numerical results using MATLAB.

**Keywords:** multipoint nonlocal boundary value problem, difference schemes, convergence, stability, numerical analysis.

**References:**

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# APPLIED MATHEMATICS 2

## On the controllability of the multidimensional phase system

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**Abstract:** We will study the issues of controllability multidimensional phase system. The problem is reduced to a system of nonlinear differential equations, for which we develop a method proposed in [1]. Consider the following model of the phase system, "synchronous generator- steam turbine," described by differential equations of the form

$$\frac{d\delta}{dt} = S,$$
$$T_j \frac{dS}{dt} = P_T - KS - \left[\frac{E^2}{z_{11}} sin\alpha_{11} + \frac{EU}{z_{12}} \sin(\delta - \alpha_{12})\right], \qquad (1)$$
$$T_P \frac{dP_T}{dt} = -P_T + \rho_o P_0 - \frac{P_0}{\sigma_0} S + u,$$

where $P_T$-power steam turbine; $T_P$-time constant cycle regulation of the steam turbine; $\rho_o, P_0$-defined constants ($\rho_o$-start opening the windows spool, $P_0$-rated power turbines); $\sigma_0$-statism Archway (automatic speed control); u - control effects MUT (turbine control mechanism); δ - angle EMF generator; S-slip generator; $T_j$- constant inertia of the rotating masses; $K > 0$ - damping coefficient; E-EMF generator design; U-busbar voltage infinite capacity $z_{11}$- inherent resistance of the generator; $z_{12}$-mutual impedance between the generator and tires; $\alpha_{11}$-additional angle own resistance; $\alpha_{12}$-additional angle relative resistance.
For the system of regulation of the steam turbine, we give the following parameters: $\rho_o = 0.994, T_P = 251.2, P_0 = 10420, \sigma_0 = 0.06$.
**Keywords:** electric power system, nonlinear system, phase system, controllability, synchronous generator.

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# The method of calculation of flooding zones on an territory with the use of digital elevation model on the basis of successive pools

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**Abstract:** Floods inflict an enormous damage to the environment and present a threat for people's life that stipulates actuality of fight against these elements. For many areas of Kazakhstan the most dangers are presented by floods in the period if spring tide and drifting of ice on the rivers, and also flood of the localities, related to destruction of dams of storage pools, washing away of protective dikes. Harm that is brought to humanity is enormous, especially if to take into account not only the line but also indirect damage.

In our case the set of laws and rules, on which moves water, that is set by a hypsography from a digital map and laws of hydrodynamics of streams of water locomotive on a surface depending on hydrodynamic laws.

Changing of the states in contiguous pools takes place simultaneously and in parallel, and time goes discretely. In spite of seeming simplicity of motion of stream, characters of mutual relations between pools can be various and different. And here in a role rules of cellular automats can enter. Cellular automats can apply on the role of universal instrument, allowing to describe the mechanism of processes that is described other methods can't be. Otherwise speaking, cellular automats – it's methodology of presentation of task that puts before itself an aim breaking up a large task on the great number of discrete, shallow tasks so that problem definition for one element simultaneously is all problem definition for all elements.

For determination of volume of run out water from the river it is necessary to know speed of flow of the river and area of living section of the river. Using the well-known formula of Shezi for determination of middle speed of streams at the set even turbulent motion of liquid in area of quadratic resistance for the case of free water there is speed of water in river-beds:

$v_a = C\sqrt{R \cdot I}$,

where $v_a$— average flow rate , meters per second; $C$— coefficient of resistance of friction on length (coefficient of Shezi), being integral description of forces of resistance; $R$— hydraulic radius, $m$; $I$—hydraulic slope $m/m$.

**Keywords:** flooding zones, digital elevation model, successive pools, Shezi.

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# Modeling of dispersed particles sedimentation in suspension at available liquid filtration

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**Abstract:** In the process of filtering the suspension through a porous medium the liquid and solid phases are separated. The need to study the movement of fluids in porous media containing dispersed phase occurs when solving many practical problems [1-3]. The latter include issues of cleaning fluids from impurities by filtration. For the analysis of such phenomena semi-empirical models most widely are used. However, as shown by comparison with experimental data, in many cases the kinetics of colmatage is significantly affected the rate of filtration. The feature of such problems is the presence of moving (generally unknown in advance) interfaces between the individual phases, on which put the boundary conditions. Analysis of the literature shows that, despite the large number of studies on hydro-separation processes in heterogeneous environments, the solution of this problem is still far from complete.

To describe the behavior of the suspension we use the system of equations for one-dimensional motion of two-phase mixture [4], and to describe the process of fluid flow through the layer of sediment used Darcy's law. Thus, we have a complete system of equations describing the laws of motion of the interfaces between the pure liquid, suspension and sediment. Importantly, nonstationarity problem in this case is implicitly manifested through the boundary conditions. We consider some special cases of neutrally buoyant and equilibrium suspensions. The formulas for the filtering time through sediment were obtained and analyzed. In general case the solution of equations were obtained and analyzed numerically.

**Keywords:** suspension, sedimentation, filtration

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# Computing analysis of one-dimensional flotation process

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**Abstract:** The current method of flotation is widely used for the separation of particulate mixtures in various industries [1, 2], which accounts for the urgency of the problem. Simulation of the flotation process is a rather complicated task, as the process takes place in a three-phase medium. Theoretical study of the flotation process is mainly based on the kinetic model, which does not take into account many important hydrodynamic factors. In this regard, it is necessary to construct a mathematical model of the flotation process, taking into account the influence of hydrodynamic factors [3]. As an illustration of the application of the constructed model, we consider a one-dimensional stationary case of flotation. In this paper are analyzed some calculated results obtained for air bubbles in the suspension of water with chalcocite mineral particles of radius 10 μm. The height of the slurry bed is 2 m, the speed of the pulp at the inlet is 0.01 m/s. For solution of the system of equations is used numerical fourth-order Runge-Kutta method. Analysis of the calculation results showed that the equation for the velocity of the bubble is singularly perturbed. In the presence of the flotation rate of bubble decreases monotonically. The bubbles of radius 60 μm due to their surface mineralization slowed so that, starting from a certain critical height they are moving at almost the speed of the suspension.

**Keywords:** suspension, flotation, computer simulation

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# Mathematical transformations of Erlang formula

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In this report the classical formula of Erlang formula for queuing system with refusals is analyzed. With the help of mathematical transformations its different presentations, more convenient to calculate the probability of load losses were obtained. A recurrent relation of calculating the probabilities of load losses and presentation of Erlang formula in integral kind were obtained.

Intensive theoretical researches and their practical introduction for the effective operation of high-speed computer networks in different areas of industrial activity put forward a number of the most pressing issues, related to the modernization of technological processes in the improvement of telecommunications as one of the developing economic sectors of the Republic of Kazakhstan. Such theoretical studies in the field of analysis and synthesis of computer networks are based primarily on the use of well-known mathematical methods and models. The target set of methods of information distribution is mainly due to the application of the mathematical theory of probability. The study of such methods has led to the formation of the theoretical trend, known as queuing system theory and has found wide application in many fields of science and engineering. Usage of the theory of probability in the field of information distribution is closely related to the calculation of the statistical parameters of the information network and is caused by load sensing and solution of analytical tasks of calculation of the characteristics of quality of service of network users.

