On generalization of Hermite-Hadamard inequality for bivariate log-convex stochastic processes

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Abstract: In the paper, the author defines a stochastic process $S : \Xi \times \Sigma \to \mathbb{R}_+$ that called "bivariate log-convex" on $\Xi \subseteq \mathbb{R}^2$. Then, the author establishes the Hermite-Hadamard type integral inequalitiy for these processes, and generalizes classical Hermite-Hadamard inequality for integrals to these processes as follows:

$$\begin{aligned} &\frac{\delta_2-\theta_2}{2n}\sum_{i=1}^{n}\int_{\theta_1}^{\delta_1} S\big((\omega, A(\rho_{i-1},\rho_i)\big),\cdot\big)\mathrm{d}\omega \\ &+\frac{\delta_1-\theta_1}{2n}\sum_{i=1}^{n}\int_{\theta_2}^{\delta_2} S((A(\omega_{i-1},\omega_i),\rho),\cdot)\mathrm{d}\rho \leq \int_{\theta_1}^{\delta_1}\int_{\theta_2}^{\delta_2} S((\omega,\rho),\cdot)\mathrm{d}\omega\mathrm{d}\rho \\ &\leq \frac{\delta_2-\theta_2}{2n}\sum_{i=1}^{n}\int_{\theta_1}^{\delta_1} L\left(S\left((\omega,\rho_{k-1}),\cdot\right), S\left((\omega,\rho_k),\cdot\right)\right)\mathrm{d}\omega \\ &+\frac{\delta_1-\theta_1}{2n}\sum_{i=1}^{n}\int_{\theta_2}^{\delta_2} L\left(S\left((\omega_{k-1},\rho),\cdot\right), S\left((\omega_k,\rho),\cdot\right)\right)\mathrm{d}\rho, \end{aligned} \tag{1}$$

where $\omega_i = \theta_1 + i\frac{\delta_1-\theta_1}{n}$, $\rho_i = \theta_2 + i\frac{\delta_2-\theta_2}{n}$, $i = 0, 1, 2, \ldots, n$; *and* $n \in N$.

Keywords: Log-convexity; bivariate stochastic process; mean-square integral; Hermite-Hadamard inequality

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References

[1] K. Nikodem, On convex stochastic processes, Aequationes Math., vol. 20 no 2-3, 184–197, 1980.

[2] N. Okur, Multidimensional general convexity for stochastic processes and associated with Hermite-Hadamard type integral inequalities, Thermal Sci., vol. 23, Suppl. 6, 1971–1979, 2019.