A note on mathematical theory of epidemics: SIR modeling of the COVID-19

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Abstract: A considerable amount of research works has been devoted for the epidemic outbreak caused by coronavirus COVID-19. The high rate of the infection spread and the number of fatalities makes the understanding of the current epidemiological models more important than ever before. The most relevant mathematical models relating to the spread of a pandemic is the susceptible-infectious-removed (SIR) [3], [6] model, susceptible-exposed-infectious-removed (SEIR) [4], [5], [7] model, the susceptible - infectious - susceptible (SIS) [1], [2] model, the susceptible-unquarantined-quarantined-confirmed (SUQC) [9] model. For long-time predictions, more complicated mathematical models are necessary which makes the procedure difficult if reliable data are limited as the complicated models need more effort to calculate unknown parameters. In this study, we consider well known SIR model which is proposed by McKendrick and Kermack [8] by the following nonlinear system of ordinary differential equations.

$$(1) \qquad \begin{cases} \dfrac{dS}{dt} = -\beta IS \\[2ex] \dfrac{dI}{dt} = \beta IS - \gamma I \\[2ex] \dfrac{dR}{dt} = -\gamma I \end{cases}$$

In this model a fixed population with only three compartments is considered: susceptible (S), infected (I), and recovered (R), respectively. The unknowns used for this model consist of three classes:
a) $S(t)$ represents the number of individuals not yet infected with disease at time t or those susceptible to the disease,
b) $I(t)$ denotes the number of individuals who have been infected with the disease, and are capable of spreading the disease to those in the susceptible category, and
c) $R(t)$ is the compartment of the individuals who have been infected and then recovered from the disease.

There are various approaches to understand the predictions of this model and the behavior of its solutions. Kermack and McKendrick [8] reduced this problem to a single differential equation and derived an approximate solution for the removal rate, dR/dt, in terms of a hyperbolic secant function by neglegting some terms. In this study, we obtain an approximate solution by taking into consideration the term neglegted in Kermack and McKendrick [8]. The results of the numerical experiments has similar features with the well-known strategies which shows that it can be used to forecast COVID-19 epidemic situation. This present work will be of great support to the epidemiology to investigate the validity of these models which is already being considered and used by different researchers.

Keywords: COVID-19, susceptible, infectious, recovered, mathematical model, SIR

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