# On suspension design and passengers comfort

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**Abstract:** In this work, we consider the dynamical model of a half car with passengers. It is assumed that the characteristic of the passive suspension is linear. The vibration behavior of the system has been studied by numerically simulating

the model over different roads with single bumps. By varying the mass of the passengers within a given range the variation and range of the vibration levels of the dynamical system has been analyzed. Initially, the vibration level of the system is minimized using genetic algorithm and the suspension parameters are determined. This is done by considering constant mass of the passengers. With these parameters the vibration analysis is further done by varying the mass of the passengers in a suitable interval.

**Keywords:** optimization, suspension, genetic algorithm, dynamical system, half car.

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# A method for solving equations of the turbulent atmospheric diffusion

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**Abstract:** In this presentation, we consider the problem in the frames of the meso scale range of the gas impurities distribution in the atmosphere: calculation of the gaseous impurity moderate quantity discharge, which density is higher than the air density, and the problem of discovery of the ‘active’ gas concentration distribution in the atmosphere. The concentration of distribution is modeling by the turbulent diffusion equation, which consists of two functions, accordingly describing the ‘heavy’ and the ‘active’ gas qualities.

The concentration of distribution is modeled by the following equation [1,2]:

$$\frac{\partial c}{\partial t}+V_x\frac{\partial c}{\partial x}-V_z\frac{\partial c}{\partial z}+(\alpha_1(t)+\alpha_2(t))c=\frac{\partial}{\partial x}(K_x\frac{\partial c}{\partial x})+\frac{\partial}{\partial y}(K_y\frac{\partial c}{\partial y})+\frac{\partial}{\partial z}(K_z\frac{\partial c}{\partial z})+f(x,y,z,t). \quad (1)$$

In the subjacent surface, we have condition

$$(K_z\frac{\partial c}{\partial t}+V_z c)\Big|_{z=z_0}=\left(v_s c\right)\big|_{z=z_0}. \quad (2)$$

Here, $t$ – time; $C$-concentration; $V_x$, $V_y$, $V$ – the gas velocity components; $K_x$, $K_y$, $K_z$ - turbulent diffusion coefficients in case the impurity discharge source is pointed nature having the coordinates.

Problem (1) - (2) is solved by splitting Marchuk method. Distribution of concentration is obtained at the initial stage of the process. A comparison numerical results with the results of other authors is given.

**Keywords:** Mathematical Modelling, turbulent diffusion, concentration distribution, active gas, meso scale range of discharge, free atmosphere.

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# Modeling and calculation of the trajectory of droplets in a multistage channel mass-transfer apparatus

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**Abstract:** The research paper is focused on the development of mathematical model of the drops movement in the gas flow in the mass transfer apparatus multi-stage channel. Dynamic characteristics of the gas in the laminar regime is obtained by numerical solution of the Navier-Stokes equations. Gas velocity in the turbulent regime is found by the solution of the Reynolds equations. Solvability of the mathematical models and convergence of the used numerical systems have been illustrated. The equation of motion drops has the form [1]:

$$\frac{d\left(m_d\vec{W}_d\right)}{dt} = -\xi\left(\frac{\rho}{2}\right) f_d w^2 \vec{e} + m_\kappa \vec{g}. \tag{1}$$

Using by the method of [1], for determining the values of $w(t)$ and $\gamma(t)$, we have

$$
\begin{aligned}
\frac{dw}{dt} &= -Kw^2 - g\sin\gamma - \left(P_x\cos\gamma + P_y\sin\gamma\right) - w\left(e'_x\cos\gamma + e'_y\sin\gamma\right) \\
&\quad - wA - A\left(W_g^x\cos\gamma + W_g^y\sin\gamma\right), \\
\frac{d\gamma}{dt} &= -\frac{g\cos\gamma}{w} + \frac{\left(P_x\sin\gamma - P_y\cos\gamma\right)}{w} - e'_y\cos\gamma \\
&\quad + e'_x\sin\gamma - \frac{A\left(W_g^x\sin\gamma - W_g^y\cos\gamma\right)}{w}.
\end{aligned}
\qquad (2)
$$

The components of gas velocity $W_g^{\tilde{o}}$, $W_g^{\acute{o}}$ and associated quantities $P_x$, $P_y$, $e'_x$, $e'_y$ had been calculated in [2].

Numerical solution of equations (2) determine the path of movement of droplets for different diameters, the path length of the droplets and their stay in the contact zone. The obtained data were used to calculate the overall mass transfer between the gas and liquid droplets.

**Keywords:** mathematical model, mixture of gas and droplets, multistage channel

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# An inverse problem of mathematical modeling of the extraction process of polydisperse porous materials

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**Abstract:** The processes of the extraction from the solid poly-disperse porous materials are widely distributed in the food, chemical, pharmaceutical and medical industries. Herewith the increased requirements are presented to the quality of the extract. To effectively carry out the process of extraction from solid poly-disperse porous materials it is necessary to use machines and processes which allow provide the optimal values of the hydrodynamic and physical and chemical parameters, in particular, the contact surfaces among the phases, the difference in the concentration of the target component in the raw material and the extractant, duration of extraction.

In this paper, we consider one family of problems simulating the determination of target components and density of sources from given values of the initial and final states. The mathematical statement of these problems leads to the inverse problem for the diffusion equation, where it is required to find not only a solution of the problem, but also its right-hand side that depends only on a spatial variable. A specific feature of the considered problems is that the system of eigenfunctions of the multiple differentiation operator subject to boundary conditions of the initial problem does not have the basis property. We prove the unique existence of a generalized solution of the problem.
**Keywords:** Inverse problems, Diffusion equation, Initial state, Final state, Not strongly regular boundary conditions, Samarskii-Ionkin boundary conditions, Biorthogonal Fourier series, Riesz basis, Extraction process, Polydisperse porous materials.
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# A modelling of thermal source's efficiency via data envelopment analysis

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**Abstract:** This research utilizes relational data envelopment analysis (DEA) to construct a model to analyze the efficiency and effectiveness of thermal sources of Canakkale in TURKEY. One input and two outputs are used in analysis, covering 9 natural thermal sources. The results show that one natural thermal source is efficient.
**Keywords:** data envelopment analysis (DEA), efficiency, thermal source.
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# Mathematical Modelling of a five stages $n^2+n$ voltage multiplier's output in the time-domain

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**Abstract:** The time-domain behavior of the $n^2+n$ voltage multiplier was unknown. In this manuscript, we present an optimized approximate mathematical model for the output voltage of a $n^2+n$ voltage multiplier. The suggested mathematical model includes two components of an exponential recovery equation and a sinusoidal equation which are performing as a function of time and frequency.

