On the Unique Existence of a Classical Solution to the Inverse Coefficient Problem for a Two-Dimensional Parabolic Equation

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Abstract. This paper is devoted to the study of a nonlocal inverse boundary-value problem for a two-dimensional parabolic equation with the integral overdetermination condition. The purpose of the work is to recover the unknown coefficient and the solution of the considered problem. To investigate the solvability of the inverse problem, we first reduce the original problem to some auxiliary equivalent one. Then using the contraction mappings principle the unique existence of the solution of an equivalent problem is proved. Further, on the basis of the equivalency of these problems the theorems about the existence and uniqueness of classical solution of the original inverse problem are proved for the smaller value of time.

Practical requirements often lead to the problem of determining the coefficients or the right hand side of the differential equations for some known data about its solutions. Such problems are called inverse problems in mathematical physics. Inverse problems arise in various fields of human activity, such as seismology, mineral exploration, biology, medical visualization, computed tomography, Earth remote sensing, desalination of seawater, movement of liquid in a porous medium, nondestructive control, etc. Problems of the solvability of inverse problems for a two-dimensional heat equation were considered in the papers of Ismailov [1], Ivanchov [2], Kinash [3], Zaynullov [4], and many others. But the statement of the problem and the proof techniques used in this study are different from representations in these papers.

In the domain defined by $D_T = Q_{xy} \times [0,T]$, where $Q_{xy} = \{(x,y) : 0 < x < 1,\ 0 < y < 1\}$, we consider the problem of determining the unknown functions $u(x,y,t) \in C^{2,2,1}(D_T)$ and $a(t) \in C[0,T]$ such that the pair $\{u(x,y,t), a(t)\}$ satisfies the following system [5]:

$$u_t(x,y,t) - c(t)(u_{xx}(x,y,t) + u_{yy}(x,y,t)) = a(t)u(x,y,t) + f(x,y,t), \ \ (x,y,t) \in D_T, \tag{1}$$

$$u(x,y,0) + \delta u(x,y,T) + \int_0^T p(t)u(x,y,t)dt = \varphi(x,y), \ \ (x,y) \in \bar{Q}_{xy}, \tag{2}$$

$$u(0,y,t) = u(1,y,t) = 0, \ \ 0 \le y \le 1, \ \ 0 \le t \le T, \tag{3}$$

$$u(x,0,t) = u(x,1,t) = 0, \ \ 0 \le x \le 1, \ \ 0 \le t \le T, \tag{4}$$

$$\int_0^1 \int_0^1 K(x,y)u(x,y,t)dxdy = h(t), \ \ 0 \le t \le T, \tag{5}$$

where $\delta \ge 0$ is known number, $(x_0, y_0) \in Q_{xy}$ is some fixed point, $0 < c(t), f(x,y,t), \varphi(x,y), p(t)$, and $h(t)$ are unknown functions of $x, y \in [0,1]$ and $t \in [0,T]$.

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