

The convergence of attractors

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Abstract: The question of the attractor's existence is closely related to theorems on the existence of solutions. An effective method for studying the solvability of fluid dynamic problems and the existence of their attractors is the approximation-topological method. This method was proposed by V.G. Zvyagin and developed by him with his collaborators V.T. Dmitrienko, D.A. Vorotnikov, M.V. Turbin, A.V. Zvyagin and others. It consists of the fact that in the beginning the original initial-boundary value problem is approximated by some problem with better topological properties. Then, based on the topological degree of completely continuous, or condensing vector fields, or mappings satisfying the alpha condition and a priori estimates of the solutions, the solvability of these approximation problems is established. Approximation equations contain approximation parameters, and these equations move into the original, unperturbed equations as the approximation parameter tends to zero. Once the solvability of the approximation problems is proved, the solvability of the original problem is established using the passage to the limit.

For the solutions to the original problem usually it is possible to obtain some estimates of the dissipative type. Based on them, the trajectory space is constructed, for which the existence of trajectory and global attractors of both the approximation problem and the original problem is proved. Approximation equations have usually more natural properties of continuous dependence of solutions on the right-hand side and initial conditions, i.e. for small changes in the initial conditions and the right-hand side of the equation, a small change in the set of solutions is obtained. The presence of this property makes it possible to apply various approximate methods to find attractors of approximation problems and to study their convergence to attractors of the original system. Such an approach can be useful for the approximate calculation of the attractors for the original problem.

Keywords: Approximation-topological method, attractor's existence, trajectory and global attractors

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