**Keywords:** Mathematical Modelling, voltage multiplier, time-domain, optimization, non-linear least squares.

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# Nonlinear matrix modeling of macro system asymptotic behavior

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**Abstract:** A class of the dynamical systems is proposed as nonlinear matrix models. Let $F$ be the map of the form [1]

$$F : R^n \to R^n, \quad F\,y = \Phi(y)\,Ay, \tag{1}$$

where $R^n$ is $n$- dimensional real vector-space, $\Phi(y)$ is a scalar function, $A$ is a matrix of $n$ order. Select $X \subseteq R^n$ , $FX \subseteq X$ then map $F$ is in general, non- invertible and generates the dynamical system $\{F^m, X, Z_+\}$. Here $F^m$, $m \in Z_+$ is a cyclic

semi-group of maps, $X$ is a phase space of the system, $Z_+$ is the set of nonnegative integers. The systems $\{F^m, X, Z_+\}$ are used as mathematical models for determining the macro system dynamics in the presence of limiting factors. In the models, $n$ is a number of macro system components, $y \in X$ is a vector of component characteristics, $X = \left\{ y \in R^n \middle| y \geq 0, \|y\| \leq a \right\}$, $a < \infty$ ($y \geq 0$ means nonnegativity of its coordinates), $A$ is a matrix of component interrelations, $\Phi(y)$ is a limiting function (limiting factor).

There are some problems with the nonlinear matrix modeling one of which concerns reducing the system size. Since the number $n$ may be very large then constructing appropriate algorithms for determining the system asymptotic behavior is essential. We constructed computer algorithm for determining the asymptotic behavior of macro systems governed by the systems $\{F^m, X, Z_+\}$. It is based on the results of qualitative theory which we develop for this class of the dynamical systems. Stabilized $p$- periodic macro system structure, the number of parameters ($\leq p$) by which the macro system dynamics is described, and the limit set of macro system component characteristics for any nonzero initial vector $y$ are obtained by the algorithm. Here the macro system structure (at the time $m$) is a vector $\left\|F^m y\right\|^{-1} F^m y$ for any nontrivial trajectory $\{F^m y\}$. The algorithm is very useful at $p > n$ as well as at $p > n^2$ .

**Keywords:** dynamical system, nonlinear matrix models, computer simulation

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# APPLIED MATHEMATICS 3

## Error control methods for digital devices

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**Abstract:** Most modern communication systems work by means of the transmission digital information. It is known that due to the presence of the noise in the communication channels in the process of working of the digital device occur the errors, which distort the information. If the reason is the fail of the element of the device, the permanent distortion of information occurs and in the case of the noises one definite code combination replaces the other, that leads to the failure of the devise. Because of the occurrence of errors, it is necessary to provide the device with the system of control of the correctness of the circularizing information. The effective way of ant blackout is the noiseless coding, which is based on introducing artificial redundancy in sending message. In such control systems the following method of ant blackout is used: coding with errors detection, coding with errors correction. Methods of errors detection and correction use minimal code distance i.e. minimum number of symbols, in which all of the code combinations differ from each other. In common case, for detection $r$ errors – minimal code distance $d0 = r + l$; for synchronous error detection and correction: $d0 = r + s + l$; where $s$ - the number of detected errors. For codes only detecting errors $d0 = 2s + 1$. For computer system error detections linear periodic code is widely used. It is accepted to describe periodic number with the help of the generator polynomial $G(X)$ involution $m = n - k$, where $m$ - the number checking symbols in code word. In the article the process of periodic codes for system errors detection, the choice of the constitutive polynomial according to the adjusted code dimension and detection adjustment capacity are observed.

**Keywords:** coding, noiseless code, periodic code, generator polynomial, errors detection.

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# Quick multiplying multidigit number with use the modification of the algorithm fast transformation Fourier

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**Abstract:** In this study, we develop approach Shtrassen [1] to quick multiplying multidigit number in a part of the use for its realization of the original modification algorithm of the fast transformation Fourier (FTF) [2].
For realization on computer asymmetric cryptographic algorithm it is necessary to have a library effective on speed algorithm and programs of the execution operation with multidigit number.
Time to operations of the multiplying on processor with fixed by length of the word is proportional to the square of the length operand $O(n)^2$. By construction of consecutively-parallel hardware provision, that time can be reduced to $O(n)$. In this case amount of required logical elements will be proportional to a length operand $O(n)$. Time of the work for the most high-speed realization is proportional to $O(\log n)$, but it requires $O(n)^2$ logical elements.
For the method Karacuby, its difficulty is $O(n^{\log_2 3})$, where $\log_2 3 \approx 1.58$ [3]. The method polynomial since running time of the order $n^{1+\varepsilon}, (\varepsilon > 0)$. The algorithm Tooma-Kuka has difficulty of the order $O(n 2^{\sqrt{2\log_2 n}} \log_2 n)$ [4]. The algorithm Shenhage – Shtrassen allows to multiply two -$n$ a class numbers, executing for $O(n \log n \log\log n)$ step (the bit operation) [1].
Note that these methods are based on the information of the multiplying $n$- a class numbers to multiplying numbers with smaller number category.
**Keywords:** multidigit number, modification, cryptographic, hardware provision.
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## A friendly introduction to condensing operators

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**Abstract:** Condensing operators contain both compact and contracting operators as special cases. Therefore, Darbo's celebrated fixed point theorem [1] and Sadovskij's generalization [2] bridge the gap between Schauder's fixed point principle and the Banach-Caccioppoli contraction mapping theorem. The aim of this talk is to illustrate the usefulness of the Darbo-Sadovskij theorem by means of a large variety of applications, ranging from integral equations over boundary value problems to spectral theory, for details see [3]. The presentation is throughout elementary and requires only a modest background in functional analysis and operator theory.
**Keywords:** measure of noncompactness, condensing operator, fixed point theory
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## The wave field in a rectangular area with discontinuities in boundary conditions

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**Abstract:** In this study, we consider the plane elastic isotropic media with a rectangular cross section of finite size. At the initial time on a certain part of the front

of the rectangular field of applied external dynamic is П - shaped loading and the remainder of this boundary is stress-free. Another side of the rectangular area is defined by boundary conditions. For plane strain conditions, the problem is solved numerically by using the spatial characteristics [1-2]. Especially, we consider the body such that the singular points of the rectangular area of the front boundary conditions have discontinuity of the first kind. Namely, in these critical points the system of equations for the unknown functions are obtained. The stability calculation algorithms for a sufficiently long time is established by numerical implementation. Result of the study in its final form brought to the numerical solution.
**Keywords:** dynamic loading, plane deformation, singular point, elastic.
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# Influence of heterogeneity of the character fixing the border on the spread of two-dimensional waves

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**Abstract:** In this study planar elastic isotropic medium with rectangular cross section of finite size is considered. At the initial time, on the front border of the rectangular area, absolute rigid body having speed is impinged. Lateral sides of the rectangular area are free from stress. On the lower boundary inhomogeneous boundary conditions when the plane deformation problem has been solved numerically using the method spatial characteristics [1-4] are given. We have developed a numerical algorithm for the calculation of stress and displacement velocity at points of discontinuity of the boundary conditions that are special because of the abrupt change in the boundary conditions. Results of the study in its final form are brought to the numerical solution.
**Keywords:** isotropic medium, plane strain, a singular point, wave processes, numerical algorithm.
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# On strongly GA-convex functions and stochastic processes

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**Abstract:** In this study, we introduce strongly GA-convex functions and stochastic processes. We provide related well known Kuhn type results and Hermite-Hadamard type inequality for strongly GA-convex functions and stochastic processes.

