

# A note on some fixed point theorems for generalized expansive mappings in cone metric spaces over Banach algebras

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**Abstract:** In this paper, we prove some fixed point theorems for generalized expansive mappings in non-normal cone metric spaces over Banach algebras. Moreover, we give an example to elucidate our result. Our results are significant extension and generalizations of recent result of B. Jiang et al., (J. Comput. Anal. Appl., 21(6), 1103 - 1114, 2016) and many well-known results in the literature. The following is our main result which generalizes Theorem 3.1 in [1], Theorem 2.2 in [2], Theorem 2.1 in [3], and Theorems 2.3, 2.6 in [4].

Let  $(Y, \rho)$  be a complete cone metric space over Banach algebra  $\mathcal{A}$ , a unit  $e$  and  $\mathcal{P}$  be a solid cone in  $\mathcal{A}$ . Let the mapping  $\mathfrak{T} : Y \rightarrow Y$  be a surjective and satisfies the generalized expansive condition:

$$\rho(\mathfrak{T}y, \mathfrak{T}z) + \vartheta_1[\rho(y, \mathfrak{T}z) + \rho(z, \mathfrak{T}y)] \succcurlyeq \vartheta_2\rho(y, z) + \vartheta_3\rho(y, \mathfrak{T}y) + \vartheta_4\rho(z, \mathfrak{T}z),$$

for all  $y, z \in Y$ , where  $\vartheta_j \in \mathcal{P}$  ( $j = 1, 2, 3, 4$ ) such that  $(\vartheta_1 + \vartheta_2 - \vartheta_4)^{-1} \in \mathcal{P}$ , and a spectral radius  $\delta[(\vartheta_2 + \vartheta_3 - \vartheta_1)^{-1}(e + \vartheta_1 - \vartheta_4)] < 1$ . Then  $\mathfrak{T}$  has a fixed point in  $Y$ .

**Keywords:** Cone metric space over Banach algebras, generalized expansive mapping, fixed point,  $c$ -sequence

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