On the ternary semigroups of homeomorphic transformations of bounded closed sets with nonempty interior of finite-dimensional Euclidean spaces

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Abstract: This report is devoted to a characterization of bounded closed sets with nonempty interior of finite-dimensional Euclidean spaces by ternary semigroups of homeomorphic maps.

A ternary semigroup is a nonempty set T together with a ternary operation $[abc]$ satisfying the associative law $[[abc]\,de] = [a\,[bcd]\,e] = [ab\,[cde]]$ for every $a, b, c, d, e \in T$. Let R be a finite-dimensional Euclidean space with the standard topology. Let Ω_1 and Ω_2 be two bounded closed sets of R such that $Int\,(\Omega_i) \neq \emptyset$ for $i = 1, 2$. Let $B_i\,(\Omega_i)$ denote the set of all homeomorphic maps a from Ω_i to Ω_j for which there is an $n-$sized element $E_a \subset \Omega_j$ and a closed set $F_a \subset \Omega_j$ such that $a\Omega_i \subset F_a \subset IntE_a$, where $i, j = 1, 2$ $(i \neq j)$. The set $B\,(\Omega_1, \Omega_2) = B_1\,(\Omega_1) \times B_2\,(\Omega_2)$ is a ternary semigroup with respect to the ternary operation

$$[(a_1, b_1)\,(a_2, b_2)\,(a_3, b_3)] = (a_1 b_2 a_3, b_1 a_2 b_3)\,.$$

Theorem 1. Let R and R' be finite-dimensional Euclidean spaces. Let Ω_1 and Ω_2 be bounded closed sets of R and let Ω_1' and Ω_2' be bounded closed sets of R' such that $Int\,(\Omega_i) \neq \emptyset$ for $i = 1, 2$. The ternary semigroups $B\,(\Omega_1, \Omega_2)$ and $B\,(\Omega_1', \Omega_2')$ are isomorphic if and only if the spaces Ω_i and Ω_i' are homeomorphic $(i = 1, 2)$.

Keywords: Euclidean space, ternary semigroup

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References

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