**Keywords:** strongly GA-convexity, stochastic process, Kuhn type result, Hermite-Hadamard type inequality.

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# A novel modified simple equation method and its application to some nonlinear evolution equation systems

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**Abstract:** In this paper, the modified simple equation (MSE) method [1,2] is used to construct exact solutions of the nonlinear Drinfeld–Sokolov system, Maccari system and Coupled Higgs equation The exact solutions obtained by the proposed method indicate that the approach is easy to implement and computationally very attractive. Also we can see that when the parameters are assigned special values, solitary wave solutions can be obtained from the exact solutions [3-6]. All calculations in this study have been made with the aid of the Maple packet program.
**Keywords:** exact solutions, modified simple equation method (MSE), the nonlinear Drinfeld-Sokolov system, Maccari system, Coupled Higgs equation.

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# Bessel functions in problems of inhomogeneous soil consolidation

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**Abstract**: Soil modulus of deformation which continuously increases with depth is called continuously inhomogeneous. In this paper, the inhomogeneity is represented as follows:

$$E = E_m(\alpha + \beta z)^m \quad (\alpha > 0,\ \ E_m > 0,\ \ \alpha + \beta z > 0\,),$$

where $E_m, \alpha, \beta, m$ are experienced parameters.

On the basis of this dependence, consolidation problems of elastic and elastic-creeping inhomogeneous soils applied to the restricted area sealing are solved. These solutions make it possible to calculate the values of the pore pressure, the sum of the principal stresses and vertical displacements of the points of the upper surface of the sealed inhomogeneous soil mass.

In these solutions strongly compressible saturated clay soils are also considered that at the initial time of the load, the load $q$ is applied instantly to the ground, which is equal in magnitude of structural strength compression $p_{стр}$ is immediately perceived by soil skeleton.

In addition, Darcy's law is violated, i.e. the initial pressure gradient is considered. The resulting formulas are presented as a combination of Bessel functions of the first and second kinds. It is now possible to determine the value of any given Bessel functions and to calculate the pressure in the pore fluid and predict rainfall velocity in the sealed array. Similar problems in various formulations are investigated in [1-2].

**Keywords:** soil, consolidation voltage deformation modulus, pore pressure.

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# A second order of accuracy finite difference scheme for the integral-differential equation of the hyperbolic type

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**Abstract:** In this study, a second order of accuracy difference scheme approximately solving the initial-value problem for the integral-differential equation of the hyperbolic type in a Hilbert space $H$ is presented. The stability estimates for the solution of this difference scheme are established. In application, the second order of accuracy difference scheme for solving the local boundary problem for the multidimensional integral-differential equation of the hyperbolic type with two dependent limits is constructed. Theoretical results are supported by numerical examples.

**Keywords:** finite difference method, integral-differential equation of the hyperbolic type.

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# Numerical solution for high-order linear complex differential equations in a circular domain

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**Abstract:** In this study, a collocation approach and Legendre polynomials are used to get numerical solution of high-order linear complex differential equations in circular domain. The solutions in the form of power series are obtained. Method is applied on test problems. Results demonstrate in tables that they are quite applicable. All of the numerical computations have been computed on computer using a code written in Matlab.
**Keywords:** Linear complex differential equations, Legendre polynomials, collocation method, numerical solution.
**References:**
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# On the optimality of one power system

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**Abstract:** We will consider a mathematical model that describes the transients in the electrical system, and represents the following system of nonlinear differential equations:

$$\frac{d\delta_i}{dt} = S_i,\ H_i\frac{dS_i}{dt} = -D_iS_i - E_i^2Y_{ij}\sin\alpha_{ij} - P_i\sin(\delta_i - \alpha_i)$$

$$- \sum_{j=1, j\neq i}^{l} \mathrm{P_{ij}}\left(\delta_{ij} - \alpha_{ij}\right) + u_i, \quad i = \overline{1,l}, \quad t \in [0,T] \quad (1)$$
$$\delta_{ij} = \delta_i - \delta_j, \quad P = E_i U Y_{i,n+1}, \quad P_{ij} = E_i E_j Y_{i,j},$$
where $\delta_i$ is angle of rotation of the rotor of i-th generator with respect to some synchronous rotational axis (the axis of rotation of constant voltage tires, it makes 50 rev / sec); $S_i$ is sliding of the i-th generator; $H_i$ is constant inertia of i-th machine; $u_i$ is electromotive force i-th machine; $Y_{ij}$ is mutual conductance of i-th and j-th system branches; U=const is the voltage on constant voltage tires; $Y_{i,n+1}$ is characterizes the contact (conductivity) of i-th oscillator with constant voltage tires; $D_i$=const≥0 is mechanical damping, $\alpha_{ij}\alpha_i, \alpha_j$ are constants, taking into account the influence of active resistances in the generator stator circuits. The complexity of model (1) consists in considering analysis $\alpha_{ij}$ with the following property $\alpha_{ij} = \alpha_{ji}$. We believe that the conditions $\delta_{ij} = -\delta_{ji}$ are realized. In this case, the model (1) is not conservative; so it isn't possible to build for it the Lyapunov function in the form of the first integral. This creates an additional difficulty for its research. System (1) is traditionally called the positional model, and it belongs to non-conservative class.

**Keywords:** electric power system, nonlinear system, phase system, control synthesis, Bellman-Krotov function

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# A numerical comparison for recent modifications of Adomian decomposition methods for nonlinear fractional KdV equations

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**Abstract:** The explicit solutions to the modified fractional Korteweg–de Vries (KdV) equation with initial condition are calculated by using the recent modified decomposition methods. To illustrate the application of these methods, numerical results are obtained and compared with classical and modified fractional KdV

equations. The numerical results demonstrate that the many of them are relatively accurate and easily implemented.

In the recent years, remarkable progress has been made in the construction of the approximate solutions for nonlinear fractional partial differential equations (nfPDE) [1-7]. In some sense fractional differential equations could represent various real-life problems. Because of this usefulness finding the exact and numerical solutions' methods for these types of equations has become very important. The studies in finding exact solutions to nfPDE, when they exist, are very important for the understanding of most fractional nonlinear physical phenomena.

In this part, we will introduce the reader to a certain nonlinear partial differential equation (PDE) which is characterized by the solitary wave solutions of the classical nonlinear equations that lead to solitons [8-12]. We meant with the classical nonlinear equations of interest usually admit for the existence of a special type of the traveling wave solutions which are either solitary waves or solitons. In this study, we will implement various versions of ADM [13-22] for a few solutions arising from the semi-analytic work of the fractional Korteweg-de Vries (fKdV) equation and modified fractional Korteweg-de Vries (mfKdV) equation for a comparable way.

