On the periodicity of solutions of a system of rational difference schemes

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Abstract: In this paper, we have investigated the periodicity of the well-defined solutions of the system of difference equations:

$$u_{n+1} \equiv -\frac{u_{n-1}+v_n}{\alpha v_n u_{n-1}-1},\ v_{n+1} \equiv -\frac{v_{n-1}+u_n}{\alpha u_n v_{n-1}-1},\ w_{n+1} \equiv -\frac{u_n}{v_n} \tag{1}$$

where

$$u_0, u_{-1}, v_0, v_{-1}, w_0, w_{-1} \in \mathbb{R}\backslash\{0\} \text{ and } \ \alpha > 0. \tag{2}$$

Note that system (1) can be written as system

$$x_{n+1} \equiv -\frac{x_{n-1}+y_n}{y_n x_{n-1}-1},\ y_{n+1} \equiv -\frac{y_{n-1}+x_n}{x_n y_{n-1}-1},\ z_{n+1} \equiv -\frac{x_n}{y_n} \tag{3}$$

by the change of variables ,

$$u_n \equiv -\frac{x_n}{\sqrt{\alpha}},\ v_n \equiv -\frac{y_n}{\sqrt{\alpha}},\ w_n \equiv -z_n. \tag{4}$$

That's why, we will consider system (4) instead of system (1) for the remaining part of the paper.

Main Result.

Theorem 0.1. *Let $y_0 = a$, $y_{-1} = b$, $x_0 = c$, $x_{-1} = d$, $z_0 = e$, $z_{-1} = f$ be nonzero arbitrary real numbers and $\{x_n, y_n, z_n\}$ be a solution of system (3). Also, assume that $ad \neq 1$, $bc \neq 1$ and $(b+c) \neq 0$ and $(d+a) \neq 0$. Then, all solutions of system (3) are as following:*

$$x_n = \begin{cases} \frac{d+a}{ad-1}, & n = 6k+1 \\ b, & n = 6k+2 \\ a, & n = 6k+3 \\ \frac{b+c}{cb-1}, & n = 6k+4 \\ d, & n = 6k+5 \\ c, & n = 6k+6 \end{cases} \quad \text{for } k \in \mathbb{N}_0 \tag{5}$$

$$(6)\qquad y_n = \begin{cases} \frac{b+c}{cb-1}, & n=6k+1 \\ d, & n=6k+2 \\ c, & n=6k+3 \\ \frac{d+a}{ad-1}, & n=6k+4 \\ b, & n=6k+5 \\ a, & n=6k+6 \end{cases} \quad for\ k\in\mathbb{N}_0$$

$$(7)\qquad z_n = \begin{cases} \frac{c}{a}, & n=6k+1 \\ \frac{(d+a)(cb-1)}{(ad-1)(b+c)}, & n=6k+2 \\ \frac{b}{d}, & n=6k+3 \\ \frac{a}{c}, & n=6k+4 \\ \frac{(b+c)(ad-1)}{(cb-1)(d+a)}, & n=6k+5 \\ \frac{d}{b}, & n=6k+6 \end{cases} \quad for\ k\in\mathbb{N}_0$$

Proof. We prove the theorem by induction for k. □

Keywords: Difference equation; system; solutions; periodicity.

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