A new sufficient conditions on the generalized spectrum method to deal with spectral pollution

Ammar Khellaf [1]

[1] *University of Guelma, Universit 8 Mai 1945 Guelma, Algeria*

E-mail: [1] *amarlasix@gmail.com*

Abstract: In this work we solve the spectral pollution. We suggest a modern method based on generalized spectral techniques, where we show that the propriety L is hold with norm convergence. We describe the theoretical foundations of the method in details, as well as we illustrate its effectiveness by numerical results.

It is well known that the spectral pollution is the weakness of projection methods for an unbounded operator. Thus, our technic is an alternative method based on disrupting an unbounded operator by a bounded one until that the spectral properties transform on controlled case. The generalized spectrum takes its place as the desired case. We will show that every unbounded operator contains a decomposition of two bounded operators which carry all the spectral properties. Through this decomposition and basing upon its numerical approximations the phenomenon of spectral pollution will be resolved.

The natural framework of our research is a complex separable Banach space $(X, ||.||)$. Our numerical results are applied on harmonic oscillator operator, which is defined over $L^2(\mathbb{R})$ by

$$Au = -u'' + x^2 u. \tag{1}$$

Generalized spectrum. Let T and S two operators in $\mathrm{BL}(X)$, we define the generalized resolvent set by

$$\mathrm{re}(T, S) = \{\lambda \in \mathbb{C} : (T - \lambda S) \text{ is bijective}\}. \tag{2}$$

The generalized spectrum set is $\mathrm{Sp}(T, S) = \mathbb{C} \setminus \mathrm{re}(T, S)$. For $z \in \mathrm{re}(T, S)$, we define $R(z, T, S) = (T - zS)^{-1}$, the generalized resolvent operator.

We define $\lambda \in \mathbb{C}$ as a generalized eigenvalue when $(T - \lambda S)$ is not injective, then the set $E(\lambda) = \mathrm{Ker}(T - \lambda S)$ is the generalized spectral subspace. We say that λ has an finite algebraic multiplicity if there exist α where $\dim \mathrm{Ker}(T - \lambda S)^\alpha < \infty$. If the operator S is invertible, we have $\mathrm{Sp}(T, S) = \mathrm{Sp}(S^{-1}T)$.

Theorem 1. *Let $\lambda \in re(T, S)$ and $\mu \in \mathbb{C}$ where $|\lambda - \mu| < ||R(\lambda, T, S)S||^{-1}$, then $\mu \in re(T, S)$.*

Theorem 2. *The set $Sp(T, S)$ is closed in $\mathbb{C}$.*

Theorem 3. *The function* $R(\cdot, T, S) : re(T, S) \to BL(X)$ *is analytic, and its derivative given by* $R(\cdot, T, S)SR(\cdot, T, S)$.

References

[1] E.B. Davies and M. Plum, Spectral Pollution, arXiv:math/0302145v1, (2002)
[2] H. Guebbai, Generalized Spectrum Approximation and Numerical Computation of Eigenvalues for Schrödinger's Operators 45,60 (Accepted in Lobachevskii Journal of-Mathematics 2013).