**Keywords:** Adomian decomposition method, the Rach–Adomian–Meyers modified decomposition method, Waxwaz's modified decomposition method, fractional mKdV, the solitary wave solutions, modified Riemann–Liouville derivative.

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# On the solvability of the problem of the optimal boundary control of thermal processes described by the Volterra integro-differential equations

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**Abstract:** In this paper, we investigate the problem of tracking, where it is required to minimize the functional

$$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2 dxdt + \beta \int_0^T u^2(t)dt, \quad \beta > 0,$$

on the set of solutions of the following boundary value problem

$$V_t - AV = \lambda \int_0^t K(t,\tau)V(\tau,x)\,d\tau + g[t,x], \quad x \in Q \subset R^n, \ \ 0 < t \le T,$$

$$V(0,x) = \psi(x), \quad x \in Q,$$

$$\Gamma V(t,x) \equiv \sum_{i,j=1}^n a_{ij}(x) V_{x_j}(t,x) cos(\delta, x_i) + a(x)V(t,x) = b[t,x]p[t,u(t)],$$

$$x \in \gamma, \ \ 0 < t \le T.$$

Here, $\gamma$ is a piecewise smooth boundary of the region $Q$, $\delta$ is a normal, which is conducted at the point $x \in \gamma$, $A$ is an elliptic operator; $g[t,x] \in H(Q_T)$, $Q_t = Q \times (0,T)$, $b[t,x] \in H(\gamma \times (0,T))$, $p[t,u(t)] \in H(0,T)$, $\psi(x) \in H(Q)$ are given functions, $u(t) \in H(0,T)$ is boundary control, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; $T$ is a fixed moment of time; $\lambda$ is a parameter.

**Keywords:** Volterra integro–differential equation, nonlinear integral equation, optimal control.

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# On the solvability of the problem of the distributed optimal control of oscillation processes described by the Fredholm integro-differential equations

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**Abstract:** In this paper the problem of tracking were studying, where it is required to minimize the functional

$$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2 dxdt + \beta \int_0^T \int_Q p^2 [t,x,u(t,x)] dxdt,$$

$\beta > 0,$

on the set of solutions of the following boundary value problem

$$V_{tt} - AV = \lambda \int_0^T K(t,\tau)V(\tau,x)\,d\tau + f[t,x,u(t,x)], \ \ x \in Q \subset R^n, \ \ 0 < t \le T,$$

$$V(0,x) = \psi(x), \quad x \in Q,$$

$\Gamma V(t,x) \equiv \sum_{i,j=1}^{n} a_{ij}(x) V_{x_j}(t,x) cos(\delta, x_i) + a(x)V(t,x) = 0,$
$x \in \gamma, \;\; 0 < t \le T.$
Here, $\gamma$ is a piecewise smooth boundary of the region $Q$, $\delta$ is a normal, which is conducted at the point $x \in \gamma$, $A$ is an elliptic operator; $f[t,x,u(t,x)] \in H(Q_T)$, $Q_T = Q \times (0,T)$, $\psi(x) \in H(Q)$ are given functions, $u(t,x) \in H(Q_T)$ is distributed control, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; $T$ is a fixed moment of time; $\lambda$ is a parameter. It is established that the optimal control $u = u^0(t)$ is defined as the solution of a nonlinear integral equation with discontinuous kernel and satisfies the additional conditions in the form of inequality. Solution is obtained in the form of a triplet $(u^0(t),\; V^0(t,x),\; J[u^0(t)])$, where $u^0(t)$ is the optimal control, $V^0(t,x)$ is the optimal process, $J[u^0(t)]$ is the minimal value of the functional.
**Keywords:** functional, Fredholm integro – differential equation, the optimality condition, nonlinear integral equation, optimal control.
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# On the solvability of the problem of the optimal control of thermal processes described by the Fredholm integro-differential equations

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**Abstract:** We study the problem of tracking, where it is required to minimize the functional

$$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2 dxdt + \beta \int_0^T u^2(t)dt, \quad \beta > 0,$$

on the set of solutions of the following boundary value problem
$V_t - AV = \lambda \int_0^T K(t,\tau)V(\tau,x)\, d\tau + g[t,x] f[t,u(t)], \;\; x \in Q \subset R^n, \;\; 0 < t \le T,$
$V(0,x) = \psi(x), \quad x \in Q,$
$\Gamma V(t,x) \equiv \sum_{i,j=1}^{n} a_{ij}(x) V_{x_j}(t,x) cos(\delta, x_i) + a(x)V(t,x) = 0,$
$x \in \gamma, \;\; 0 < t \le T.$

Here, $\gamma$ is a piecewise smooth boundary of the region $Q$, $\delta$ is a normal, which is conducted at the point $x \in \gamma, A$ is an elliptic operator; $g[t,x] \in H(Q_T)$, $Q_T = Q \times (0,T)$, $f[t,u(t)] \in H(0,T)$, $\psi(x) \in H(Q)$ are given functions, $u(t) \in H(0,T)$ is control function, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; $T$ is a fixed moment of time; $\lambda$ is a parameter.

**Keywords:** quadratic functional, Fredholm integro-differential equation, optimal control.

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# On the solvability of the problem of the optimal control of thermal processes described by the Volterra integro-differential equations

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**Abstract:** In this paper, we study the problem of tracking, where it is required to minimize the functional

$$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2 \, dxdt + \beta \int_0^T |u(t)| dt, \quad \beta > 0,$$

on the set of solutions of the following boundary value problem

$V_t - AV = \lambda \int_0^t K(t,\tau) V(\tau,x)\, d\tau + g[t,x] f[t,u(t)], \;\; x \in Q \subset R^n, \;\; 0 < t \leq T,$

$V(0,x) = \psi(x), \quad x \in Q,$

$\Gamma V(t,x) \equiv \sum_{i,j=1}^n a_{ij}(x) V_{x_j}(t,x) cos(\delta, x_i) + a(x) V(t,x) = 0,$

$x \in \gamma, \;\; 0 < t \leq T.$

Here, $\gamma$ is a piecewise smooth boundary of the region $Q$, $\delta$ is a normal, which is conducted at the point $x \in \gamma, A$ is an elliptic operator; $g[t,x] \in H(Q_T)$, $Q_t = Q \times (0,T)$, $f[t,u(t)] \in H(0,T)$, $\psi(x) \in H(Q)$ are given functions, $u(t) \in H(0,T)$ is control function, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; $T$ is a fixed moment of time; $\lambda$ is a parameter.

