

# Singularly perturbed first-order equations in complex domains that lose their uniqueness under degeneracy

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**Abstract:** In this paper we consider a singularly perturbed first-order equation in complex domains, the degenerate equation of which has several isolated solutions. Cases when degenerate equations have unique solutions are considered in [1-2].

Let consider the equation:

$$(1) \quad \varepsilon z'(t, \varepsilon) = f(t, z(t, \varepsilon)),$$

where  $\varepsilon > 0$  is a small parameter,  $\mathbb{C}$  is a set of complex numbers,  $\Omega$  is a simply connected domain, with the initial condition  $z(t_0, \varepsilon) = z^0$  where  $t_0 \in \Omega \subset \mathbb{C}$  and is internal point. Constants independent of  $\varepsilon$  are denoted by  $M_0, M_1, M_2 \dots$

For  $\varepsilon = 0$ , from (1) we get the degenerate equation

$$(2) \quad f(t, \xi, (t)) = 0,$$

Let the equation (2) has solutions  $\xi_j(t), j = 1, 2, \dots, n$ .

**Definition 1.** If  $\forall (k \neq m) \wedge \forall t \in \Omega (|\xi_k(t) - \xi_m(t)| > M_0)$ , then  $\xi_k(t)$  and  $\xi_m(t)$  are said isolated solutions in  $\Omega$ . U1. Let  $f(t, z)$  the analytic function with respect to the variables  $(t, z)$  in some domain  $D$  the variables  $(t, z)$ .

**Definition 2.** If: 1.  $\forall t \in \Omega_j \subset \Omega$  there exists  $z(t, \varepsilon)$  the solution of problem (1)-(2) 2.  $\forall t \in \Omega_j (\lim_{\varepsilon \rightarrow 0} z(t, \varepsilon) = \xi_j(t))$  then the domain  $\Omega_j$  are said the domain of attraction of the solution  $\xi_j(t)$ .

**Theorem.** Suppose that the condition U1 are satisfied. Then for each  $\xi_j(t)$  there exists:

1. The solution  $z_j(t, \varepsilon)$  of equation(1)satisfying condition  $z_j(t_0, \varepsilon) = z_j^0, |z_j^0 - \xi_j(t_0)| \leq M_1 \varepsilon$ .
2. The domain  $\Omega_j \subset \Omega$  and  $\forall t \in \Omega_j (z_j(t, \varepsilon) \rightarrow \xi_j(t) \text{ by } \varepsilon)$

**Keywords:** singularly perturbed equations, degenerate equation, analytical functions, isolated solutions, domain of attraction

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