Singularly perturbed first-order equations in complex domains that lose their uniqueness under degeneracy

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Abstract: In this paper we consider a singularly perturbed first-order equation in complex domains, the degenerate equation of which has several isolated solutions. Cases when degenerate equations have unique solutions are considered in [1-2].
Let consider the equation:

$$\varepsilon z'(t,\varepsilon) = f\big(t, z(t,\varepsilon)\big), \tag{1}$$

where $\varepsilon > 0$ is a small parameter, C is a set of complex numbers, Ω is a simply connected domain, with the initial condition $z(t_0, \varepsilon) = z^0$ where $t_0 \in \Omega \subset C$ and is internal point. Constants independent of ε are denoted by $M_0, M_1, M_2...$

For $\varepsilon = 0$, from (1) we get the degenerate equation

$$f\big(t, \xi, (t)\big) = 0, \tag{2}$$

Let the equation (2) has solutions $\xi_j(t), j = 1, 2, , n$.
Definition 1. If $\forall (k \neq m) \wedge \forall t \in \Omega(|\xi_k(t) - \xi_m(t)| > M_0)$,then $\xi_k(t)$ and $\xi_m(t)$ are said isolated solutions in Ω. U1. Let $f(t,z)$ the analytic function with respect to the variables (t,z) in some domain D the variables (t,z).
Definition 2. If:1.$\forall t \in \Omega_j \subset \Omega$ there exists $z(t,\varepsilon)$ the solution of problem (1)-(2) 2.$\forall t \in \Omega_j\big(\lim_{\varepsilon\to 0} z(t,\varepsilon) = \xi_j(t)\big)$ then the domain Ω_j are said the domain of attraction of the solution $\xi_j(t)$.
Theorem. Suppose that the condition U1 are satisfied. Then for each $\xi_j(t)$ there exists:
1. The solution $z_j(t,\varepsilon)$ of equation(1)satisfying condition $z_j(t_0,\varepsilon) = z_j^0, | z_j^0 - \xi_j(t_0) | \leq M_1\varepsilon$.
2. The domain $\Omega_j \subset \Omega$ and $\forall t \in \Omega_j (z_j(t,\varepsilon) \to \xi_j(t) by \varepsilon)$
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