Exact Solution of Schrödinger Equation in 2D deSitter and Anti-deSitter Spaces for Kratzer Potential plus a Dipole

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Abstract: In this work we gave the exact solution of Schrödinger equation for Kratzer Potential plus a Dipole potential$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{D_r}{r^2} + \frac{D_\theta \cos\theta}{r^2}\right)$[1] in 2D deSitter and Anti-deSitter spaces ,this deformed space defined by the following commutation relations[2][3]$[X_i, X_j] = 0$, $[P_i, P_j] = i\hbar\tau\lambda\epsilon_{ijk}L_k$, $[X_i, P_j] = i\hbar\left(\delta_{ij} - \tau\lambda X_i X_j\right)$ with $\tau = -1, +1$

the noncommutative operators X_i and P_i satisfying the modified algebra

$$X_i = \frac{x_i}{\sqrt{1+\tau\lambda r^2}} and\ P_i = -i\hbar\sqrt{1+\tau\lambda r^2}\partial_{x_i}$$

when the deformed Schrödinger equation in polar coordinate is

$$\begin{bmatrix} -\frac{\hbar^2}{2\mu}\left[\left(1+\tau\lambda r^2\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right) + \tau\lambda r\frac{\partial}{\partial r}\right] + \\ \frac{q}{4\pi\epsilon_0}\left(\frac{Q\sqrt{1+\tau\lambda r^2}}{r} + \frac{\left(1+\tau\lambda r^2\right)\left(D_r + D_\theta\cos\theta\right)}{r^2}\right) \end{bmatrix}\psi = E\psi$$

After the separation we get tow equations the radial equation and the angular equation , the angular equation is Mathieu equation like and its solution is Mathieu function ,we used Nikiforov–Uvarov method [**?**] to solve the radial equation

The energy eigenvalue is given in their exact forms and the corresponding radial wave functions are given in terms of Romanovski polynomials and the angular wave functions are expressed in terms of Mathieu function [**?**].

$$E_n = -\frac{\hbar^2}{8\mu}\left[\left(\frac{\alpha Q}{n+\frac{1}{2}+\nu}\right)^2 + 4\lambda\left(\left(n+\frac{1}{2}+\nu\right)^2 - \left(\nu^2+\frac{1}{4}\right)\right)\right]$$

$$\Psi_n\left(r,\theta\right) = C_n^{`}\left(\sqrt{\lambda}r\right)^{\left(\delta_1^{`}-\frac{1}{2}\right)} e^{\frac{\eta}{2\delta_1^{`}}\tan^{-1}\left(\frac{\sqrt{1-\lambda r^2}}{\sqrt{\lambda}r}\right)} R_n^{\left(-\delta_1^{`},\frac{\eta}{\delta_1^{`}}\right)}\left(\frac{\sqrt{1-\lambda r^2}}{\sqrt{\lambda}r}\right)\Theta(\theta)$$

We have also studied the effect of the spatial deformation parameter on the bound states.

Keywords:2D Schröding er equation Kratzer Potential, Dipole potential,deSitter and Anti-deSitter spaces,Nikiforov–Uvarov method,Mathieu function.

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