About optimal feedback control problem for motion model of nonlinearly viscous fluid

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Abstract: The motion of an incompressible nonlinearly viscous fluid in a bounded domain $\Omega \subset R^n$, $n = 2, 3$, on the time interval $[0, T]$ $(T < \infty)$ is described by the following initial–boundary value problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \text{Div } \big[2\mu\big(I_2(v)\big)\varepsilon(v)\big] + \text{grad } p = f, \tag{1}$$

$$\text{div } v = 0, \qquad v|_{t=0} = v_0(x), \qquad v|_{\partial\Omega\times[0,T]} = 0. \tag{2}$$

Here $v(x, t)$ is a vector–function of the velocity of a fluid particle at a point $x \in \Omega$ at a time $t \in [0, T]$; p is a pressure function in a fluid; f is the density of external forces; ε is the strain rate tensor $\varepsilon(v) = \big(\varepsilon_{ij}(v)\big)$, $\varepsilon_{ij}(v) = \frac{1}{2}\big(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\big)$, tensor $I_2(v)$ is determined by the equality $I_2^2(v) = \varepsilon(v) : \varepsilon(v) = \sum_{i,j=1}^{n} \left[\varepsilon_{ij}(v)\right]^2$.

In this presentation solutions existence to the feedback control problem for a nonlinearly viscous fluid model (**??**)–(**??**) is studied. Also the existence of an optimal solution to the problem under consideration that gives a minimum to a given bounded quality functional is proved.

Keywords: Optimal feedback control problem, existence theorem, nonlinear viscous fluid

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