

On Reich - type contraction in cone pentagonal metric spaces over Banach algebras

Abba Auwalu^{1,2}, Evren Hınçal²

¹ *Sule Lamido University Kafin Hausa, Nigeria*

abba.auwalu@slu.edu.ng, abba.auwalu@neu.edu.tr

² *Near East University Nicosia, Turkey*

evren.hincal@neu.edu.tr, evrenhincal@yahoo.co.uk

Abstract: In the present paper, we studied and proved a Reich - type fixed point theorem in Cone pentagonal metric spaces over Banach algebras which extend and generalized the results in [1], [2], [3], and many well-known results in the literature. Some examples were also given to elucidate our results:

Let (\mathcal{X}, ρ) be a complete cone pentagonal metric space over Banach algebra \mathcal{A} with a unit element e and K be a solid cone in \mathcal{A} . Suppose that $F : \mathcal{X} \rightarrow \mathcal{X}$ is a mapping satisfying the following condition:

$$\rho(Fu, Fv) \preceq \vartheta_1\rho(u, v) + \vartheta_2\rho(Fu, u) + \vartheta_3\rho(Fv, v), \text{ for all } u, v \in \mathcal{X},$$

where $\vartheta_j \in K$ ($j = 1, 2, 3$) such that ϑ_1 commutes with $\vartheta_2 + \vartheta_3$, ϑ_2 commutes with ϑ_3 . Then F has a unique fixed point u^* in \mathcal{X} . Moreover, for any point $u_0 \in \mathcal{X}$, the iterative sequence $\{F^k u_0\}$ ($k \in \mathbb{N}$) converges to u^* .

Keywords: Cone pentagonal metric space; Banach algebras; contraction mapping; fixed point

2010 Mathematics Subject Classification: 47H10, 54H25

REFERENCES

- [1] S.K. Malhotra, S. Shukla, R. Sen, Some fixed point theorems for ordered Reich type contractions in cone rectangular metric spaces, *Acta Math. Univ. Comenianae* Vol. LXXXII, 2, 165 - 175, 2013.
- [2] S. Shukla, S. Balasubramanian, and M. Pavlovic, A generalized Banach fixed point theorem, *Bull. Mal. Math. Sci. Soc.* 39, 1529 - 1539, 2016.
- [3] A. Auwalu, E. Hincal, A generalized Banach contraction principle on cone pentagonal metric spaces over Banach algebras, *Fourth International Conference on Analysis and Applied Mathematics (ICAAM)*, Nicosia - Cyprus, 2018.