**Keywords:** piecewise linear functional, Volterra integro–differential equation, optimal control.

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# On the solvability of the problem of the optimal boundary control of thermal processes described by the Fredholm integro-differential equations

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**Abstract:** We study the problem of tracking, where it is required to minimize the functional

$$J[u(t)] = \int_0^T \int_Q \left[V(t,x) - \xi(t,x)\right]^2 dxdt + \beta \int_0^T u^2(t)dt, \quad \beta > 0,$$

on the set of solutions of the following boundary value problem

$V_t - AV = \lambda \int_0^T K(t,\tau)V(\tau,x)\, d\tau + g[t,x], \quad x \in Q \subset R^n, \ \ 0 < t \le T,$

$V(0,x) = \psi(x), \quad x \in Q,$

$\Gamma V(t,x) \equiv \sum_{i,j=1}^n a_{ij}(x)V_{x_j}(t,x)cos(\delta,x_i) + a(x)V(t,x) = b[t,x]p[t,u(t)],$

$x \in \gamma, \ \ 0 < t \le T.$

Here, $\gamma$ is a piecewise smooth boundary of the region $Q$, $\delta$ is a normal, which is conducted at the point $x \in \gamma$, $A$ is an elliptic operator; $g[t,x] \in H(Q_T)$, $Q_t = Q \times (0,T)$, $b[t,x] \in H\big(\gamma \times (0,T)\big)$, $p[t,u(t)] \in H(0,T)$, $\psi(x) \in H(Q)$ are given functions, $u(t) \in H(0,T)$ is control function, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; $T$ is a fixed moment of time; $\lambda$ is a parameter.

**Keywords:** integro-differential equation, boundary control, the optimality condition.

**References:**

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[2] A.K. Kerimbekov, *On solvability of the nonlinear optimal control problem for prosesses described by the semi-linear parabolic equations*, Proceedings World Congress on Engineering, vol. 1, pp. 270–275, 2011.

# Estimation of parameters in logistic distribution with RSS and comparing with SRS

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**Abstract:** Logistic function was used as a growth curve for the first time in 1838. Logistic model was used in economic growth, population growth studies, regression analysis natural sciences. Because of the importance of this distribution, we need optimal estimators for the involved parameters. In this paper we will estimate the parameters of the Logistic distribution based on Simple Random Sampling and Ranked Set Sampling, and we will compare these two methods.
**Keywords:** logistic distribution, order statistics, simple random sampling, ranked set sampling, momen estimation, maximum likelihood estimation.
**References:**
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# Commutativity conditions of some second order time-varying linear differential systems

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**Abstract:** The realization of many engineering systems consists of cascade connection of systems of simple orders, which is very important in design of electrical and electronic systems. Although the order of connection of the systems mainly

depends on the special design approach engineering genuity, traditional synthetic methods, when the sensitivity, stability, linearity, noise disturbance, robustness effects are considered the change of the order of connection without changing the main function of the total systems (commutativity) may lead positive results. Therefore the commutativity is very important from the practical point of view.
The subject has appeared to be a tutorial paper in the literature; in spite of including the general commutativity conditions, this paper is lack of focusing on the special engineering problems from the commutativity point of view.
The aim of this presentation is to investigate the commutativity properties and its results for the well-known second-order time-varying differential equations each of which describes some particular physical systems, which will constitutes of the original value of the project. The expected outcomes will lead new design trends in engineering as to improve the total system performance covering sensitivity, stability, disturbance and even robustness.
In particular, the commutativity of systems described by Lommel, Halm and Legurre type differential equations are investigated and requirements for commutativity are derived. It is shown that under certain circumstances these systems have commutative pairs some of which have explicit analytical solutions.
**Keywords:** Commutativity, Time-varying Systems, Differential Equations, Initial Conditions, Stability, Sensitivity, Disturbance, Robustness
**References:**
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# Variational-B-spline iteration method for solution of Lane-Emden equation

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**Abstract:** In this paper, the variational method based on the linear B-spline functions are presented for the solution of Lane-Emden equation. Some properties of B-spline functions are presented and utilized to solve the Lane-Emden equation

numerically. Illustrative examples are given to demonstrate the validity and applicability of the presented technique.

**Keywords:** B-spline function, Lane-Emden equation, operational matrix of derivative, operational matrix of integration.

**References:**

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## Efficient quadrature rules for solving nonlinear fractional integro-differential equations of the Hammerstein type

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**Abstract:** The aim of this paper is to solve nonlinear fractional integro-differential equations of the Hammerstein type. The basic idea is to convert fractional integro-differential equations to Volterra integral equations of the second kind. Then, the obtained Volterra integral equation will be solved by some suitable quadrature rules. Numerical tests for demonstrating the convergence and accuracy of the method are given.

**Keywords:** quadrature rules, fractional integro-differential equations.

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# The differential transform method for solving the model describing biological species living together

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**Abstract:** F. Shakeri. and M. Dehghan in [1] presented the variational iteration method for solving the model describing biological species living together. Here, we suggest the differential transform (DT) method for finding the numerical solution of this problem. To this end, we give some preliminary results of the DT and by proving some theorems, we show that the DT method can be easily applied to mentioned problem. Finally several test problems are solved and compared with variational iteration method.

**Keywords:** biological species living together, differential transform method, Volterra integro-differential equations, variational iteration method.

**References:**

[1] F. Shakeri, M. Dehghan, *Solution of a model describing biological species living together using the variational iteration method*, Mathematical and Computer Modeling, vol. 48, pp. 685-699, 2008.
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# Numerical solutions of linear differential-algebraic equation systems via Hartley series

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**Abstract:** In this paper, Hartley series are presented first. Then, the operational matrix of integration together with the product and coefficient matrices are presented. They are used to transform linear differential equation systems to a set of linear algebraic equations. Finally, numerical examples are given.
**Keywords:** Hartley series, linear differential equation systems, linear differential-algebraic equation systems, approximation.
**References:**
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# Definition of characteristics of output streams of the random data in radio telemetering systems with compression

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**Abstract:** At compression of the data in radio telemetry systems arriving on an input of the device of compression of the data the stream of essential references represents random process. Therefore at analytical research of the device of shaping of several streams of its oblate data is considered possibly as system of a queuing [1]. Thus, it is possible to solve a problem of search of optimum parameters on boundary of area of their modification [2].
As coming requirements in the form of essential references are maintained by the groups which are not exceeding certain number $s$ holding time distribution $s$ on Erlang looks like requirements:

$$f(t) = \mu^k t^{k-1} e^{-\mu t} / (k-1)! \qquad (1)$$

where $\mu = k / M[t]$- intensity of service of one requirement, $k$ - the distribution parameter of Erlang.
As a result of a problem solution of the probability distribution of a condition of system is received that and relative the average length of turn and its variance essentially depend on character of the distribution law of the holding time determined in parameters $k$ and $s$ of simultaneously maintained essential references.
**Keywords**: Compression of the data, essential references, queuing system, flow of priority messages.

