Implicit Method of Second Order Accuracy on Hexagonal Grids for Approximating the First Derivatives of the Solution to Heat Equation on a Rectangle

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Abstract: The Dirichlet type boundary value problem of heat equation

$$\frac{\partial u}{\partial t}=\omega\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2}\right)+f\left(x_1,x_2,t\right) \tag{1}$$

on a rectangle, where $\omega>0$, is constant and f is the heat source is considered. We propose a two stage implicit method for the approximation of the first order derivatives of the solution $u\left(x_1,x_2,t\right)$ with respect to the spatial variables x_1 and x_2. At the first stage a two layer implicit method on hexagonal grids with order of accuracy $O\left(h^2+\tau^2\right)$ given in [1] is used to approximate the solution $u\left(x_1,x_2,t\right)$. At the second stage we propose special difference boundary value problems on hexagonal grids for the approximation of $\frac{\partial u}{\partial x_1}$ and $\frac{\partial u}{\partial x_2}$ of which the boundary conditions are defined by using the obtained solution from the first stage. It is proved that the given implicit schemes of the special difference boundary value problems are unconditionally stable. It is also showed that the solution of these difference boundary value problems converge to the corresponding exact derivatives $\frac{\partial u}{\partial x_1}$ and $\frac{\partial u}{\partial x_2}$ on the grids of order $O\left(h^2+\tau^2\right)$ where, h and $\frac{\sqrt{3}}{2}h$ are the step sizes in space variables x_1 and x_2 respectively and τ is the step size in time. The method is applied on test problems and the obtained numerical results justify the given theoretical results.

Keywords: Finite difference method, Hexagonal grid, Stability analysis, Two dimensional heat equation, Approximation of derivatives.

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References

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