On uniform difference schemes and asymptotic formulas for the solution of Shrödinger's type nonlocal boundary value perturbation problems

Allaberen Ashyralyev[1,2,3], Ali Sirma[4]

[1] *Department of Mathematics, Near East University, Nicosia, TRNC, Mersin 10, Turkey*

[2] *Peoples Friendship University Russia, Ul Miklukho Maklaya 6, Moscow 117198, Russia*

[3] *Institute of Mathematics and Mathematical Modeling, 050010, Almaty, Kazakhstan,*

aallaberen@gmail.com

[4] *Department of Industrial Engineering, Haliç University, Istanbul, Turkey*

alisirma@halic.edu.tr

Abstract: The abstract nonlocal boundary value problem

$$\begin{cases} i\varepsilon u'(t) + Au(t) = f(t), 0 < t < T, \\ u(0) = \int\limits_0^T \alpha(s)u(s)ds + \varphi \end{cases}$$

for Shrödinger equations in a Hilbert space H with the self adjoint positive definite operator A and with an arbitrary $\varepsilon \in (0,\infty)$ parameter multiplying the derivative term is considered. An asymptotic formula for the solution of this problem with a small ε parameter is established. The high order of accuracy single-step uniform difference schemes for the solution of this problem are presented. The convergence estimates for the solution of these difference schemes are established.

Keywords: Asymptotic formula, uniform difference schemes, Schrödinger problem

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27.

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