Cordoba-Cordoba type inequality on homogenous Lie groups

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Abstract: Let $s \in [0, 1]$ and $x \in \mathbb{R}^n,\ n \geq 2$. In the work [1], authors show that the following inequality for the fractional Laplacian

$$2f(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^2(x), \tag{1}$$

where $(-\Delta)^s$ is the fractional Laplacian, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

This inequality is using for the maximum principle of the quasi-geostrophic equations. Also, in the works [2] generalized the Cordoba-Cordoba inequality,

$$pf(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^p(x), \tag{2}$$

where $(-\Delta)^s$ is the fractional Laplacian, $p > 0$, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

In the work [3], author generalized these inequalities for the fractional Laplacian. Our main aim of this talk is to establish analogues of the Cordoba-Cordoba inequality and its generalizations for the fractional sub-Laplacian on the homogenous Lie groups.

In this talk, we show an analogue of the Cordoba-Cordoba type inequality for the fractional sub-Laplacian on the homogenous Lie groups. Also, we show generalized analogueof the Cordoba-Cordoba type inequality on the homogenous Lie groups.

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