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# MATHEMATICAL EDUCATION

## Mathematics in the eyes of academics and students of engineering faculty

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**Abstract:** Engineers are known as the people who have knowledge about mathematics, science, and technology and as the people who solve the problems with their knowledge in real life situations. They define *mathematics* as *the language of engineering*. Although the importance of mathematics in *engineering education* is widely accepted, it has been determined in recent studies that engineering faculty students, especially those who start their educational processes in recent years, do not have sufficient mathematical knowledge. This problem has been emphasized that some academics do not attach enough importance to the basic sciences (mathematics, physics, chemistry, and so on) [1-3]. Studies about engineering show the relation between the daily life and engineering education. So, mathematics is related with daily life. For this reason, the views of engineering faculty on mathematics and the benefits of it are important for engineering education. The aim of the study is that examining and determining the views of the academics and students from different departments of the engineering faculty on mathematics and its benefits in terms of different variables (mathematics as a discipline, mathematics in daily life, mathematical knowledge and the applications in students' own fields and relations among them). In this context, a literature-supported questionnaire contains three sub-dimensions. It was applied to 123 academics from different departments of Faculty of Engineering at Gumushane University and to 648 students of the same department. Then, semi-structured interviews were performed with three academics and students from each department. Most of the academics and students consider that mathematics is a beneficial field as a problem solving technique and a mental activity to develop the mental skills. Academics and students from different departments made use of different skills to define mathematics; like mathematical, mental, communicational, practical skills, etc. It has been determined that participants generally preferred the skills relevant with their own fields, their learning areas, applications and benefits of

mathematics. While most of the students think that “mathematics was discovered by human beings while they were trying to understand the world they lived in”, less students think that “mathematics was developed by human beings to cover their practical needs”. The academics, in general, stated that both points of views are true, but most of them chose the second one. Except for the academics and students from the mathematical engineering department, academics and students from other departments consider mathematics as a “tool.”
**Keywords:** mathematics, academics and students thoughts, engineering faculty.
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## Enhancing ideological and humanistic orientation in teaching mathematics

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**Abstract**: In this study, we consider enhancing ideological and humanistic orientation in mathematics education to develop students' ideas about the correct reflection of reality in mathematics and to disclose them of the dialectical character of the development of its basic concepts, ideas, techniques and laws.
We cover issues related to the theory of knowledge, issues related to the process of mathematization of scientific knowledge, the role of applied mathematics, questions the relation between mathematics and dialectics, and historical questions of mathematics. This allows, on the one hand, to introduce students with the most important ideological and humanistic issues of school mathematics. On the other hand, it allows to study special mathematical questions with an emphasis on their ideological and humanistic side generalization logical-methodological character.
**Keywords**: ideological and humanistic orientation, teaching mathematics, history of mathematics, logic and methodological generalization, the theory of knowledge.
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# History of the formation of profile training mathematics in Kazakhstan and prospects of development

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**Abstract:** We analyzed the current state of profile training in Kazakhstan and abroad. At the moment, the whole school system of Kazakhstan is undergoing fundamental changes in structural and organizational, also in content and methodologically. Before, the comprehensive school has the task of preparing of the competitive graduates. One way to implement this task was the introduction to the 2006-2007 year profile training in high classes of secondary school.
Study of the problems of formation and development profile training in mathematics in Kazakhstan is connected with the problem to enhance scientific and methodological soundness profile differentiation in mathematical education of senior pupils. It is also aimed to solve the following problems: selection of content for the system of additional education in high classes; development programs profiled basic and advanced training courses; ensure a comprehensive readiness of senior pupils to continue their studies in high school mathematics; development of methodical support of elective courses and methods of enhancing pupils' cognitive activity; formation of general and professional competencies of future teachers of mathematics; and mathematics teachers to teach in specialized classes.
**Keywords:** profile education, mathematics, education system of Kazakhstan, elective courses, methodological support of the course.
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# The peculiarities of the elective course "Elements of the theory of probability" in terms of specialized education

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**Abstract:** In modern developing world, numerous people face with problems. Most of them are associated with the analysis of the influence of random factors and decisions in situations with a probabilistic basis.
The article presents examples about the problems of applied nature which allowed the analysis of the decision to come to the following conclusions:
1. Problems of this nature are presented in the elective course "Combinatorics Probability and Statistics" which develops probabilistic-statistical thinking of students and contributes the formation of the modern worldview. It also helps the destruction of rigid ideas about the connections between events and phenomena, and helps to make decisions in situations with a probabilistic framework and variable character.
2. Separation and accounting features of methodical teaching an elective course can solve the problems associated with the improvement of the overall direction of applied mathematics education. Multilevel design content based on differentiation in the interests, needs and opportunities for students to develop the elements of probability theory.
**Keywords:** mathematics, modernization of education, specialized education, the elective course, technology training, probability theory and statistics, improving.
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# Ways of improving the modernization of education in Kazakhstan and problems of specialized education

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**Abstract:** Today, the great importance to the modernization of general education has diversifying abilities, skills, self-education of children, and the youth formation of willingness to adapt to changing of social and economic conditions of life. These problems can be solved on the basis of differentiated content, organizational forms, teaching methods, educational needs, interests, aptitudes, and abilities of students.

Establishment of specialized education is a relevant and important step in the development of Kazakhstan's school system. Comparative analysis of specialized education in foreign countries shows the effectiveness of the implementation of specialized education into educational process. Thus, differentiation of education in schools is common in many countries. We have analyzed the experience of implementation of specialized education in countries such as the US, Japan, Germany, China, Turkey, and Russia.

The analysis showed that the study of educational conditions in the UK, France, Germany, South Korea and Japan can state that the competition of countries in the economic field is reduced the struggle for supremacy in the field of science and technology. Hence, competition in the training of highly qualified technicians is a competent system solution in the field of practical problems.
Modernization of education as a factor in the strategic development of the country should become a priority for Kazakhstan. Education cannot be limited by "tightening," it must be proactive. The new socio-economic conditions of the educational institutions exercise their rights in terms of academic and economic autonomy. It also expands the range of educational services and provide to citizens and enterprises of different organizational forms. Thus, specialized education is time requirement.

Establishment of specialized education is a basic part of continuing education that is directed to implement the learner-centered educational process. It contributes to the empowerment of the individual.
**Keywords:** Mathematics, modernization of education, specialized education, the elective course, technology training, probability theory and statistics, improving

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# Scientifically-methodical system of extensive training of the discipline “Elements of the spectral theory of operators” for future teachers of mathematics

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**Abstract:** This article described the scientific and methodical system of expanded training to a special course for undergraduates. The training program disclosed the need of special courses to prepare teachers according to the purpose of a special course. Its appointment and received results from studying of a special course were also considered.

Elective courses provided an opportunity to improve the quality of training specialists that were able to be ready to change. That change was happening in an educational institutions of higher education and introduced new specialization relevant regional requirements [1].

Learning objectives elements of the spectral theory in the master are:

1. Teaching modern methods of the basis properties for solutions of spectral problems;
2. Ability to apply mathematical methods of research in solving applied problems;
3. Disclosure humanitarian potential of learning elements of the spectral theory of operators;
4. Achieving independent work with a special mathematical literature, ways to apply acquired knowledge in their professional activities.

These goals make future teachers high requirements. In the content of the elective course "Elements of spectral theory" taught in higher education institutions to prepare future mathematics teachers include the emergence and development of the basic concepts (properties of bases , ordinary first order differential operators, own functional systems overall spectral problem, boundary conditions, and so on) definitions, characteristics and properties. The relationships with other disciplines are development of the elementary theory of generalized spectral problem, research methods, metodolonicheskie and historical aspects of this science [2].

