On the convergence of high-precision finite element method schemes for the two-temperature plasma equation

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Abstract: Mathematical models of physical problems of short-wave oscillations of a two-temperature plasma in an external magnetic field are generally described by the equation [1]

$$\frac{\partial^2}{\partial t^2}\left(\Delta_3 u-\rho^2 u\right)+\omega^2\frac{\partial^2}{\partial t^2}\left(\Delta_2 u\right)+\theta^2\Delta_1 u=f(x,t),\ (x,t)\in Q_T, \tag{1}$$

where $\rho^2, \omega^2, \theta^2 -$ const >0, depending on Debaevskiy radius or from the Alfven-speed, ω^2 - Langmuir frequency, $\Omega=\{0\le x_k\le l_k, k=1,2,3\},\ Q_T=\{(x,t): x\in\Omega, t\in(0,T]\}, \Delta_3=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2+\partial^2/\partial x_3^2, \Delta_2=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2,\ \Delta_1=\partial^2/\partial x_3^2$. Equation (1) is supplemented with the following initial and boundary conditions:

$$u(x,t)\Big|_{\partial\Omega}=\frac{\partial u}{\partial n}\Big|_{\partial\Omega}=0,\forall t\in[0,t],\ u(x,0)=u_0(x),\ \frac{\partial u(x,0)}{\partial x}=u_1(x). \tag{2}$$

Approximating the spatial variables in (1) and (2) on the basis of the finite difference method or the finite element method, we obtain a system of ordinary differential equations

$$D\frac{d^2u_h(t)}{dt^2}+Au_h(t)=f_h(t),\ u_h(0)=u_{0,h},\ \frac{du_h}{dt}(0)=u_{1,h}. \tag{3}$$

The operators D, A operate from H_h in H_h. They correspond to the matrix finite element method $D=(a_3(\phi_l,\phi_m))_{l,m-1}^M$ and $A=(a_2(\phi_l,\phi_m))_{l,m-1}^M+(a_1(\phi_l,\phi_m))_{l,m-1}^M$, where $a_m(u,v)$ some bilinear forms. Besides, $u_{k,h}=P_hu_k(x)$, $k=0,1$, where P_h - design operator $P_h: H\to H_h$.

Further, to solve the problem (3), a multiparametric scheme of the fourth-order finite element method of time accuracy is applied [2]:

$$D_\gamma\dot{y}_t+Ay^{(0.5)}=\Phi_1,\ D_\alpha y_t-D_\beta\dot{y}^{(0.5)}=\Phi_2, \tag{4}$$

$$y(0)=u_{0,h},\ \dot{y}(0)=u_{1,h}. \tag{5}$$

Here it is indicated $y=y^n=y(t_n),\ \dot{y}=\dot{y}^n=\frac{dy}{dt}(t_n),\ D_\gamma=D-\gamma\tau^2A,\ D_\beta=D-\beta\tau^2A,\ y_t=\left(y^{n+1}-y^n\right)/\tau,\ \dot{y}_t=\left(\dot{y}^{n+1}-\dot{y}^n\right)/\tau,\ y^{(0.5)}=\left(y^{n+1}+y^n\right)/2,$

$\dot{y}^{(0.5)} = (\dot{y}^{n+1} + \dot{y}^n)/2,\ y^n, \dot{y}^n \in H_h,\ n = 0, 1, ...$ Further $\Phi_k = \int_0^1 f(t_n + \tau\xi)v_k(\xi)d\xi,\ k = 1, 2,\ \xi = (t - t_n)/\tau,\ v_1(\xi) = 1,\ v_2(\xi) = s_1 v_2^{(1)}(\xi) + s_2 v_2^{(2)}(\xi)$, $v_2^{(1)}(\xi) = \tau(\xi - 1/2),\ v_2^{(2)}(\xi) = \tau(\xi^3 - 3\xi^2/2 + \xi/2),\ s_1 = 180\beta - 40\alpha,\ s_2 = 1680\beta - 280\alpha$.

A high order of accuracy of the scheme is achieved by special discretization of time and spatial variables. The accuracy in space h^3. The stability and convergence of the constructed algorithms are proved. A priori estimates in various norms are obtained, which are used in the future to obtain consistent estimates of the accuracy of the scheme under weak assumptions about the smoothness of solutions to differential problems. Scheme (4), (5) has certain advantages over other schemes. a) a scheme of high order of accuracy (higher than two); b) in addition to the solution itself, its derivative (speed) is simultaneously found with the same accuracy; c) since the schemes are two-layer, you can use a variable step without loss of accuracy.

Keywords: Finite element method, high order of accuracy, convergence.

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