A new numerical algoritm for solving Sturm-Liouville problems with discontinuities in an interior point

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Abstract: Boundary value problems for Sturm-Liouville type equations are used to understand various type of physical problems in nature. To understand how nature works, we need to solve these Sturm-Liouville problems using some analytical methods. [1] In most cases, it is impossible or difficult to solve these modeled problems under realistic boundary conditions analytically. So we prefer different numerical methods to solve these problems. [3] The numerical methods, such as differential transform method, the shooting method, the homotopy pertubation method, the Adomian decomposition method, finite difference method and etc. are powerful approximate methods for solving various kinds of initial and/or boundary value problems. The main advantage of finite difference method (FDM) is that it can be applied efficiently to a rather wide class of boundary value problems. [2] In this work, we will adapt the FDM to discontinuous Sturm-Liouville problems the main feature of which is the nature of the equations and the boundary conditions imposed. Namely, the boundary conditions contains not only boundary points of the considered interval, but also an interior point of discontinuity at which given additional transmission conditions, so our problem is the nonclassical once. Based on Finite Difference method and our own approaches a new numerical algoritm is introduced for such type transmission problems.

We consider Sturm-Liouville equation,

(1) $$y^{''}(x) + p(x)y^{'}(x) + q(x)y(x) = f(x), \quad x \in [a, b]$$

subject to separate boundary conditions

(2) $$y(a) = \alpha, \quad y(b) = \beta$$

with additional transmission conditions at the interior point $c \in (a, b)$, given by

$$y(-c) = my(+c) \qquad y'(-c) = ny'(+c) \tag{3}$$

where p(x), q(x) and f(x) are continuous functions on [a,b] and α, $\beta\, m$,n are real constants,$m \neq 0, n \neq 0$.

It is our main goal here to develop FDM to deal with the discontinuous boundary-value problems involving additional transmission conditions at the the point of discontinuity. By comparision with the exact solutions we show that the our own numerical algoritm, which based on FDM is an efficient method for solving Sturm-Liouville type problems under supplementary transmission conditions.

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References

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