**Keywords:** eigen values, eigen functions, boundary conditions, adjoined function.

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# The key teaching issues on mathematics and physics integration

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**Abstract:** The interdisciplinary integration is considered as one of the realistic aspects of teaching. It was described in a great number of scientific and educational researches. That's why this approach should be kept in mind as an alternative way.

The interdisciplinary integration has a long history. It was founded by Kamenski, Ushinski, Danilov, and Esipov. The psychological features of this field were illustrated by such scientists as Pavlov, Samarin, Sechenov, and Vygotski. The impact of integration on the teaching process was determined by Kabanova-Meller, Zverev, Fedorova, Kulagin, and Maksimova [1].

Many school teachers design their lesson plans of this combination and find it as a practical method of teaching. Mathematics and physics are related to the most difficult disciplines of secondary education. The handling of this unit has given beneficial results. In fact, through many experiments researchers (Usova, Multanovski, Kozhekina, Sinyakov, and Tazhmaganbetov [2]) expound on the merits of this link as a learning activity.

Nevertheless, there are some barriers for beneficial effect on proficiency in mathematics and physics integration. The following issues should be detailed:

1. Supplementary materials, binders, plans, handouts, programs for mathematics and physics integration are not made and they are not published.
2. The same notions have different explanations in both disciplines as: alternating-variable quantity, vector-vector quantity, multiplier-proportional multiplier, functional dependence, collinear vectors.

The purpose of our research is to offer practical activities for solving these challenges.

**Keywords**: Mathematics, physics, inter disciplinary integration, methodical bases.

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# Task solving: An effective form of students' mathematical knowledge forming

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**Abstract:** The great role in the process of mathematical knowledge mastering, forming of knowledge, abilities and experiences, necessary for studying mathematics, and using it in practice for preparing students to practical activity, psychologists, didactics, and methodists conduct to education tasks effective using [1]. Tasks of mathematics education at the same time appears in two aspects: as the facility and as the aim of education.

The notion «task» itself is multifaceted. The detailed analysis of definition of the notion «task» is situated in Kolyagin [3] and Fridman [4]. They point to single tasks definition absence because of investigating this notion from different points of view: logical, mathematics, cybernetic, psyche, and pedagogic. On the basis of the investigations of this question analysis one can pick out two approaches of

task definition. The majority of the authors include the subject into task notion itself [5, 6, 3, 7]. On the basis of psychologists' and deducts' investigations analysis it was picked out that the structure of experiences forming should be reflected in the system to make this system an effective quality for process of forming experiences of full value control.
**Keywords:** mathematical knowledge, forming abilities and experiences, operations, actions, algorithms.
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# Educational task - generalized aim of education activity

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**Abstract:** During school years labor, playing and education activities are distinguished. But in general life and activity do not consist of only separate activity forms. Some of them have important role for today; while some of them are more important for future development. So it is not necessary to talk about psyche dependence of activity, but of leading activity [l].The leading activity is an activity which causes the main changes in physical peculiarities of child personality at definite point of his development [2].
The education activity can be understood as a students' activity which is directed to acquire the theoretical knowledge about the studying subject and general ways of accomplishing the tasks. Thanks to activity children can develop and form their

personalities. [3]. First of all, the educational activity is the result of what the changes in the students take place. Tins is the activity of self-changing, its products are changes [2]. Generally we can say that these changes are acquisition of new knowledge, in other words, new ways of action by the students.
The main unit of education activity is an education task which formed as a task before *the* class, *for* example «To realize and adopt receptions of education activity of solving *logarithm* equalizations». The education task is tightly connected with generalization in the content; *it b*rings the student to generalized attitudes in studying sector of knowledge and new ways of action mastering.
The general aim of education activity can be reached with the help of the system of particular sub aims which help to choose the particular (private) task from definite number of subject tasks. Studying tasks in case of coincidence of particular studies can aim to build structure or to keep maintenance of subject tasks or with both of them.
**Keywords:** education activity, notion cognition activity, particular education.
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# Methodological foundations of teaching methods of mathematics

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**Abstract:** In this study we consider methodological issues study methods of teaching mathematics and the possibility of further improvement. We consider the problem of methodology of teaching mathematics as a problem of the theory of methods of teaching mathematics in general.
Methodological and methodical training of teacher begins with a study of textbooks, teaching and methodical manual subject, but the available text books and educational manuals on teaching mathematics are not sufficiently reflected methodology techniques[1, 2]. Therefore, this study opens up subject and object of methodical science, research methods are described, gives information on the nomination of hypotheses, designing forecasts, development and formulation of solutions of methodical tasks, conducting experimental work, etc.

**Keywords:** the methodology of mathematics, teaching methodology of mathematics, methodological preparation of teachers, subject and object of methodical science, research methods.

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# Technology of neurolinguistic programming in mathematics teaching

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**Abstract:** In pedagogy, learning process being generated the necessary thought processes and operations develops attention and memory.

In most cases, a psychologist or a teacher identifies the student who has a low level of generalization or inability to establish causal relationships. But the reason for his inability, and how it relates to the fact that it is poorly taught mathematics or Russian language, there is no psychologist or teacher in most cases that can explain it.

Probably, every school psychologist should have faced with the fact that there are students showing good results in all tests, including tests on the now fashionable learning, but having serious problems with studies, despite the sufficient motivation and diligence. What are the causes of this?

Finding answers to these and other questions led us to one of the modern trends of psychology – neurolinguistic programming (NLP). We seem to have found one of the keys for both diagnosis and correction of most student's problems.

In our opinion, 70% of the children who have learning difficulties are associated primarily with mismatching their modal type to teaching methods.

NLP concepts and approaches may become the common language, which will be able to find greater understanding between teacher and student. Currently, they say, as it was in different languages.

NLP is the art and science of personal excellence. It is art because everyone brings their own unique personality and style in what he does, and it is impossible to

reflect in words or technologies. It is science because there is a method and process of discovering patterns, used by outstanding individuals in any field for achieving outstanding results.
NLP is a practical art, allowing achieves those results, which we sincerely strive in this world.
Within neurophysiological concepts here assumes that each person has his own perception of the main channel and storage of information, its so-called "representational system." It is believed that through the channel leading man enters the main stream media.
Representation determines how organized our experience and how we describe the world. This, according to NLP, occurs in images (visual system), in the auditory system sounds and sensations (kinesthetic). Some studies also noted a way of obtaining information as intelligence and, accordingly, allocated rational or digital system.
A teacher should be able to identify the style of the student to use the methods that are most appropriate for the student or for a given group of students.
Advantage of NLP over most other approaches to teaching methods is that is the focus of formal features of internal learning processes and not the specific content of the information. Another feature of NLP is the desire to mobilize domestic resources unconscious students to form a new, more effective ways of teaching.
**Keywords:** NLP, represent, kinesthetic.
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