Integro-differential Bellman-Egorov equation in the synthesis problem of optimal control of thermal processes

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The synthesis problem of optimal control is investigated for thermal processes described by Fredholm integro-differential equations in this paper.

An optimal control problem is considered, where it is required to minimize the quadratic functional

$$I[u(t)] = \int_Q \Big\{ [v(T,x) - \xi(x)]^2 + \beta \int_0^T p^2[t,u(t)]dt, \beta > 0,$$

on the set of solutions of boundary value problem

$$v_t - Av = \lambda \int_0^T K(t,\tau)v(\tau,x)dx + g(t,x)f[t,u(t)], x \in Q, 0 < t \leq T,$$

$$v(0,x) = \psi_1(x), \ x \in Q,$$

$$\Gamma v(t,x) \equiv \sum_{i,j}^{n} a_{ij} v_{x_j} cos(\nu, x_i) + a(x)v = 0, x \in \gamma, 0 < t \leq T.$$

Here $v(t,x)$ is a state of function, $u(t)$ is a control function; A is an elliptic operator; $Q \subset R^n$ is a region with piecewise-smooth boundary γ ; ν is a normal vector passing through a point $x \in \gamma$; T is a fixed time; other functions are assumed to be given.

In determining the optimal control as a function of the state of the controlled process, i.e. $u(t) = u[t, v(t,x)]$, according to the Bellman scheme and the method of Professor A.I. Egorov [1] the following integro-differential equation is obtained:

$$-\frac{\partial S[t,v(t,x)]}{\partial t} = \min_{u \in U} \Big\{ \beta p^2[t,u(t)] + \int_Q m(t,x)g(t,x)dx f[t,u(t)] -$$

$$- \int_Q m(t,x) \int_Q D(\lambda,x,y)v(t,y)dydx + \lambda \int_Q m(t,x) \int_0^T K(t,\tau)v(\tau,x)d\tau dx \Big\} \quad (1)$$

with the additional condition

$$S[T, v(T,x)] = \int_Q [v(T,x) - \xi(x)]^2 dx, \quad (2)$$

where function $m(t,x)$ is a gradient of the Bellman functional $S[t,v(t,x)]$

We investigate the problem of Cauchy—Bellman—Egorov (1) - (2), we define structure of solutions to control (1) and structure of synthesizing optimal control, and classes of functions $f[t,u(t)]$ and $p[t,u(t)]$ are indicated for which it is possible to lead the solution of the synthesis problem to numerical calculations.

References

[1] Egorov A.I. (1974). Optimal stabilization of systems with distributed parameters. Materials of the IFIP International Conference on Optimization Technology. Preprint 8, Novosibirsk.